Observational Learning and Demand for Search Goods

Ken Hendricks
University of Texas at Austin

Alan Sorensen
Stanford University & NBER

Tom Wiseman
University of Texas at Austin

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Abstract

In many differentiated good markets like music, books, and movies, the choice set of available products is overwhelmingly large and growing as new products flow into the market each month. Consumers are not aware or poorly informed about many of the available products. They learn about products and their preferences for them from the purchasing decisions of other consumers and through costly search. We use a variant of the sequential search models of Banerjee [2], Bikhchandani, Hirshleifer, and Welch [5] and Smith and Sorensen [18] to study market demand in these kinds of markets. The option to search prior to purchase leads to different dynamics and outcomes than the standard herding models. The results explain both the unpredictability of sales conditional on quality and the inequality of sales across products. The model also yields testable predictions regarding the impact of product quality, search costs, and price on the likelihood of a high-quality product ending up with low sales (i.e., a "bad" herd). We validate the model using data from an experimental study by Salganik, Dodds, and Watts [15].

1 Introduction

In many differentiated good markets, the number of available products is overwhelmingly large. The sheer size of the choice set implies that consumers are often unaware or poorly informed about many products, especially new products. According to the marketing literature, the response of consumers is often to reduce their choice problem by selecting a much smaller set of products. Eliaz and Spiegel [9] refer to such sets as "consideration sets." After selecting consideration sets, consumers decide which products to purchase based upon their information about the products and their preferences. When the products are search goods, consumers can learn their preferences for the goods prior to purchase by acquiring informative signals about the goods. Market demand in these markets then depends not only upon consumers' knowledge of the product space and their preferences, but also upon the process by which they select a consideration set. This process is driven in part by observational learning, which occurs when the behavior of individuals is influenced by observing the choices of other people. Consumers tend to consider the products they hear about, and they hear about the products that other consumers buy. As a result, a product's success or lack of success reinforces itself, causing market demand to be unpredictable and sales across products to be substantially more skewed than in a world where consumers costlessly know about all available products and their preferences for them.

Our main goal in this paper is to examine the impact of observational learning on market demand for search goods. Search goods are products whose quality can be ascertained by consumers prior to purchase. We use a variant of the learning models introduced by Banerjee [2] and Bikhchandi, Hirshleifer and Welch [5] and subsequently generalized by Smith and Sorensen [18]. An infinite number of consumers with heterogenous preferences arrive sequentially and have to decide whether or not to buy a new product whose quality can either be high or low. Consumers do not know the quality of the product or their idiosyncratic preferences for it.[1] Prior to making their purchasing decision, consumers observe the purchasing history of previous consumers or some summary statistic of that history. They can also engage in costly search, which informs them about their preferences for the product. For example, in the market for recorded music, search would involve listening to songs on the internet or at the listening post in the store. A positive search

[1] Smith and Sorensen [18] also admit heterogeneous preferences but assume that consumer know the idiosyncratic component.

cost provides a rationale for consumers *not* to consider the product if they believe they are unlikely to buy it. More generally, it captures the idea that, even though cost per product may be quite small, the number of products is too large for a consumer to search every product.[2]

When consumers search prior to purchase, the purchasing decision of each consumer is informative to subsequent consumers and influences their search decisions. However, it does not perfectly reveal which search actions were taken. If a consumer does not buy a product, it may be because she decided not to consider it or because she did consider it and did not like it. We show that, in the long-run, two outcomes are possible. One is that a herd forms on the "search" action. In this case, the market learns the quality of the product and demand converges to its "true" market share (i.e., the share that it would obtain if product quality is known). This outcome can only occur if product quality is high. The other outcome is that a herd forms on the "no search" action. In this case, the market does not learn the true quality of the product and sales converge to zero. This outcome is certain to occur if product quality is low and occurs with positive probability if product quality is high. Thus, observational learning with search prevents a population of consumers from considering low quality products but can lead them not to consider, and therefore not to buy, high quality products. We refer to this event as a "bad" herd. These results differ from those of the standard herding model where the market never learns the true quality of products and market shares never converge to their correct market shares since eventually everybody either buys or does not buy with positive probability in each state.

The fact that observational learning can generate "bad" herds is not surprising. The more interesting issue is how the probability of a "bad" herd depends upon such factors as product quality, price, signal quality, and search costs. Robust comparative statics on the likelihood of "bad" herds are surprisingly difficult to obtain in herding models. The key restriction is the posterior likelihood ratio that the product is low rather than high has to be monotone increasing in the prior likelihood ratio following each action. We provide sufficient conditions on the joint distribution of signals and preferences for the monotonicity condition to hold. Given this condition, the likelihood ratio for a high quality product is forever trapped between two stationary points and we are able to obtain a closed form solution for the probability that the ratio does not converge to zero (i.e., market learns true

[2] We are working on a model with more than one product. The choice dynamics in a multi-product are more complex because of the interactions in the learning process across products.

quality). We then show that the probability of a "bad" herd increases with product price and search costs, and decreases with product quality.

Rosen [13] argued that the reward function to quality is convex because, in equilibrium, more talented artists can sell more units at higher unit prices. Our results suggests another source of convexity: in the long-run, consumers are more likely to learn about higher quality products and are more likely to buy them. Thus, small differences in product quality can lead to large differences in expected sales even when prices do not vary with quality. It can also explain why prices of products like albums, books and videos do not vary with quality. A small increase in price can have a disproportionate effect on expected sales since it decreases market share *and* increases the probability of a "bad" herd. We also show that the impact of a decrease in search costs is smaller on higher quality products. This result implies that a decline in search costs due to the Internet has a larger impact on the sales of niche products than on hit products.

There is a large empirical literature that tries to quantify the effect that the choices of others have on an individual's choices and identify the source of the effect (see Cai, Chen, and Fang [8] for discussion and references). One possible source is observational learning but another plausible mechanism is that individuals want to conform.[3] However, none of these studies have studied the implications of social influence on choice dynamics and outcomes. The exception is an ingenious experimental study conducted by Salganik, Dodds, and Watts [15] (hereafter referred to as SDW). SDW created an artificial music market in which hundreds of participants arrived sequentially at a website to listen to 48 songs by unknown artists. After listening to a song, participants could choose to download it. Listening to a song is analogous to product search in our model, and downloading is analogous to purchasing the product. In the control experiment, the songs were shown in random order, with no information about the previous participants' listening or downloading choices. The fraction of consumers who download a song conditional on listening to it (SDW called this fraction the song's "batting average") can be interpreted as a measure of the song's intrinsic quality. In the treatment experiments, of which they were eight, the songs were listed by download rank with download counts. The authors find that download rates in the treatment experiments were more highly skewed than in the control experiment, and quite unpredictable, particularly for songs with higher batting averages. The worst songs

[3] Cai, Chen, and Fang [8] also discuss a third mechanism, the saliency effect. The choices of other consumers may matter simply because those choices are highlighted.

never did well.

We use data from the SDW study to validate our model. The authors interpreted their results as evidence of social influence on download decisions but did not directly address the issue of the source of the social influence: did the download information affect choices primarily through observational learning (i.e., information) or through a desire to conform (i.e., preferences)? The observational learning hypothesis implies that download rate should affect listening probabilities but not the download probabilities conditional on listening. The conformity hypotheis implies that download rates should effect both probabilities. We examine these hypotheses using a probit analysis. We find that a higher download rank and count causes the song's listening probability to increase but the download information has no impact on the conditional download probability after controlling the song's intrinsic quality (i.e., its batting average in the control experiment). The download rates in the control experiment allows us to resolve the identification problem that Manski [11] calls the "reflection problem". Our analysis also controls for "framing" effect that can arise from ordering the songs by download rates. Thus, we conclude that participants download the songs that others download primarily because they tend to listen to the songs that others download. We also examine the relationship between song quality and long-run outcomes. The herding model predicts that the listening and downloading rates for the better songs will either converge to zero or to some positive fraction bounded away from zero. The outcomes in the eight treatment experiments are largely consistent with this prediction.

Hendricks and Sorensen [10] provide indirect evidence of the importance of observational learning on market demand for albums. Using detailed albums sales data for 355 artists, they show that many albums that flopped when they were released succeeded after the artists released another album that was a hit. If consumer preferences do not change over time, then the spillover reflects the arrival of new information that led many consumers to consider buying the debut album. In other words, the debut album suffered from a "bad herd," and the artist's new hit album caused consumer beliefs about the debut album to change enough that many of them were willing to consider buying it. Hendricks and Sorensen estimated a model of album demand based on this simple idea and found that it fits the data remarkably well. The parameter estimates indicate that almost all consumers know their preferences for a debut album that is a major hit, but only 32% of consumers know about their preferences for a debut album that achieves the median level of sales. This finding implies that album sales would have been substantially less skewed in a world where all consumers knew their preferences for the debut albums. For example, the authors estimated that the sales of the top artist in the sample would have exceeded the median artist's sales by a factor of 30 instead of the observed factor of 90.

The paper is organized as follows. In Section II, we describe our basic model. In Section III, we characterize the equilibrium dynamics and outcomes. In Section IV we present the comparative results. Section V examines two extensions of the basic model that make it more relevant for applications. Section VI studies an application of the model to data from the experimental study by SDW. Section VII concludes.

2 The Basic Model

In this section we present a sequential choice model in which heterogenous consumers arrive randomly and have the option of searching before deciding whether or not to purchase a product of unknown quality. Search involves acquiring a costly, private signal about preferences for the product. Each consumer's purchasing decision is observable to later consumers but her search decision is not observable.

An infinite sequence of consumers indexed by t enter in exogenous order. Each consumer makes an irreversible decision on whether or not to purchase the product. Consumer t's utility for the product is given by

$$V_t = X + U_t$$

where X denotes the mean utility or quality of the product and U_t is the idiosyncratic component. Here U_t is identically and independently distributed across consumers with zero mean. Let F_U denote the distribution of U. There are two quality levels: $X = H$ and $X = L$, where $H > L$. We will refer to H as the high quality state and L as the low quality state. We normalize $L = 0$. Consumer t does not know X or U_t. There is a common prior belief that assigns a probability μ_0 to the event that $X = H$. The price of the product is p. Consumers' utility is quasilinear in wealth, so consumer t's net payoff from purchasing the product is $V_t - p$.

Consumer t has two available actions. Buying the product involves risk since the ex post payoff may be negative. She can reduce the likelihood of this event by choosing to *Search (S)* before making her purchasing decision. Search involves paying a cost c to obtain a private, informative signal about V_t, and then purchasing if the expectation of V_t conditional on the signal exceeds p and not purchasing otherwise. For notational simplicity, it will be convenient to assume that the signal is perfectly informative and reveals V_t precisely.[4] Note that search remains a valuable option for consumer t even if she has learned X. The other action that she can choose is to *Not Search and Not Buy (N)*. Let $a_t \in \{N, S\}$ denote the action chosen by consumer t.

Given the consumer's purchasing rule following search, the expected value of search conditional on state X is

$$w(X) = \int_{P-X}^{\infty} (X - p + u) dF_U(u).$$

Hence, the payoff to a consumer from action S in state X is $w(X) - c$. We impose the following restrictions on the payoffs from search in each state.

A1: (i) $w(H) - c > 0$ and $0 > H - p$; (ii) $w(0) - c < 0$.

Condition (i) of Assumption A1 states that, conditional on H, the payoff to S is positive and the payoff from buying without search is always dominated. It implies that consumers never purchase without search even when they know that the state is H. Condition (ii) states that the consumer's payoff to S is negative if she knows that the state is L. It implies that the consumer's optimal action in state L is N.

Consumer t's action generates a purchasing outcome $b_t \in \{0, 1\}$. Here $b_t = 0$ is the outcome in which consumer t does not purchase the good and $b_t = 1$ is the outcome in which consumer t purchases the product. Outcome 0 occurs if consumer t chooses N or if she chooses S and obtains a realization of V_t such that her net payoff from purchase is negative. Outcome 1 arises if consumer t chooses S and obtains a realization of V_t such that her net payoff from purchase is positive.

Before taking her action, consumer t observes a private signal about the quality of the product. (In a later section we consider the case where the private signal is informative about both X and U.) Smith and Sorensen [18] have shown that there is no loss in generality in defining the private signal that a consumer receives as her *private belief*. Here we denote the signal by σ and define it as the probability that the state is H. Conditional on the state, the signals are identically and independently distributed across consumers and drawn from a distribution F_X, $X = H, L$. We assume that F_L and F_H are continuous and differentiable with densities f_L and f_H. Under the assumption that both states are equally likely, the

[4] The important restriction is that the signal is informative about X and not just U.

unconditional distribution of σ is $F = (F_L + F_H)/2$ with density f. Smith and Sorensen (2000) show that defining the private signal in this way implies that the joint distribution of X and σ possesses the monotone likelihood ratio property. As is well known, this property implies that the conditional distributions F_L and F_H as well as their hazard and reverse hazard rates are ordered. Private beliefs are *bounded* if the convex hull of the common support of F_L and F_H consists of an interval $[\underline{d}, \overline{d}]$ where $\underline{d} > 0$ and $\overline{d} < 1$.

In addition to the private signal, consumer t also observes the purchasing decisions of consumers 1 through $t-1$. She does not observe the private signals they received, the actions they chose or, if they searched, the information they obtained, and if they purchased, the payoffs they realized. The private signals imply that the consumers' search decisions are also private information. This feature of the model has important implications for outcomes and learning dynamics. The space of possible t-period purchase histories is given by $\Omega_t = \{0, 1\}^{t-1}$ and a particular history is denoted by ω_t. The initial history is defined as $\omega_1 = \varnothing$. The assumption that consumers observe the entire ordered action history ω is obviously quite strong. In most markets, consumers are likely to know only the number of past purchases. Later we consider situations in which consumers only observe some summary statistics of the purchasing history. Note that the assumption that the private signals are not informative about U implies that t-period history is also not informative about U_t.

Given any history ω_t, consumer t updates her beliefs about X using Bayes rule. Let $\mu_t(\omega_t)$ represent her posterior belief that the state is H conditional on history ω_t. Since ω_t is publicly observable, μ_t is also the *public* belief in period t. Given public belief μ_t and private signal σ_t, consumer t's *private* belief that the state is H is

$$r(\sigma_t, \mu_t) = \frac{\sigma_t \mu_t}{\sigma_t \mu_t + (1 - \sigma_t)(1 - \mu_t)}. \tag{1}$$

In studying the dynamics of beliefs and actions, we follow Smith and Sorensen [18] and work with the public likelihood ratio that the state is L versus H rather than public beliefs. Define

$$l_t = \frac{1 - \mu_t}{\mu_t}.$$

and let l_0 denote the prior likelihood ratio. Using this transformation of variables in equation (1), consumer t's private belief that the state is H becomes $r(\sigma_t, l_t)$. Her expected payoff to S is

$$\pi(S; \sigma_t, l_t) = r(\sigma_t, l_t)w(H) + (1 - r(\sigma_t, l_t))w(0) - c. \tag{2}$$

Recall that L is normalized to zero. Therefore, if the consumer chooses N, her payoff is zero. We look for a Bayesian equilibrium where everyone computes posterior beliefs using Bayes rule, knows the decision rules of all consumers and knows the probability laws determining outcomes under those rules.

A *cascade* on action $a \in \{S, N\}$ occurs when a consumer chooses a regardless of the realization of her private signal σ. Because of the distinction between actions and outcomes, we have to careful in defining a herd. We say that a *herd* on action a occurs at time n if each consumer $t \geq n$ chooses action a. Note that while a herd on N implies that all future outcomes are the same (all 1's and 0's, respectively), a herd on S does not. The outcome for a consumer who chooses S depends not only on X (which is common across consumers) but also on the realization of the idiosyncratic component U. In fact, a herd on S precludes the event that all future outcomes are the same (almost surely) - if the outcome does not vary with the realization of U, then it is not worthwhile paying c to search.

A related concept is outcome convergence. Let $\lambda_t \in [0,1]$ be the fraction of the first $t-1$ consumers whose outcome was 1 (purchase). Outcome convergence is the event that λ_t converges to some limit $\lambda \in [0,1]$. A herd implies outcome convergence. A herd on N leads to $\lambda = 0$; and a herd on S leads to $\lambda = 1 - F_U(p - X)$.

How does our model differ from the standard herding model? In the standard herding model, actions are observable; in our model, they are not. Instead, consumers observe outcomes, which are signals about the actions taken. More precisely, when consumer t does not purchase the product, subsequent consumers do not know whether it is because she chose N or because she chose S and obtained a realization on V_t such that her net payoff from purchase was negative. If consumer t purchases the product, then subsequent consumers can infer that she chose S. As we shall see, the endogeneity of the signal generated by the action taken has important implications for outcomes and for the learning dynamics.

3 Outcomes and Learning Dynamics

In this section we characterize the equilibrium outcomes and dynamics. We begin by defining thresholds. Let $\widehat{r}$ represent the private belief at which a consumer is indifferent between S and N. From equation (2),

$$\widehat{r} = \frac{c - w(0)}{w(H) - w(0)}. \tag{3}$$

Assumption A1 implies that $\widehat{r} \in (0,1)$. Using equations (1) and (3), we can then define the private signal at which a consumer is indifferent between S and N (assuming it is interior) as

$$\widehat{\sigma}(l) = \frac{(c - w(0))l}{w(H) - c + (c - w(0))l}. \tag{4}$$

Thus, given l, the consumer's optimal action is to choose S if $\sigma \geq \widehat{\sigma}$ and to choose N if $\sigma < \widehat{\sigma}$. We will refer to $\widehat{\sigma}$ as the *search threshold*.

Next we define the cascade regions. Let $\underline{l}$ denote the largest value of the public likelihood ratio such that a consumer is certain to choose S. From equation (4), $\underline{l}$ satisfies $\widehat{\sigma}(\underline{l}) = \underline{d}$. Solving this equation for $\underline{l}$ yields

$$\underline{l} = \frac{\underline{d}(w(H) - c)}{(1 - \underline{d})(c - w(0))}. \tag{5}$$

Let $\overline{l}$ denote the lowest value of the public likelihood ratio such that a consumer is certain to choose N. Once again, using equation (4), $\overline{l}$ satisfies $\widehat{\sigma}(\overline{l}) = \overline{d}$. Solving this equation for $\overline{l}$ yields

$$\overline{l} = \frac{\overline{d}(w(H) - c)}{(1 - \overline{d})(c - w(0))}. \tag{6}$$

Thus, we can partition the values of the public likelihood ratio into three intervals. When $l < \underline{l}$, there is a cascade on S; when $\underline{l} \leq l \leq \overline{l}$, the consumer searches with probability $1 - F(\widehat{\sigma}(l))$ and does not search with probability $F(\widehat{\sigma}(l))$; and when $l > \overline{l}$, there is a cascade on N.

We now characterize the dynamics of the public likelihood ratio. Suppose $\underline{l} < l_t < \overline{l}$. Then the probability that consumer t buys the product in state X is

$$\Pr\{b_t = 1 | X, l_t\} = (1 - F_X(\widehat{\sigma}(l_t)))(1 - F_U(p - X)).$$

It is the probability that consumer t searches in state X times the probability that she gets a realization of U that lies above the purchasing threshold $p - X$. The probability that consumer t does not buy the product in state X is

$$\Pr\{b_t = 0 | X, l_t\} = F_X(\widehat{\sigma}(l_t)) + (1 - F_X(\widehat{\sigma}(l_t)))F_U(p - X).$$

The first term is the probability of the event that consumer t does not search in state X; the second term is the probability of the event that consumer t searches in state X and gets a value of U that lies below the purchasing threshold. Using Bayes' rule, the public likelihood ratio in period $t+1$ is given by

$$l_{t+1}(b_t) = \begin{cases} \left[\dfrac{(1-F_L(\widehat{\sigma}(l_t)))(1-F_U(p))}{(1-F_H(\widehat{\sigma}(l_t)))(1-F_U(p-H))}\right] l_t & \text{if } b_t = 1 \\ \left[\dfrac{F_L(\widehat{\sigma}(l_t)) + (1-F_L(\widehat{\sigma}(l_t)))F_U(p)}{F_H(\widehat{\sigma}(l_t)) + (1-F_H(\widehat{\sigma}(l_t)))F_U(p-H)}\right] l_t & \text{if } b_t = 0 \end{cases}. \tag{7}$$

The dynamic system consists of a pair of non-linear, first-order difference equations with initial condition l_0. When $l_t < \underline{l}$, $F_L(\widehat{\sigma}) = F_H(\widehat{\sigma}) = 0$, and the dynamic system reduces to a pair of *linear,* first-order difference equations. When $l_t > \bar{l}$, $F_L(\widehat{\sigma}) = F_H(\widehat{\sigma}) = 1$ and $l_{t+1} = l_t$.

Smith and Sorensen [18] show the following:

Lemma 1 *Conditional on state H, the public likelihood ratio is a martingale. It converges to a random variable with support in $[0, \infty)$ so fully wrong learning has probability zero. Conditional on state L, the inverse public likelihood ratio is a martingale. It converges to a random variable with support in $[0, \infty)$.*

The martingale property rules out convergence to nonstationary limit beliefs such as cycles or to incorrect point beliefs.

For notational convenience, define

$$\psi_0(l_t) = \frac{F_L(\widehat{\sigma}(l_t)) + (1-F_L(\widehat{\sigma}(l_t)))F_U(p)}{F_H(\widehat{\sigma}(l_t)) + (1-F_H(\widehat{\sigma}(l_t)))F_U(p-H)}$$

$$\psi_1(l_t) = \frac{(1-F_L(\widehat{\sigma}(l_t)))(1-F_U(p))}{(1-F_H(\widehat{\sigma}(l_t)))(1-F_U(p-H))}$$

With this notation, equation (7) becomes

$$l_{t+1}(b_t) = \begin{cases} \psi_1(l_t) l_t & \text{if } b_t = 1 \\ \psi_0(l_t) l_t & \text{if } b_t = 0 \end{cases}. \tag{8}$$

The following lemma follows from our assumptions on the conditional distributions F_H and F_L.

Lemma 2 *(i) ψ_0 is continuous, $\psi_0(l) > 1$ on $[0, \bar{l})$, and $\psi_0(l) = 1$ on $[\bar{l}, \infty)$. (ii) $\psi_1(l) < 1$ and continuous on $[0, \bar{l})$ with $lim_{l \uparrow \bar{l}} \psi(l) > 0$.*

A particular useful property for comparative static purposes is when the posterior likelihood ratio following each action is monotone increasing in the prior likelihood ratio. Smith and Sorensen [19] find that their herding model has this property if the density of the private belief log-likelihood ratio is log-concave. In our model, log-concavity is sufficient to establish that the "buy" difference equation is strictly increasing in l_t for $l_t < \bar{l}$ but it is not sufficient for the "no buy" difference equation. In the former case, there is no distinction between outcome and action: when consumer $t+1$ observes the outcome "buy", she knows that consumer t chose action S. This is not true when consumer $t+1$ observes the outcome "no buy": it is consistent with consumer t choosing either action N or S. As a result, we need to impose an additional restriction on the distribution of U [and σ] to ensure monotonicity.

For any private belief σ, let γ denote the natural log of the corresponding likelihood ratio:

$$\gamma = \ln\left(\frac{1-\sigma}{\sigma}\right).$$

Denote the unconditional distribution of γ by F^γ, with density f^γ.

A2: (i) The density of the log likelihood ratio, f^γ, is strictly log-concave; and either (iia)

$$\left[\frac{F_U(p)}{F_U(p-H)}\right] \leq \left[\frac{F_L(\sigma)}{F_H(\sigma)}\right] \Big/ \left[\frac{1-F_L(\sigma)}{1-F_H(\sigma)}\right] \text{ for all } \sigma \in [\underline{d}, \overline{d}], \text{ or}$$

(iib) the unconditional density of the private signal f is bounded above by $\left[\dfrac{F_U(p)F_U(p-H)}{1-F_U(p-H)}\right]$.

Assumption A2(iia) ensures that, at any public belief, a decision not to search (after receiving the private signal) is more suggestive of state L than deciding to search but then not buying. Note that the right-hand side of the inequality is strictly greater than one. Assumption A2(iib) bounds the change in the probability of search that results from a fixed change in the public belief.

Lemma 3 *Assumption A2 implies that $\psi_0(l_t)l_t$ and $\psi_1(l_t)l_t$ are strictly increasing in l_t.*

Figure 1 illustrates a dynamic system that satisfies Assumption A2. The cascade set for S is the interval $[0, \underline{l}]$ and the cascade set for N is the interval $[\bar{l}, \infty)$. The "no buy" difference equation intersects the diagonal at 0 and at $\bar{l}$ and lies everywhere above the diagonal in between these two values. It is linear on the cascade set for S and strictly increasing on the interval $(\underline{l}, \bar{l})$. The "buy" difference equation intersects the diagonal at 0 and lies below the diagonal for positive values of l_t. It is linear on the cascade set for S and increasing on the interval $(\underline{l}, \bar{l})$. Active dynamics occur when the prior is such that $l_0 \in (0, \bar{l})$.

We can adapt Smith and Sorensen's [18] arguments to derive the following results.

Proposition 4 *Suppose* $0 < l_0 < \bar{l}$. *(a) Outcome convergence occurs almost surely. In state* H, $\lambda = 1 - F_U(p - H)$ *with positive probability and* $\lambda = 0$ *with positive probability; in state* L, $\lambda = 0$. *(b) In state* H, *beliefs converge to the truth when* $\lambda = 1 - F_U(p - H)$*; otherwise, the limit belief converges to* $\bar{l}$ *and learning is incomplete. Beliefs can never enter the cascade set for* N *from outside. (c) Actions in state* H *converge almost surely to* S *when* $\lambda = 1 - F_U(p - H)$; *otherwise they converge almost surely to* N.

The key feature of the dynamics is that the public likelihood ratio never enters the cascade set for N from outside. For any $l_0 < \bar{l}$, the dynamics are forever trapped between the stationary points 0 and $\bar{l}$ and convergence is always asymptotic. A sequence of "buy" outcomes causes the likelihood ratio to decrease in ever smaller increments towards 0; a sequence of "no buy" outcomes causes the likelihood ratio to increase in ever smaller increments to $\bar{l}$. A herd always eventually starts. In state H, the herd can form with positive probability on either N or on S. If it forms on S, then the market eventually learns the true state from the frequency of purchases. In state L, the herd can only form on N. Intuitively, if a herd formed on S, then the purchase frequency reveals the state is L, which contradicts the assumption that it is not optimal to search in state L.

To illustrate the role of Assumption A2, we consider a special case in which the private signal is completely uninformative about X. In this case, the distributions F_L and F_H are degenerate at $\sigma_t = 1/2$, the thresholds $\underline{l} = \bar{l}$ where

$$\bar{l} = \frac{(w(H) - c)}{(c - w(0))},$$

and each consumer's private belief is equal to the public belief. Hence, consumer t is certain to search when $l_t < \bar{l}$ and is certain not to search when $l_t > \bar{l}$. Given any $l_t < \bar{l}$, equation (7) reduces to

$$l_{t+1}(b_t) = \begin{cases} \dfrac{1 - F_U(p)}{1 - F_U(p - H)} l_t \text{ if } b_t = 1 \\ \dfrac{F_U(p)}{F_U(p - H)} l_t \text{ if } b_t = 0. \end{cases}$$

Otherwise, $l_{t+1} = l_t$.

Figure 2 presents the phase diagram for the two linear difference equations. The slope of the "buy" equation is less than 1 which implies that, given any $l_t < \bar{l}$, a sequence of "buy" outcomes causes the public likelihood ratio to converge in ever smaller increments to 0. Thus, conditional on H, there is a positive probability that beliefs converge to the truth. The slope of the "no buy" equation exceeds 1. The downward discontinuity in $\psi_0(l_t)l_t$ at $\bar{l}$ arises from the fact that probability of search is not continuous: it is equal to 1 if $l_t < \bar{l}$ and zero otherwise. The important property is not the discontinuity but the fact that $\psi_0(l_t)l_t$ is not monotone increasing at $\bar{l}$. When this is the case, a sequence of "no buy" outcomes causes the likelihood ratio to increase in ever larger increments and, after a finite number of periods, "jump" beyond $\bar{l}$. Convergence is not asymptotic and, given any $l_t < \bar{l}$, the set of rest points for the likelihood ratio consists of 0 and a nondegenerate subset of the stationary points $\{l \geq \bar{l}\}$. As we shall see in the next section, models with these kinds of dynamics yield relatively few testable predictions.

The above analysis shows how search modifies the results of herding models. In the standard herding model, the market never learns the true quality of products since eventually everybody either buys or does not buy and both outcomes can occur with positive probability in each state. Thus, the market share of high quality products never converges to their correct market shares, and the market share of low quality products converges to the wrong market share with positive probability. By contrast, when consumers can search prior to purchase, the market can learn the true quality of high quality products and outcomes can converge to the correct market shares. They are certain to do so for low quality products and with positive probability for high quality products. High quality products can still fail in our model due to the herding effect: outcomes converge to 0 with positive probability. Thus, our model generates a stochastic mapping between quality and market outcomes that allows market shares to vary with quality but explains why high quality products can still fail.

4 Comparative Statics

In this section we study the comparative static properties of our model. The goal is to derive testable predictions that can be applied to data. We will be interested in two sets of questions: how does cumulative sales vary with product price, quality, and search costs and how does long run sales vary with these parameters? Throughout this section, we will assume that the number of consumers is very large but finite.

Cumulative sales of a product depends upon two factors: the number of consumers who search and the fraction of these consumers who purchase the product. Let M denote the equilibrium number of consumers who search. It is a random variable whose realizations depend upon whether and when a herd forms on N. Let $F_{M|X}$ denote the distribution of M conditional on state X. Applying the law of large numbers, the fraction of M consumers that purchase the product is given by the probability of purchase, $1 - F_U(p - X)$. This probability is clearly decreasing in price and increasing in product quality. Unfortunately, similar comparative static results on $F_{M|X}$ (and therefore on cumulative sales) are difficult to obtain.

When price increases or, equivalently, H decreases, the probability of purchase falls, which in turn reduces the value of search. Thus, given any sequence of realizations of idiosyncratic shocks, zero outcomes are more likely to occur because search is less likely and purchasing conditional on search is less likely. However, more zero outcomes do not necessarily cause public beliefs to be more pessimistic. The change in purchasing and search rules affects the informativeness of the signal generated by the purchasing outcomes, and consumers take these changes into account when they update their beliefs about the state. It is not difficult to construct examples of sequences in which the number of consumers who search is actually higher at higher prices or lower quality. As a result, it is not possible to determine the effects of changes in price or product quality on the distribution of M or its moments.

The one case in which we are able to obtain a comparative static result on M is when private signals are completely uninformative. In this model, consumers are certain to search until public beliefs enter the cascade set for N. The purchasing rule also does not depend upon c. Consequently, given any sequence of realizations of idiosyncratic shocks, outcomes are unaffected by an increase in c as long as public beliefs are less than $\bar{l}$. Thus, the impact of an increase in search costs on M is completely determined by its impact on $\bar{l}$. The following proposition establishes that distribution of M is stochastically decreasing in c.

Proposition 5 *Suppose private signals are completely uninformative. Then* $F_{M|X}(m;c) \leq F_{M|X}(m;c')$ *for* $c > c'$, $X = L, H$.

We turn next to long run sales. The main prediction of our model (which does not depend upon Assumption A2) is that, in the long run, sales of high quality products either converge to 0 or to their true market shares. The question of interest is how the probability of a "bad" herd varies with the parameters of the model. Recall that, since the likelihood ratio process $\langle l_t \rangle$ is a bounded martingale conditional on state H, the $E[l_\infty] = l_0$. This property of martingales is quite useful when public beliefs cannot enter the cascade set for N from outside. When this is the case, supp$l_\infty = \{0, \bar{l}\}$, from which it follows that

$$\Pr\{l_\infty = \bar{l}\} = \frac{l_0}{\bar{l}}. \tag{9}$$

Thus, the sign of the changes in product quality, search costs, and price on the probability of a "bad" herd (i.e., long-run sales are 0) are determined by the sign of their impact on the value of $\bar{l}$. Differentiating $\bar{l}$ with respect to H, c and p yields the following results:

Proposition 6 *Suppose the state is H. Then the probability of a limit cascade on N is (a) strictly decreasing in H; (b) increasing and convex in c; (c) increasing in p.*

The proposition yields several predictions. An increase in search costs increases the likelihood that long-run sales of a high quality product is zero and hence reduces its expected long-run sales. The impact of the increase is larger at higher cost levels. An increase in price also increases the probability of zero long-run sales but the sign of the second derivative of $\bar{l}$ with respect to p depends upon the distribution of the private signal. This issue is important because, if $\bar{l}$ is convex, then the model could explain why search goods like music albums and books sell for the same prices even when the distributors know that the quality of their products vary.

A number of papers (e.g., [7]) have argued that the decline in search costs due to the Internet has disproportionately increased sales of niche products and reduced the concentration of sales. The next proposition provides support for this claim.

Proposition 7 *Suppose the state is H. Then the impact of an increase in c (or an increase in p) on the probability of a limit cascade on N is smaller (in absolute value) for higher quality products.*

The results follows from differentiating $\bar{l}$ with respect to H and c (and H and p). Proposition 7 implies that a decrease in search costs has a larger impact on long-run sales of niche products (i.e., medium quality products) than on high quality products.

Finally, the probability of a limit cascade on N does not depend upon the precision of the private signal. The only property of F that matters (aside from continuity) is $\overline{d}$, the upper bound of its support. This is a striking result, which has important implications for the kind of information that the market should reveal about consumers' purchasing decisions.

The comparative static results depend critically upon Assumption A2. When public beliefs can enter the cascade set on N from outside, the support of l_∞ consists of 0 and a nondegenerate subset of the cascade set. We can use the martingale property to bound the probability of a "bad" herd but cannot predict how this bound will vary with the parameters of the model. Pastine and Pastine [12] obtained similar "perverse" results when they studied the effect of changing the accuracy of signals on the probability of incorrect herds in a herding model where beliefs can enter the cascade set from outside.

The main prediction of our model (which does not depend upon Assumption A2) is that, in the long run, sales of high quality products either converge to 0 or to their true market shares. When sales converge to 0, a seller has an incentive to invest in a signal (e.g., advertisements) that can change public beliefs and lead the market to correct its mistake.

Proposition 8 *The introduction of a sufficiently positive, public signal after beliefs have converged (a) leads to an increase in short-run sales and, with positive probability, a herd on S if the initial herd is on N; (b) has no effect on sales if the initial herd is on S.*

In their study of recorded music, Hendricks and Sorensen [10] show that the release of a hit new album by an artist can substantially raise sales of the artist's catalog albums. They attribute the spillover to consumer learning. The new release causes some consumers to discover the artist or to revise their beliefs about the artist's catalog albums and purchase them. In the context of our model, the release of a hit new album can be interpreted as a strongly positive, public signal about the catalog album. If public beliefs about catalog albums have previously converged to $\bar{l}$, then the release of a new album that is a hit can cause public beliefs to move away from $\bar{l}$ and sales can re-converge with some probability to the true market share.

5 Extensions

In this section we consider two extensions of the basic model. In the first extension, we relax the assumption that signals are informative about the state but not about tastes. In practise, the signal that consumers obtain is likely to provide information about both components of utility. For example, when a consumer hears a song on the radio, he may recognize the song to be high quality but not like it because he does not like the genre. Similarly, when a consumer searches for product by submitting a query on a search engine, he observes not only the positions of advertisers on the page but also their ad text, which are informative signals about the idiosyncratic component of the match. In the second extension, we relax the assumption that consumers observe all past purchases. In most applications, consumers observe only aggregate statistics of the purchasing history such as total sales.

5.1 Private Signals of Utility

Suppose each consumer t gets a (noisy) private signal $Z = z_t$ of his utility V_t for the product rather than a signal only of the product's quality X. In particular, let

$$Z_t = V_t + \varepsilon_t$$

where ε_t is independent of X and U_t and is i.i.d. across consumers. The distribution of ε is denoted F_ε. For simplicity, suppose that both F_ε and F_U have smooth densities, and that the support of the sum

$$\eta = u + \varepsilon$$

is the real line (so that no signal perfectly reveals the state). Let F_η denote the distribution of η; it is the convolution of F_ε and F_U. The expected value of search conditional on state X after signal s is then defined as:

$$w(X, z) = \int_{p-X}^{\infty} (X - p + u) dF_{U|\eta=z-X}(u);$$

$F_{U|\eta=c}$ is the distribution of the taste component conditional on $\eta = c$:

$$F_{U|\eta=c} = \int_{-\infty}^{\infty} f_U(c - \varepsilon) f_\varepsilon(\varepsilon) d\varepsilon.$$

We need to impose restrictions on payoffs in order to focus on the interesting cases. In particular, we assume that (i) a consumer who knows that the product is low quality always chooses to *Not Search and Not Buy* regardless of the realization of his signal and (ii) a consumer who knows that quality is high always prefers *Not Search and Not Buy* to *Buy Without Search* but for sufficiently high realizations of the signal, he prefers to *Search.*

That assumption, which is analogous to Assumption A1 in our baseline case, is formalized as Assumption A1':

A1': (i) $w(H,z)-c>0$ for high enough z and $H-p+E[U|X=H,z]<0$ for all z; (ii) $w(H,z)-c<0$ and $0>L-p+E[U|X=L,z]$ for all z.

We also need to order the distributions of the signal. We assume that the distribution of the private signal Z conditional on quality H first-order stochastically dominates the distribution conditional on L, so Z is informative about quality. Let $\sigma(z)$ denote the *belief component* of the signal. It is defined as

$$\sigma(z)=\frac{f_\eta(z-H)}{f_\eta(z)+f_\eta(z-H)},$$

the ratio of the density of z in state H to the total density of z. We assume in addition that the distributions of signals in the two states satisfy the monotone likelihood ratio property, so that $\sigma(z)$ is an increasing function, and that the support of $\sigma(z)$ is bounded:

A3: (i) $\dfrac{f_\eta(z-H)}{f_\eta(z)}$ is increasing in z; and (ii) $\sigma(z)\in[\underline{d},\overline{d}]$ for all $z\in\Re$.

With these assumptions in place, we can proceed to describe the dynamics. A consumer t who starts with public belief μ_t and observes private signal z_t updates his belief to

$$r(\sigma(z_t),l_t)=\frac{\sigma(z_t)}{\sigma(z_t)+(1-\sigma(z_t))l_t}.$$

The consumer's expected payoff from search is then

$$\pi(l_t,z_t)=r(\sigma(z_t),l_t)w(H,z_t)+(1-r(\sigma(z_t),l_t))w(0,z_t).$$

Note that $\pi(l,z$ is decreasing in l and increasing in z, so for each public likelihood ratio l_t consumer t follows a cutoff strategy, choosing to search if z_t exceeds a threshold $\widehat{z}$ that is increasing in l_t. Let

$$\alpha(X,z)=\frac{\int\limits_{u=p-X}^{\infty}[1-F_\varepsilon(z-X-u)]f_U(u)du.}{1-F_\eta(z-X)}$$

denote the probability in state X that a consumer's utility exceeds the price p conditional on the event that his signal exceeds s. From Bayes rule, then, the updated public likelihood ratio in period $t+1$ after a purchase by consumer t ($b_t = 1$) or not purchase ($b_t = 0$) is, respectively,

$$l_{t+1}(l_t) = \begin{cases} \varphi_1(l_t)l_t \text{ if } b_t = 1 \\ \varphi_0(l_t)l_t \text{ if } b_t = 0 \end{cases}$$

where the functions φ_0 and φ_1 are defined as

$$\begin{aligned} \varphi_0(l_t) &= \frac{\alpha(0, \widehat{z}(l_t))[1 - F_\eta(\widehat{z}(l_t))]}{\alpha(H, \widehat{z}(l_t))[1 - F_\eta(\widehat{z}(l_t) - H)} \\ \varphi_1(l_t) &= \frac{[1 - \alpha(0, \widehat{z}(l_t))][1 - F_\eta(\widehat{z}(l_t))] + F_\eta(\widehat{z}(l_t))}{[1 - \alpha(H, \widehat{z}(l_t))][1 - F_\eta(\widehat{z}(l_t) - H)] + F_\eta(\widehat{z}(l_t) - H)}. \end{aligned}$$

As in the baseline case, the public likelihood ratio is martingale, conditional on H. The fixed points of the dynamics are $l_t = 0$ and $l_t \geq \bar{l}$ where here $\bar{l}$ is the largest value of l such that $\widehat{z}(l_t)$ is finite. It follows from Assumption A1'(ii) and A3(ii) that $\bar{l}$ is finite. In fact, we can obtain a closed form solution for $\bar{l}$ using arguments similar to those in the basic model:

$$\bar{l} = \frac{\overline{d}(\widehat{w}(H) - c)}{(1 - \overline{d})(c - \widehat{w}(0))} \tag{10}$$

where $\widehat{w}(X) = \lim_{s \longrightarrow \infty} w(X, s)$.

As in the baseline case, if consumer t purchases the product, future consumers know that he received a signal whose value was above his cutoff, and that he learned that his utility exceeded the price p. If he does not purchase, then future consumers know that either his signal was below the cutoff, or his utility was less than p. The complication relative to the baseline case is that, given quality, the probability of purchase after search is not independent of the private signal (since the signal is now correlated with the idiosyncratic taste component U). The correlation introduces a selection effect into the model: consumers who search are more likely to like the product and buy it.

The correlation also leads to less-than-clean sufficient conditions for the monotonocity property that is needed for the comparative statics. We need to establish that both $\varphi_0(l_t)l_t$ and $\varphi_1(l_t)l_t$ are increasing in l_t. As an analog to Assumption A2, we assume that the density of the log of the likelihood ratio and that either (i) not searching is a more negative signal than searching but not buying, or (ii) the density of the belief component $\sigma(z)$ is bounded above.

A2': (i) The density of the log of the likelihood ratio implied by s, $(1 - \sigma(z)/\sigma(z)$, is strictly log-concave; and (ii)

$$\frac{1-\alpha(0,z)}{1-\alpha(H,z)} \leq \left[\frac{F_\eta(z)}{F_\eta(z-H)}\right] \Big/ \left[\frac{1-F_\eta(z)}{1-F_\eta(z-H)}\right] \text{ for all } z \in \Re.$$

To cope with the complications created by the dependence of the probability of purchase after search on the search probability, we introduce an addition assumption:

$$A4: \frac{1-\alpha(0,z)}{1-\alpha(H,z)} \geq \frac{\partial\alpha(0,z)}{\partial z} \Big/ \frac{\partial\alpha(H,z)}{\partial z} \geq \frac{\alpha(0,z)}{\alpha(H,z)}.$$

Assumption A4 has the flavor of a monotone likelihood ratio property for the probability of purchase conditional on search in the two states.

Lemma 9 *Assumptions A2' and A4 imply that $\varphi_0(l_t)l_t$ and $\varphi_1(l_t)l_t$ are increasing in l_t.*

Having established the monotonicity of the dynamics of the public likelihood, we can extend the results of the baseline case concerning long-run behavior (Propositions 4-8) to the case of private signals of both common and idiosyncratic components.

5.2 Aggregate Histories

The assumption that all prior purchases are observable to consumers plays a pivotal role in the analysis. In particular, it implies that public likelihood ratio conditional on state H is a martingale, which allows us to use the Martingale Convergence Theorem to establish convergence of beliefs and actions. When consumer t only observes total purchases K_t (an unordered purchase history), then the public likelihood ratio is no longer a martingale. Consumer t no longer knows what consumer $t-1$ knows; he knows only that K_{t-1} was either equal to $K_t - 1$ and consumer $t-1$ searched and purchased the product, or it was equal to K_t and consumer $t-1$ did not purchase the product. Therefore, we need to develop an alternative proof technology to establish convergence of beliefs (and actions) in order for our analysis to apply. Here is a sketch of the proof.

Step 1: If l_t converges, it must converge to either 0 or $\bar{l}$.

1. Suppose that l_t converges a.s. and that with positive probability $l_\infty = j$, where $0 < j < \bar{l}$.

2. Outcome is not possible when purchase history is observable because if l_t is close to j for all $t > T$, then after T search probability is constant and so purchase frequency reveals product quality.

3. Similar argument here: if convergence by period T with ex ante probability close to 1, and consumer $1000T$ observes a history consistent with $l_\infty = j$, then he believes that search probability has been constant since T and that therefore purchase frequency reveals quality.

Step 2: 0 and $\bar{l}$ are fixed points of the dynamics and thus possible limit points of l_t.

Step 3: If public LR is close to 0 or $\bar{l}$, with positive probability they stay there forever (so sufficient to show that 0 or $\bar{l}$ is approached infinitely often).

Step 4: Rule out the possibility that with positive probability l_t eventually stays in some support J with sup $j < \bar{l}$.

1. Argument similar to Step 1: $\exists T$ such that probability of $\{$convergence or $l_t \in J$ forever$\}$ by period T is high.

2. If K_{1000T} is such that $l_{1000T} \approx j$, and a consumer $1000T$ does not purchase, then next consumer believes he did not purchase (alternative is that $l_{1000T} > j$).

3. Since $j < \bar{l}$, probability of search was strictly positive, and so not purchasing shifts $l_{1000T+1}$ discretely above j. But that event has positive probability, and so j cannot be an upper bound.

6 Application

The basic setup of our model is mirrored nicely in a recent online experiment conducted by SDW[5] In the experiment, thousands of subjects were recruited to participate in artificial online music markets. Participants arrived sequentially and were presented with a list of 48 songs, which they could listen to, rate, and then download (for free) if they so chose. In

[5]See Salganik, Dodds, and Watts [15] for a brief but insightful analysis of the experiments; Salganik [14] provides a more thorough description and analysis.

real time, each participant was randomly assigned to one of nine "worlds." In the treatment worlds, of which there were eight, songs were listed by download rank: the first song listed was the one with the most downloads by previous participants in that same world, the second song listed had the second most downloads, and so on. In the control world, the 48 songs were shown in a random order, with no information about previous participants' listening, rating, or downloading behavior. The eight treatment worlds operated independently of one another, so that the researchers could observe eight separate realizations of the stochastic process.

As in our model, the products in these experiments were search goods. The songs were carefully screened to ensure that they would be unknown to the participants.[6] Choosing whether to sample a song is analogous to the decision of whether to search in our model. The cost of search in the experiment (i.e., the opportunity cost of the time spent listening to a song) was large enough that most participants listened to very few songs. Downloading a song (after listening to it) is analogous to the purchase decision in our model. Also, since participants assigned to treatment worlds were shown the number of downloads by previous participants, the information they received is essentially the same as in our model.

Table 1 summarizes the behavior of the participants in the experiment. On average, participants listened to fewer than four songs and downloaded fewer than two. The median number of listens was 1 and the median number of downloads was zero.[7] Over 90% of participants listened to 10 songs or fewer, and roughly 40% exited the experiment without listening to any songs at all. Overall, the listening and downloading behavior of the participants in the treatment worlds is similar to that of the participants in the control world.

Table 2 summarizes the outcomes for the 48 songs. In the control world, the fraction of participants who listened to a given song was roughly equal across songs, ranging from 6% to 11%. In the treatment worlds, listening probabilities varied widely, with some songs in some worlds being listened to by more than 40% of participants. The higher variance of listening probabilities naturally translated to a higher variance in downloading probabilities. Hence, downloads were much more "skewed" in the treatment worlds than in the control

[6] They were obtained from the music website *purevolume.com*, a website where aspiring bands can create homepages and post music for download. Bands that had played too many concerts or received too many hits on their homepages were excluded.

[7] It was not possible in the experiment to download a song without first listening to it.

world.

Table 2 also describes the distribution of the songs' *conditional* download probabilities—i.e., the probability that a participant downloaded the song conditional on listening to it. (SDW refer to these conditional probabilities as "batting averages.") These conditional probabilities provide the best measure of the songs' relative qualities. Unconditional download probabilities do not accurately reflect song quality because they conflate the probability of listening (which was highly variable across songs, and across worlds for a given song) with the conditional probability of downloading. On the other hand, conditional on listening to a song, the probability of downloading is clearly higher for songs with greater appeal. As shown in the table, quality varied substantially across songs: the conditional download probability was nearly 60% for the highest-quality song, and only 11% for the lowest-quality song.[8]

The distinction between listening and downloading is an important one. In our model, consumers are influenced by others' purchases only insofar as those purchases affect the decision to search. There are no social effects in the traditional sense: preferences are unaffected by previous consumers' purchases. The experimental data support this assumption. Participants in the treatment worlds were roughly 10 times more likely to listen to the top-ranked (i.e., most downloaded) song than any song ranked below 30. However, high download ranks did not appear to increase songs' "batting averages" (i.e., the probability of downloading a song conditional on listening to it).

More formally, letting L_{jt} be an indicator variable equal to 1 if participant t listened to song j, and D_{jt} an indicator equal to 1 if participant t downloaded song j (conditional on listening to it), we can ask whether L_{jt} and D_{jt} are influenced by song j's download share among participants $1, \ldots, t-1$. Table 3 reports the results from probit regressions in which L_{jt} and D_{jt} are assumed to depend on song j's current download share (i.e., song j's share of total downloads by previous participants). Because some song titles and/or artist names might be more appealing than others on average, we include the song's listening share from

[8] We use conditional download probabilities from the control world as our measure of song quality, because the probabilities from the treatment worlds may be tainted by a selection effect. For example, if in a treatment world a participant listens to a song with a very low download rank, it may indicate that something in the song's title (or the artist's name) was idiosyncratically appealing to the participant, which may increase the conditional probability of download. This possibility does not seem too important, however, as the conditional probabilities from the treatment worlds are generally very close to those from the control world.

the independent world as a control in the listening regression. The coefficient on download share is positive and highly significant, indicating that participants' decisions to listen to a song were influenced by information about previous participants' downloads.

In interpreting this result, one might argue that it partially reflects a framing effect. Because people generally tend to choose the first item when selecting from a list, the apparent influence of download information could be conflated with the impact of list position itself. Indeed, even in the control world, where songs were ordered randomly for each participant, participants were much more likely to listen to the first listed song. However, the download information can be shown to have an effect above and beyond the effect that comes from list position. SDW ran separate experiments in which download information was provided but songs were still randomly ordered; in these experiments, the provided information still had a substantial impact on listening probabilities (albeit not as large as in the experiments we analyze here).

By contrast, conditional download probabilities did *not* appear to be higher for top-ranked songs. This is a trickier issue, however, because a probit regression of D_{jt} on download share involves an obvious reflection problem. (Songs with the most downloads will naturally have higher average download probabilities.) Fortunately, the conditional download probability from the control world is a natural control variable. When included, it forces the coefficient on download share to be identified from time variation in the download share relative to what it "ought" to be (as indicated by its download probability in the treatment world). Estimates of this model are reported in the second column of Table 3. The coefficient on download share is actually negative, suggesting that participants were slightly less likely to download top-ranked songs (conditional on listening to them)..Taken together, the estimates imply that the information provided in the treatment worlds primarily affected participants' listening decisions and not their "preferences", or at least not positively. The latter result is not consistent with theories of social preferences which imply that preferences for a song should increase with its popularity.

The most likely explanation for the negative coefficient is a selection effect. In the control world, participants who chose to listen to a particular song may have done so because something in the song's title appealed to them. In other words, the title is an informative signal about their preferences and as a result, they are more likely to download the song than a randomly selected participant would be. When download information is shown, this selection effect is not as strong. Top-ranked songs are listened to by a wider selection of participants, not just those who liked the titles. Since the selection of listeners for top-ranked songs is less favorably inclined, the fraction who choose to download these songs after listening to them is lower. Conversely, lower-ranked songs are listened to by a narrower selection of participants, those who really liked the title. Since the selection of listerners for these songs are more favorably inclined, the fraction who choose to download is higher. Thus, the results reported in Table 3 accord well with the assumptions of our extended model.

To describe how the learning processes converged in the experiments, Table 4 reports the distribution of listening probabilities among the last 100 participants in each of the eight worlds. We classified songs into three quality categories based on their batting averages in the control experiment. For each song, we simply calculated the fraction of participants who listened to the song, among the last 100 participants to arrive. The results for the best 16 songs indicate that outcomes in this category were highly unpredictable. In 53% of the cases, the listening probabilities were less than 5%, but in 13% of the cases they exceeded 20%. Outcomes were more predictable for the lower quality songs. The listening probabilities for most of these songs were less than 5%, with only 3% of songs obtaining listening probabilities above 10%. Recall that roughly 2% of the participants had essentially zero search costs and listened to all 48 songs. Consequently, in contrast to our model, we would not expect to see listening probabilities converge to zero in the experiment. Overall, therefore, we interpret the patterns as being broadly consistent with the predictions of our model.

Although the stochastic nature of the learning process makes it so that the market sometimes converges to the "wrong" outcome, in general the provision of information on previous consumer's decisions should make search more efficient. To test this in the experimental data, we compare the listen rates and download rates for three different groups of participants: (1) those who were randomly assigned to the control world; (2) those who were assigned to a treatment world, and were among the *first* 100 participants to arrive; (3) those who were assigned to a treatment world, and were among the *last* 100 participants to arrive. Listening and downloading behavior among group 2 should be similar to group 1, since not much information has yet accumulated. However, we should expect search to be noticeably more efficient for the third group, since they observe substantial information on previous participants' downloads, and that information should in most cases guide them toward higher quality songs.

Table 5 reports listens per participant (total number of listens divided by total number of participants) and downloads per listen (total number of downloads divided by total number of listens) for each group. A comparison of the listening rates for the three groups suggests that consumers listen to fewer songs when they are more informed. The listening rates and download rates for the first 100 participants look roughly similar to those in the control world. But the last 100 participants have lower listening rates and substantially higher download rates. The download rates increased by over 20%. Thus it appears that search indeed became more efficient over time in the treatment worlds.

7 Conclusion

We have studied a simple choice problem in which consumers have to decide whether or not to consider a product of unknown utility. Consumers only purchase products in their consideration sets, but including a product in the set involves a small search cost. Consumers would prefer not to pay this cost if they believe they are unlikely to buy the product. The purchasing decisions of other consumers influences their beliefs about the gains from search. A poor purchasing record can feed on itself and lead consumers to wrongfully omit high quality products from their consideration sets. On the other hand, a good purchasing record can also feed on itself and lead consumers to include the product in their consideration sets. In this case, the market learns the quality of the product and the product obtains its true market share. The experimental study by Salganik et al provides evidence on the feedback mechanism and shows how it can affect outcomes. The results are largely consistent with our model.

However, their study also suggests that the dynamics of product demand are more complicated when the choice set consists of multiple products. Consumers typically search products sequentially according to the order in which they listed, downloading the songs they like, and stopping when the expected benefits of search exceed the cost. But preferences do not appear to be additive since the listening probability of a song that makes the considerations sets of most of the last 100 participants varies across the experiments. The implication is that long-run sales of high quality products that succeed and the probability that it will succeed are likely to depend upon the number of products that consumers want to purchase and the set of other high quality products that survive (i.e., have herds on S). It will also depend upon the kind of information the market provides. For example, in the market for recorded music, the market reports the sales ranks of albums but not their actual sales. We hope to explore the dynamic interactions that can occur when the choice set consists of more than one product in subsequent research.

We have largely ignored the issue of how firms may want to increase the likelihood that consumers will include their products in their consideration sets. Eliaz and Spiegler explore this issue in a static model in which firms employ costly marketing devices such as advertising to influence the formation of consumer consideration sets. They use the model to study whether firms can profitably exploit the bounded rationality of consumers and the impact of their rational behavior on product variety and marketing.

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Appendix

Proof of Lemma 2:

The continuity of ψ_0 and ψ_1 follows from the continuity of F_L, F_H, and $\widehat{\sigma}$. Since F_L and F_H satisfy the monotone likelihood ratio property, F_H first order stochastically dominates F_L, so $\psi_0(l) > 1$ and $\psi_1(l) < 1$ for $l \in [0, \bar{l})$. For $l \geq \bar{l}, F_H(\widehat{\sigma}(l)) = F_L(\widehat{\sigma}(l)) = 1$, and so $\psi_0(l) = 1$. Finally,

$$\begin{aligned}
\lim_{l \uparrow \bar{l}} \psi_1(l) &= \frac{1 - F_U(p)}{1 - F_U(p - H)} \lim_{l \uparrow \bar{l}} \left[\frac{1 - F_L(\widehat{\sigma}(l))}{1 - F_H(\widehat{\sigma}(l))} \right] \\
&= \frac{1 - F_U(p)}{1 - F_U(p - H)} \lim_{l \uparrow \bar{l}} \left[\frac{f_L(\widehat{\sigma}(l))\widehat{\sigma}'(l)}{f_H(\widehat{\sigma}(l))\widehat{\sigma}'(l)} \right] \\
&= \frac{1 - F_U(p)}{1 - F_U(p - H)} \frac{f_L(\widehat{\sigma}(\bar{l}))}{f_H(\widehat{\sigma}(\bar{l}))} > 0.
\end{aligned}$$

The last inequality follows from the fact that

$$\widehat{\sigma}'(l) = \frac{(w(H) - c)(c - w(0))}{[w(H) - c + ((c - w(0))l)]^2} \neq 0.$$

Q.E.D.

Proof of Lemma 3:

Smith and Sorensen's [19] Lemma 6 establishes that if f^γ is log-concave, then both

$$\left[\frac{F_L(\widehat{\sigma}(l_t))}{F_H(\widehat{\sigma}(l_t))} \right] l_t$$

and

$$\left[\frac{1 - F_L(\widehat{\sigma}(l_t))}{1 - F_H(\widehat{\sigma}(l_t))} \right] l_t$$

are increasing in l_t. (That is, the posterior likelihood ratio after observing only whether or not consumer t searches increases with l_t.) In our setting, the posterior likelihood ratio after observing $b_t = 1$ is

$$\psi_1(l_t)l_t = \frac{1 - F_U(p)}{1 - F_U(p - H)} \left[\frac{1 - F_L(\widehat{\sigma}(l_t))}{1 - F_H(\widehat{\sigma}(l_t))} \right] l_t.$$

Thus, $\psi_1(l_t)l_t$ is a scalar multiple of

$$\left[\frac{1 - F_L(\widehat{\sigma}(l_t))}{1 - F_H(\widehat{\sigma}(l_t))} \right] l_t,$$

and so is also increasing in l_t.

The posterior likelihood ratio after observing $b_t = 0$ is

$$\psi_0(l_t)l_t = \left[\frac{F_L(\widehat{\sigma}(l_t)) + (1 - F_L(\widehat{\sigma}(l_t)))F_U(p)}{F_H(\widehat{\sigma}(l_t)) + (1 - F_H(\widehat{\sigma}(l_t)))F_U(p - H)}\right] l_t. \quad \text{(A1)}$$

First, suppose that Assumption A2(iia) holds. Defining

$$\rho(l_t) = \frac{F_L(\widehat{\sigma}(l_t))}{F_H(\widehat{\sigma}(l_t)) + (1 - F_H(\widehat{\sigma}(l_t)))F_U(p - H)} \in (0, 1)$$

as the probability in state H that consumer t did not search condition on his not purchasing, we can rewrite equation (A1) as

$$\psi_0(l_t)l_t = \rho(l_t)\left[\frac{F_L(\widehat{\sigma}(l_t))l_t}{F_H(\widehat{\sigma}(l_t))}\right] + (1 - \rho(l_t))\left[\frac{(1 - F_L(\widehat{\sigma}(l_t)))F_U(p)l_t}{(1 - F_H(\widehat{\sigma}(l_t)))F_U(p - H)}\right]. \quad \text{(A2)}$$

We know that the terms in brackets are increasing in l_t. Thus, if we can show that

$$(i) \quad \frac{F_L(\widehat{\sigma}(l_t))}{F_H(\widehat{\sigma}(l_t))} \geq \frac{(1 - F_L(\widehat{\sigma}(l_t)))F_U(p)}{(1 - F_H(\widehat{\sigma}(l_t)))F_U(p - H)}$$

and that (ii) $\rho(l_t)$ is increasing in l_t, then we have established that $\psi_0(l_t)l_t$ is increasing in l_t. Assumption A2(iia) implies claim (i).To check claim (ii), differentiating $\rho(l_t)$ at $l_t \geq \underline{l}$ and simplifying yields

$$\begin{aligned}\rho'(l_t) &= \frac{F_U(p - H))f_H(\widehat{\sigma}(l_t))\widehat{\sigma}'(l_t)}{[F_H(\widehat{\sigma}(l_t)) + [1 - F_H(\widehat{\sigma}(l_t))]F_U(p - H)]^2} \\ &> 0\end{aligned}$$

because

$$\widehat{\sigma}'(l_t) = \frac{(w(H) - c)(c - w(L))}{[w(H) - c + (c - w(L))l_t]^2} > 0.$$

At $l_t < \underline{l}$ (so that the probability of search is one), the value of $\rho(l_t)$ is constant at 0 (and thus weakly increasing).

Next suppose that Assumption A2(iib) holds. For notational simplicity, define

$$\begin{aligned} a &= 1 - F_U(p), \\ b &= 1 - F_U(p - H) \end{aligned}$$

and

$$\widehat{l}(l_t) = \frac{1 - \widehat{\sigma}(l_t)}{\widehat{\sigma}(l_t)},$$

so equation (A1) can be rewritten as

$$\psi_0(l_t)l_t = \left[\frac{1-a+aF_L(\widehat{\sigma}(l_t))}{1-b+bF_U(\widehat{\sigma}(l_t))}\right] l_t. \tag{A3}$$

Differentiating equation (A3) with respect to $l_t \geq \underline{l}$,

$$\begin{aligned} & \left[\frac{1-a+aF_L(\widehat{\sigma}(l_t))}{1-b+bF_U(\widehat{\sigma}(l_t))}\right] + \\ & l_t \left[\frac{1-b+bF_H(\widehat{\sigma}(l_t))af_L(\widehat{\sigma}(l_t))\widehat{\sigma}'(l_t) - [1-a+aF_L(\widehat{\sigma}(l_t))]bf_H(\widehat{\sigma}(l_t))\widehat{\sigma}'(l_t)}{[1-b+bF_H(\widehat{\sigma}(l_t))]^2}\right] \\ = \quad & \frac{1}{[1-b+bF_H(\widehat{\sigma}(l_t))]^2}[1-a+aF_L(\widehat{\sigma}(l_t))][1-b+F_H(\widehat{\sigma}(l_t))] \\ & +\widehat{\sigma}'(l_t)l_t[(1-b+bF_H(\widehat{\sigma}(l_t)))af_L(\widehat{\sigma}(l_t)) - (1-a+aF_L(\widehat{\sigma}(l_t)))bf_H(\widehat{\sigma}(l_t))]. \end{aligned}$$

Substituting $f_L(\sigma) = 2(1-\sigma)f(\sigma)$ and $f_H(\sigma) = 2\sigma f(\sigma)$, and

$$\widehat{\sigma}'(l_t) = \frac{\widehat{l}(l_t)}{[\widehat{l}(l_t)+l_t]^2}$$

then yields

$$\begin{aligned} = \quad & \frac{1}{[1-b+bF_H(\widehat{\sigma}(l_t))]^2}[1-a+aF_L(\widehat{\sigma}(l_t))][1-b+F_H(\widehat{\sigma}(l_t))] \\ & +\frac{\widehat{l}(l_t)}{[\widehat{l}(l_t)+l_t]^2}2f(\widehat{\sigma}(l_t))[(1-b+bF_H(\widehat{\sigma}(l_t)))a(1-\widehat{\sigma}(l_t)) - (1-a+aF_L(\widehat{\sigma}(l_t)))b\widehat{\sigma}(l_t)] \\ \geq \quad & \frac{1}{[1-b+bF_H(\widehat{\sigma}(l_t))]^2}[(1-a)(1-b) - bf(\widehat{\sigma}(l_t))]. \end{aligned}$$

Assumption A2(iib) ensures that the last expression is positive. If $l_t < \underline{l}$, then $\widehat{\sigma}'(l_t) = 0$, and the derivative in equation (A3) reduces to

$$\frac{1-a+aF_L(\widehat{\sigma}(l_t))}{1-b+bF_H(\widehat{\sigma}(l_t))} > 0.$$

Q.E.D.

Proof of Proposition 4:

Lemma 1 shows that the public likelihood ratio converges almost surely to a random variable l_∞. The only fixed points of the Markov process on the public likelihood rato (and thus the only possible values of l_∞) are $l = 0$ and $l \geq \bar{l}$. Given belief convergence and the monotonicity of the Markov process (Lemma 3 implies monotonicity for l between $\underline{l}$ and $\bar{l}$, the process is clearly monotonic below $\underline{l}$ and above $\bar{l}$, and the process is continuous at $\underline{l}$ and $\bar{l}$), Smith and Sorensen's [19] Lemma 12a establishes that the public likelihood ratio cannot enter the cascade set $l \geq \bar{l}$ from outside, and so if $l_0 < \bar{l}$, then either $l_\infty = 0$ or $l_\infty = \bar{l}$. Because l_∞ almost surely cannot be fully wrong, in state L, $l_\infty = \bar{l}$ with probability one. Consequently, in state L actions converge to N and so $\lambda = 0$. In state H, the argument of Smith and Sorensen's [18] Theorem 1d shows that the events $l_\infty = 0$ and $l_\infty = \bar{l}$ both have positive probability. In the former case, learning is complete, actions converge to S, and $\lambda = 1 - F_U(p - H)$. In the latter case, learning is incomplete, actions converge to N, and $\lambda = 0$. Q.E.D.

Proof of Propositions 6 and 7:

A limit cascade on N is the event that $l_\infty = \bar{l}$, which has probability

$$\Pr\{l_\infty = \bar{l}\} = \frac{l_0}{\bar{l}} = \frac{(c - w(0))(1 - \underline{d})}{(w(H) - c)\underline{d}} l_0 = l_0 \frac{(1 - \underline{d}) \left(c - \int\limits_p^\infty (-p + u) dF_U(u) \right)}{\underline{d} \left(\int\limits_{p-H}^\infty (H - p + u) dF_U(u) - c \right)}.$$

Differentiating with respect to H, c, and p yields, respectively,

$$\frac{\partial \Pr\{l_\infty = \bar{l}\}}{\partial H} = \frac{-(1 - F_U(p - H))(c - w(0))(1 - \underline{d})}{(w(H) - c)^2 \underline{d}} l_0 < 0,$$

$$\frac{\partial \Pr\{l_\infty = \bar{l}\}}{\partial c} = \frac{(w(H) - w(0))(1 - \underline{d})}{(w(H) - c)^2 \underline{d}} l_0 > 0,$$

and

$$\frac{\partial \Pr\{l_\infty = \bar{l}\}}{\partial p} = \frac{(1 - F_U(p))(w(H) - c) + (1 - F_U(p - H)(c - w(0))(1 - \underline{d})}{(w(H) - c)^2 \underline{d}} l_0 > 0.$$

Further differentiating $\frac{\partial \Pr\{l_\infty = \bar{l}\}}{\partial c}$ with respect to c and H yields, respectively,

$$\frac{\partial^2 \Pr\{l_\infty = \bar{l}\}}{(\partial c)^2} = \frac{2(w(H) - c)(w(H) - w(0))(1 - \underline{d})}{(w(H) - c)^3 \underline{d}} l_0 > 0$$

and

$$\frac{\partial \Pr\{l_\infty = \bar{l}\}}{\partial c \partial H} = \frac{-(1 - F_U(p - H))[(c - w(0)) + (w(H) - w(0))](1 - \underline{d})}{(w(H) - c)^3 \underline{d}} l_0 < 0.$$

Proof of Proposition 8:

If the initial herd is on S (corresponding to $l_\infty = 0$), then subsequent buyers will choose action S (and then purchase with probability $1 - F_U(p - H)$ with or without the positive public signal. If the initial herd is on N (corresponding to $l_\infty = \bar{l}$ and $\lambda = 0$), then a positive public signal pushes teh public likelihood ratio below $\bar{l}$. The public likelihood ratio will reconverge, to $\bar{l}$ in state L but with positive probability to 0 (and positive long-run sales) in state H. Thus, in state H, a positive public signal raises expected sales when the initial herd is on N. Q.E.D.

Proof of Lemma 9:

First, consider $\varphi_1(l_t)l_t$. As in the proof of Lemma 3, Assumption A2' and Smith an Sorensen's (2001) Lemma 6 imply that

$$\frac{1 - F_\eta(\widehat{z}(l_t))}{1 - F_\eta(\widehat{z}(l_t) - H))} l_t$$

is increasing in l_t. Thus, it is sufficient to show in addition that the ratio

$$\frac{\alpha(L, \widehat{z}(l_t))}{\alpha(H, \widehat{z}(l_t))}$$

is increasing in l_t; Assumption A4 implies that it is. Similarly, Assumption A4 implies that

$$\frac{1 - \alpha(L, \widehat{z}(l_t))}{1 - \alpha(H, \widehat{z}(l_t))}$$

is increasing in l_t, so Lemma 3's proof that $\varphi_0(l_t)l_t$ is increasing in l_t goes through here as well. Q.E.D.

Table 1: Summary statistics: participants

	Treatment Worlds ($N = 5,746$)	Control World ($N = 1,446$)	Overall ($N = 7,192$)
Number of listens:			
Mean	3.52	3.90	3.60
Std. Dev.	7.13	8.38	7.40
Min	0	0	0
Median	1	1	1
Max	48	48	48
Number of downloads:			
Mean	1.41	1.51	1.43
Std. Dev.	4.35	5.05	4.49
Min	0	0	0
Median	0	0	0
Max	48	48	48

Table 2: Summary statistics: songs

	Treatment Worlds ($N = 384$)	Control World ($N = 48$)
Prob(Listen):		
Mean	.073	.081
Std. Dev.	.078	.013
Min	.019	.059
Median	.044	.078
Max	.475	.113
Prob(Download):		
Mean	.029	.032
Std. Dev.	.037	.010
Min	.004	.007
Median	.016	.031
Max	.235	.055
Prob(Download|Listen):		
Mean	.377	.386
Std. Dev.	.117	.109
Min	.087	.112
Median	.388	.381
Max	.706	.596

Table 3: Probit regressions

	(1) Listen	(2) Download
	N = 274,512	N = 20,091
Download share	1.266	-0.192
	(.013)	(.074)
Control world listening share	0.681	
	(.035)	
Control world cond. prob. download		0.754
		(.039)

Probit regressions; marginal effects reported; standard errors in parentheses.

Table 4: Stochastic mapping of quality to outcomes

	Percent of cases with listening probabilities equal to:				
	0%-5%	6%-10%	11%-15%	16%-20%	>20%
Best 16 songs	52.34	16.41	10.16	7.81	13.28
Middle 16 songs	63.28	17.19	14.06	2.34	3.12
Worst 16 songs	81.25	15.62	2.34	0.00	0.78

The table reports the fraction of cases in which listening probabilities converged to the given ranges. The "converged" probabilities are calculated as the fraction of the last 100 participants who listened to the song. So, for example, for the best 16 songs (as measured by conditional download probabilities in the control world), in 52.34% of cases only 5 or fewer of the last 100 participants listened to the song.

Table 5: Search efficiency

	Number of participants	Listens per participant	Downloads per listen
Control world	1446	3.902	0.388
Treatment worlds: First 100	800	3.740	0.383
Treatment worlds: Last 100	800	3.524	0.462

The "First 100" are participants who were among the first 100 to arrive in their randomly assigned world; the "Last 100" were among the last 100 to arrive. Since there were 8 separate treatment worlds, there are 800 participants in each of these categories.

Figure 1

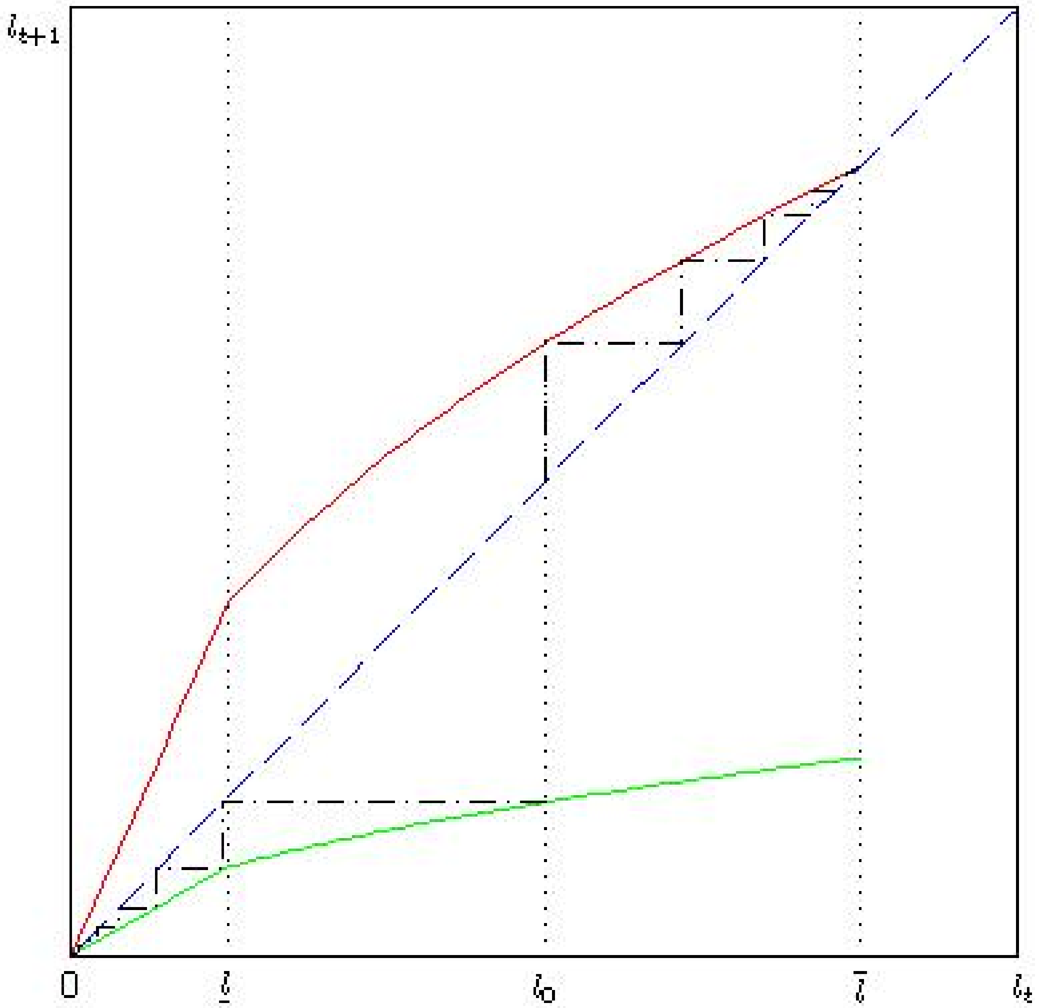

Figure 2