Competitive Nonlinear Taxation and Constitutional Choice

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Introduction

- There is an enormous variation in terms of the amount of redistribution pursued and achieved by different Governments.
- The variation in progressivity of redistribution systems can be related to
  - the institutional structure chosen at the constitutional stage
  - the initial conditions at the time of the constitutional choice.
- This paper aims to provide an analysis of the impact of key fiscal institutional choices on the difference in progressivity.
- Our analysis will clarify the *effects* of different tax regimes on redistribution as well as the *endogenous determination* of those institutions themselves.

Related Literature

- Following Mirrlees (1971), there is a growing literature on optimal income taxation with mobile labor and competition:
  - Without strategic competition between governments: e.g., Wilson (1980, 2006), Mirrlees (1982), Simula and Trannoy (2006), Krause (2007).
  - With strategic competition between governments: Piaser (2007), Brett and Weymark (2008).
- We consider competitive setting with both vertically and horizontally differentiated agents. We also incorporate constitutional choice in our analysis.
- Technically, our approach is close to Rochet and Stole (2002), and Yang and Ye (2008) on competitive nonlinear pricing.

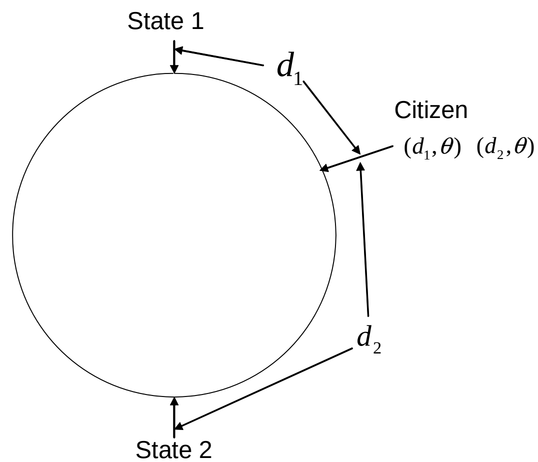
Outline of The Talk

- Three-type model
- Continuous-type model
- Extension to n-state case
- Concluding remarks

Three-type Model

- Consider two States in a potential Federation.
- Citizens differ in two dimensions:
  - Different citizens have different productivities (or abilities). This is captured by the *vertical* type $\theta \in \{\theta_H, \theta_M, \theta_L\}$. $\theta_H > \theta_M > \theta_L$. The corresponding proportions of three types are μ_H, μ_M and μ_L, respectively.
  - Different citizens have different preferences about which State to live (or to work). This is captured by the *horizontal* type d_i (the "distance" from State i in the Salop circular model). d_i is assumed to be uniformly distributed along a unit-length circle ($d_1 + d_2 = 1/2$).

Three-type Model, Cont.



- With labor (or effort) l, a citizen with productivity θ makes gross income (or production) $Q = \theta l$.
- Each State chooses a tax schedule $T(Q)$. Equivalently we may assume that each State makes a menu of offers/contracts with the form (C, Q).

Three-type Model, Cont.

- We restrict attention to deterministic contracts to restore the Revelation Principle in the competitive mechanism design setting.
- Let (C, Q) be an offer made by State i and accepted by a citizen of type (θ, d_i), $i = 1, 2$. The utility function for this citizen is given below:

$$U(C, Q, \theta, d_i) = u(C) - l - kd_i = u(C) - \frac{Q}{\theta} - kd_i$$

  - $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable.
  - k is the weight of location preferences in the utility function: the smaller the k, the less horizontally differentiated the two states are.

Three-type Model, Cont.

- The single crossing property holds in the vertical dimension only. Hence agents cannot be sorted along horizontal dimension. Without loss of generality, we can thus consider direct offers of the form $\{C(\theta), Q(\theta)\}$.
- In the unified taxation case, the tax schedules in two States are designed by the Federation.
- In the independent taxation case, each State decides on its own tax schedule independently.
- Autarky case: $C = Q$, and hence $u'(C^*(\theta)) = 1/\theta$.

Unified Taxation

- We focus on the symmetric solution in which the same tax schedule is offered for both States and the resulting "market shares" are symmetric.
- The Federation's objective is to set the pairs (C_j, Q_j), $j \in \{H, M, L\}$ to maximize the total social welfare

$$\max \mu_H \left[u(C_H) - \frac{Q_H}{\theta_H}\right] + \mu_M \left[u(C_M) - \frac{Q_M}{\theta_M}\right] + \mu_L \left[u(C_L) - \frac{Q_L}{\theta_L}\right]$$

subject to

$$\mu_H(Q_H - C_H) + \mu_M(Q_M - C_M) + \mu_L(Q_L - C_L) = 0 \quad \textit{(RC)}$$

$$\begin{aligned} u(C_H) - \frac{Q_H}{\theta_H} &\geq u(C_M) - \frac{Q_M}{\theta_H} \quad \textit{(DIC-H)} \\ u(C_M) - \frac{Q_M}{\theta_M} &\geq u(C_L) - \frac{Q_L}{\theta_M} \quad \textit{(DIC-M)} \end{aligned}$$

Unified Taxation: Main Results

- Solution does not depend on k.
- $u'(C_H) = 1/\theta_H$, $u'(C_M) > 1/\theta_M$, and $u'(C_L) > 1/\theta_L$. Hence, no distortion of consumption for type H, but the consumptions of type M and type L are both distorted downward.
- $u'(C_M) < u'(C_L)$. Hence $C_H > C_M > C_L$.
- Lemma 1: $T_H > T_M > T_L$. Hence $T_H > 0$, $T_L < 0$. The sign of T_M ambiguous.

Proof of Lemma 1: $T_H > T_M > T_L$.

Proof. Suppose $T_H \leq T_M$. That is, $Q_H - C_H \leq Q_M - C_M$. By the binding DIC-H,

$$
\begin{aligned}
& u(C_H) - u(C_M) = \frac{Q_H - Q_M}{\theta_H} \leq \frac{C_H - C_M}{\theta_H} \\
\Rightarrow \quad & u(C_H) - \frac{C_H}{\theta_H} \leq u(C_M) - \frac{C_M}{\theta_H}.
\end{aligned}
$$

But this contradicts the fact that $C_H = \arg\max_C \{u(C) - \frac{C}{\theta_H}\}$ ($u'(C_H) = 1/\theta_H$) and $C_M < C_H$. So $T_H > T_M$.

Proof of Lemma 1, Cont.

Similarly, suppose $T_M \leq T_L$, that is, $Q_M - C_M \leq Q_L - C_L$. By the binding DIC-M,

$$\begin{aligned} u(C_M) - u(C_L) &= \frac{Q_M - Q_L}{\theta_M} \leq \frac{C_M - C_L}{\theta_M} \\ \Rightarrow \quad u(C_M) - \frac{C_M}{\theta_M} &\leq u(C_L) - \frac{C_L}{\theta_M}. \end{aligned}$$

$u(C) - \frac{C}{\theta_M}$ is strictly concave, which means that $u(C) - \frac{C}{\theta_M}$ is strictly increasing in C for $C \leq C_M^*$. Since $C_L < C_M < C_M^*$, we have $u(C_M) - \frac{C_M}{\theta_M} > u(C_L) - \frac{C_L}{\theta_M}$. A contradiction. ■

Independent Taxation

- Under the independent taxation regime, each State chooses its taxation schedule simultaneously and independently.
- Our solution concept is Bertrand-Nash equilibrium: given the other State's tax schedule, each State chooses its tax schedule to maximize the total utilities of its residents, where the set of residents of each State is endogenously determined.
- We focus on symmetric equilibria in which both States choose the same tax schedule.

Independent Taxation, Cont.

- Define rent provision $v_j \equiv u(C_j) - Q_j/\theta_j$, $j = H, M, L$. Suppose the other State's taxation rule leads to v_H^*, v_M^* and v_L^*. Then the horizontal type x_j who is indifferent between the two States is determined by:

$$v_j - kx_j = v_j^* - k\left(\frac{1}{2} - x_j\right) \Rightarrow x_j = \frac{1}{4} + \frac{1}{2k}\left(v_j - v_j^*\right).$$

- x_j is the type-θ_j "market share" for State 1: all the types (θ_j, d_1) with $d_1 < x_j$ choose to live in State 1 and those with $d_1 > x_j$ choose to live in State 2.

Independent Taxation, Cont.

- Each State has the following programming problem:

$$\max \mu_H(x_H v_H - \frac{1}{2} k x_H^2) + \mu_M(x_M v_M - \frac{1}{2} k x_M^2) + \mu_L(x_L v_L - \frac{1}{2} k x_L^2)$$

where $x_j = \frac{1}{4} + \frac{1}{2k}\left(v_j - v_j^*\right), \quad j = H, M, L$

Subject to

$$u(C_H) - \frac{Q_H}{\theta_H} = u(C_M) - \frac{Q_M}{\theta_H}$$

$$u(C_M) - \frac{Q_M}{\theta_M} = u(C_L) - \frac{Q_L}{\theta_M}$$

$$\mu_H x_H(Q_H - C_H) + \mu_M x_M(Q_M - C_M) + \mu_L x_L(Q_L - C_L) = 0.$$

Independent Taxation: Main Results

- No distortion of consumption for type H, but the consumptions of type M and L are both distorted downward.
- $T_H^I > T_M^I > T_L^I$. Hence $T_H^I > 0$, $T_L^I < 0$. The sign of T_M^I is ambiguous.
- Lemma 2: $C_L^I > C_L^U$ and $C_M^I > C_M^U$.
- Proposition 1: $T_H^I < T_H^U$. That is, type H pays a lower tax in the independent State regime. $v_H^I > v_H^U$.
- Proposition 2: $T_L^I > T_L^U$. That is, type L receives less subsidy under independent taxation. $v_L^I < v_L^U$.
- Corollary: Equilibrium welfare is always greater under the unified taxation than under the independent taxation.

Proof of Proposition 1: $T_H^I < T_H^U$.

Proof. By RC, DIC-H, and DIC-M, we have

$$\begin{aligned} Q_H &= \mu_H C_H + \mu_M C_M + (1-\mu_H)\theta_H[u(C_H) - u(C_M)] \\ &\quad + \mu_L[C_L + \theta_M(u(C_M) - u(C_L))] \end{aligned}$$

Define $\Delta Q_H = Q_H^I - Q_H^U$, $\Delta C_j = C_j^I - C_j^U$, $j = H, M, L$.

Proof of Proposition 1, Cont.

$$
\begin{aligned}
\Delta Q_H &= Q_H^I - Q_H^U = \mu_M \Delta C_M + \mu_L \Delta C_L \\
&\quad - \left[(1-\mu_H)\theta_H - \mu_L \theta_M\right] \Delta u(C_M) - \mu_L \theta_M \Delta u(C_L) \\
&\leq \mu_M \Delta C_M + \mu_L \Delta C_L \\
&\quad - \left[(1-\mu_H)\theta_H - \mu_L \theta_M\right] u'(C_M^I) \Delta C_M - \mu_L \theta_M u'(C_L^I) \Delta C_L \\
&< \mu_M \Delta C_M + \mu_L \Delta C_L \\
&\quad - \left[(1-\mu_H)\theta_H - \mu_L \theta_M\right] \frac{1}{\theta_M} \Delta C_M - \mu_L \theta_M \frac{1}{\theta_L} \Delta C_L \\
&= (1-\mu_H)\frac{\theta_M - \theta_H}{\theta_M} \Delta C_M + \mu_L \left(\frac{\theta_L - \theta_M}{\theta_L}\right) \Delta C_L < 0 = \Delta C_H
\end{aligned}
$$

■

Constitutional Choice

- Suppose the taxation system is chosen by majority rule at the constitutional stage, and if $\mu_i < 1/2$ for $i = H, L$, then the independent taxation regime can be chosen if and only if it yields higher equilibrium utility for the middle class.
- Type M prefers the independent taxation system if and only if $\theta_M > \theta_M^*$ for some θ_M^* or $\mu_M > \mu_M^*$ for some μ_M^*.
  - Two countervailing incentives for the middle type
  - A country with "better" initial conditions (higher average θ) may end up with lower welfare: Suppose $\theta_H = 2, \theta_L = 1$ and $\theta_M = 1.51$. The utilitarian welfare of a Federation with $\mu_M^* - \epsilon$ is higher than that in a competitive taxation regime with $\mu_M^* + \epsilon$, for ϵ small enough.

Constitutional Choice, Cont.

- The implications of poverty: as the population of the low type increases, the middle type becomes more eager to prefer independent tax regime.
- $\theta^*_M(k)$ and $\mu^*_M(k)$ are both decreasing in k: the smaller the intensity of location preferences, the more likely that the unified regime will emerge.
- Lower k would push towards unification, but the middle class is likely to go for that only if μ_L not too large, or if μ_H not too small. This set of results fits our intuition about the situation within the European Union.

Continuum of Types

- θ is distributed on $[\underline{\theta}, \overline{\theta}]$ with density function $f(\theta)$, where $f(\theta)$ is continuous, strictly positive everywhere in the support.
- IC requires

$$\begin{aligned} V(\theta, \theta, d_i) &\geq V(\theta, \widehat{\theta}, d_i) \ \ \forall (\theta, \widehat{\theta}) \in [\underline{\theta}, \overline{\theta}]^2 \text{ iff} \\ V(\theta, \theta, 0) &\geq V(\theta, \widehat{\theta}, 0) \ \ \forall\ (\theta, \widehat{\theta}) \in [\underline{\theta}, \overline{\theta}]^2 \end{aligned}$$

- Let $v(\theta)$ denote the rent provision to type-$(\theta, 0)$. Then

$$v(\theta) = V(\theta, \theta, 0) = u(C) - \frac{Q}{\theta}.$$

Continuum of Types, Cont.

- By the Constraint Simplification Theorem, the IC conditions are equivalent to the following two conditions:

$$\begin{aligned} v'(\theta) &= \frac{Q(\theta)}{\theta^2} = \frac{1}{\theta}[u(C(\theta)) - v(\theta)] \\ Q'(\theta) &\geq 0 \end{aligned}$$

- Given $v(\theta)$, $Q(\theta)$ and $C(\theta)$ are both uniquely determined.

Unified Taxation

- The central authority's maximization program:

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\frac{1}{4}} [v(\theta) - kd_1] dd_1 f(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1}{4} v(\theta) - \frac{k}{32}\right] f(\theta) d\theta$$

$$s.t. \;\; v'(\theta) = \frac{1}{\theta}[u(C(\theta)) - v(\theta)], \;\; Q'(\theta) \geq 0$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{4}[\theta(u(C) - v) - C] f(\theta) d\theta = 0$$

- Define new state $J(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1}{4}[\theta(u(C) - v) - C] f(\theta) d\theta$. Then $J(\overline{\theta}) = 0$ and $J(\underline{\theta}) = 0$. The Hamiltonian can now be written as follows.

$$H = \left[\frac{1}{4} v(\theta) - \frac{k}{32}\right] f(\theta) + \lambda \frac{1}{\theta}\left[u(C(\theta)) - v(\theta)\right] + \mu \frac{1}{4}[\theta(u(C) - v) - C] f(\theta)$$

Unified Taxation, Cont.

The necessary conditions for optimality are as follows:

$$\begin{aligned}
\frac{\partial H}{\partial C} &= \frac{\lambda}{\theta}u'(C) + \frac{1}{4}\mu[\theta u'(C) - 1]f = 0 \\
\lambda' &= -\frac{\partial H}{\partial v} = -\frac{1}{4}f + \frac{\lambda}{\theta} + \frac{1}{4}\mu\theta f \\
\mu' &= -\frac{\partial H}{\partial J} = 0 \\
\lambda(\underline{\theta}) &= \lambda(\overline{\theta}) = 0
\end{aligned}$$

which yields

$$-\frac{u''(C)}{[u'(C)]^2}C' = 2 - \frac{1}{\mu\theta} + \frac{f'}{f}\left[\theta - \frac{1}{u'(C)}\right] \tag{1}$$

Unified Taxation, Cont.

- Consider: $\theta \sim U[0.4, 1.4]$, and $u(C) = 2\sqrt{C}$. Then (1) becomes

$$-\frac{1}{2}\mu\theta C^{-\frac{1}{2}}C' + 2\mu\theta - 1 = 0 \tag{2}$$

Solving, we have

$$\begin{aligned} C(\theta) &= \left[2\theta - 1.1314 - \frac{\log\theta}{1.2528}\right]^2 \\ Q(\theta) &= 2\theta^2 - 1.5964\theta + b \end{aligned}$$

where b is determined by the resource constraint.

  – It can be verified that $Q'(\theta) > 0$ and $T'(Q) = T'(\theta)/Q'(\theta) > 0$ for $\theta \in (.4, 1.4)$. By (RC) there must be a cutoff type $\widehat{\theta}_u \in (0.4, 1.4)$ such that $T(\theta) < 0$ for $\theta \in [0.4, \widehat{\theta}_u)$ and $T(\theta) > 0$ for $\theta \in (\widehat{\theta}_u, 1.4]$.

Independent Taxation

- Again we focus on symmetric equilibria, in which the two States choose the same tax schedule.
- Suppose State 2's rent provision is given by $v^*(\theta)$. Then if State 1 offers rent provision $v(\theta)$, the type-θ "market share" for State 1 is given by (twice of)

$$\eta(\theta) = \frac{1}{4} + \frac{1}{2k}[v(\theta) - v^*(\theta)] \tag{3}$$

Independent Taxation, Cont.

- State 1's maximization problem:

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \int_{0}^{\eta(\theta)} [v(\theta) - kd_1] dd_1 f(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left[v(\theta)\eta(\theta) - \frac{k}{2}\eta(\theta)^2 \right] f(\theta) d\theta$$

$$\text{s.t. } v'(\theta) = \frac{1}{\theta}[u(C) - v], \quad Q'(\theta) \geq 0$$

$$J'(\theta) = [\theta(u(C) - v) - C]\eta f(\theta)$$

$$J(\underline{\theta}) = 0, \; J(\overline{\theta}) \geq 0$$

where $J(\theta) = \int_{\underline{\theta}}^{\theta} [\theta(u(C) - v) - C]\eta(\theta) f(\theta) d\theta$.

- Hamiltonian:

$$H = \left(v\eta - \frac{k}{2}\eta^2 \right) f + \frac{\lambda}{\theta}[u(C) - v] + \mu\eta[\theta(u(C) - v) - C] f$$

Independent Taxation, Cont.

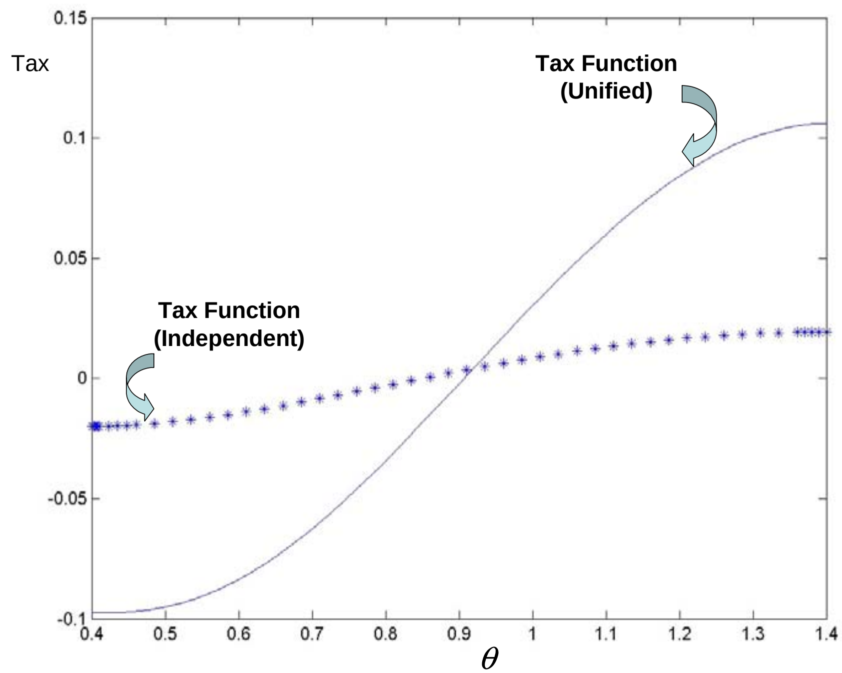
- The optimality conditions for symmetric equilibrium are:

$$
\begin{aligned}
\frac{\partial H}{\partial C} &= \frac{\lambda}{\theta}u'(C) + \mu\eta[\theta u'(C) - 1]f = 0 \\
\lambda'(\theta) &= -\frac{\partial H}{\partial v} = -\left[\frac{v}{2k} + \frac{\eta}{2}\right] f + \frac{\lambda}{\theta} - \frac{\mu}{2k}[\theta(u(C) - v) - C]f + \mu\eta\theta f \\
\mu'(\theta) &= -\frac{\partial H}{\partial J} = 0 \Rightarrow \mu \textit{ is a constant} \\
\text{where } \eta &= \frac{1}{4}. \\
\lambda(\underline{\theta}) &= \lambda(\overline{\theta}) = 0
\end{aligned}
$$

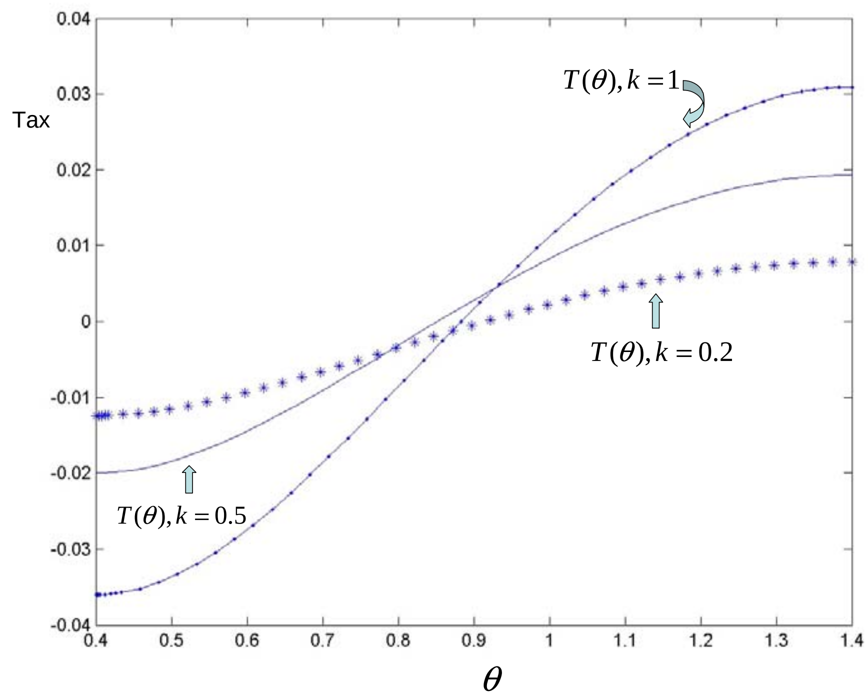
- For $\theta \sim U[0.4, 1.4]$, and $u(C) = 2\sqrt{C}$, we have

$$
\begin{aligned}
v' &= \frac{1}{\theta}[2\sqrt{C} - v] \\
C' &= 4\sqrt{C} - \frac{k + 4v + [\theta(2\sqrt{C} - v) - C]}{2k\mu\theta}\sqrt{C} \\
C(0.4) &= 0.16, C(1.4) = 1.96.
\end{aligned}
$$

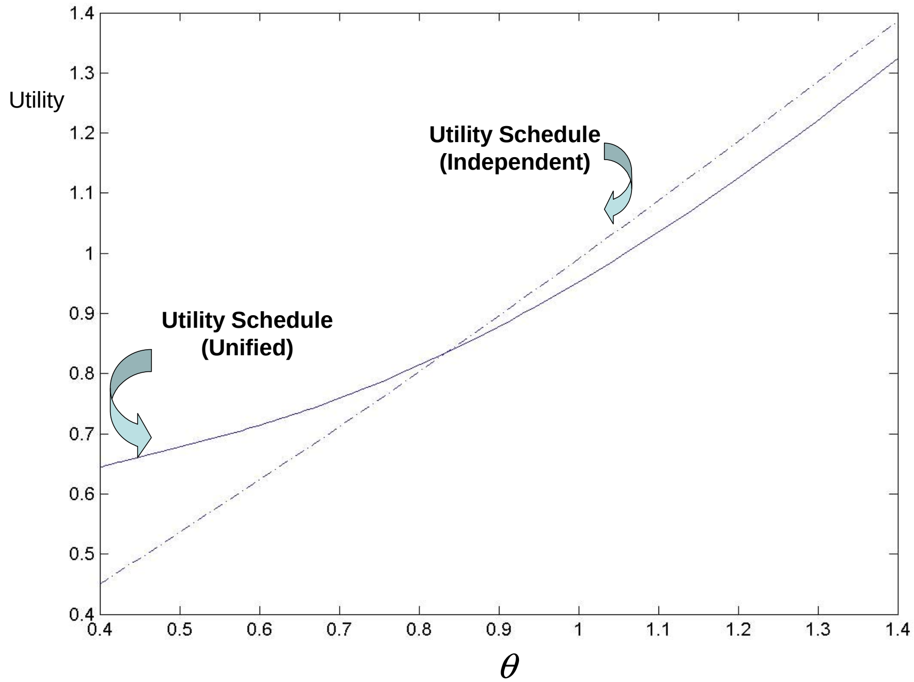
Independent system has less progressive tax



The smaller k, the less progressive tax in the independent system



Higher types are better off in the independent system



The effects of Poverty

- To examine how changes in the (type) income distribution affect the constitutional choice, we consider the distribution family

$$f_a(\theta) = \frac{1}{3(1 - a/2)}(3 - a\theta), \theta \in [1, 2]$$

  $a \in [0, 1.5)$. As a increases, the distribution is tilted toward more poor people.

- Simulation results (θ^* is the cutoff type indifferent between the two tax regimes, and θ_m is the median type):

	$a = 0$	$a = 1$	$a = 1.3$	$a = 1.4$
θ^*	1.463	1.4205	1.3705	1.362
θ_m	1.5	1.4189	1.3578	1.3284

The effects of Inequality

- To study how the degree of inequality affects constitutional choice, we consider a mean-preserving spread family of distributions:

$$f_a(\theta) = \frac{1}{3 - a/12}[3 - a(1.5 - \theta)^2], \theta \in [1, 2]$$

$a \in [0, 12)$. As a increases, the distribution becomes more concentrated around the mean (median) 1.5 (equality increases).

- Simulation results:

	$a = 0$	$a = 8$	$a = 10$	$a = 11$
θ^*	1.463	1.446	1.4345	1.4185

Extension to n-State Case

- The locations of n States ($n \geq 2$) evenly split the unit-length circle.
- For the unified taxation model, the Federation's optimization program does not involve n.
- For the independent taxation model, define

$$k' = k/n$$

then in terms of k' the FOCs or ODEs are exactly the same as their counterparts in the two-State case (where $k' = k/2$).

  - The effect of an increase in n (while holding k fixed) on the equilibrium is exactly the same as the effect of a decrease in k on the two-State equilibrium.
  - Consider $\theta_M = \theta^*_M(n) + \epsilon$: non-monotonicity of preference.

Concluding Remarks

- We provide a first analysis of nonlinear income taxation when strategic authorities compete for resident citizens along both the vertical productivity dimension and the horizontal location dimension.
- We explore the relative importance of these two dimensions for the degree of progressivity of the tax system.
- The model allows us to discuss the incentives of different classes of agents to advocate for different tax systems at the constitutional stage.
- Extensions
  - Asymmetric initial conditions
  - Empirical study