Public-Private Partnerships: Task Interdependence and Contractibility

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Abstract

We examine the proper scope of public-private partnerships in the context of a project consisting of two tasks, building and operation of a facility. We investigate the optimal arrangement regarding bundling versus unbundling and private ownership versus public ownership. Like Bennett and Iossa (2006), we assume that the innovative activity in the building stage has impacts on, among other things, the subsequent operational cost. The novelty is that we relax the nature of task interdependence and study different contractual frameworks. The general insight is that given limitation in contractibility, contrary to common sense, complementarity between tasks favors unbundling over bundling.

JEL classification: D23; H11; L33

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1 Introduction

Nowadays, it is common across countries that governmental agencies collaborate with the private sector to deliver public services; in some cases, even the whole project is contracted out to a single firm that takes responsibilities for all involved tasks, such as both building and maintaining the facility. In the literature of public-private partnerships (PPPs), as this practice is usually referred to, two issues have received

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much attention: multitasking and investment contractibility. Given multiple tasks – such as building and subsequent maintenance of a facility – an important question is whether the tasks should be handled by a single consortium (in case of bundling) or by two separate firms (in case of unbundling). This question of course depends on the contractibility of and the relationship between tasks, as is shown in the literature. In this paper, we further examine this question, improving on existing work.

Our paper is closely related to the piece by Bennett and Iossa (2006), in which two non-contractible innovation activities (or investments in short), one in the building stage and the other in the operating stage, are supposed to reduce cost and enhance quality. Assuming a sort of task externality so that the investment in the building stage may increase or decrease the cost in the operating stage, the paper shows that, with positive externality, it is more efficient for the tasks to be bundled; with negative externality, it is more efficient for the tasks to be separated. In another paper where both operational costs and service quality are contractible, Martimort and Pouyet (2008) also show similar results.

However, the relationship between tasks can be richer. They may be interdependent, being substitutes, such that making more of one investment will decrease the returns of making more of another investment. For example, a hospital may be built in a more specified manner so that, while the subsequent operational cost is generally lower (i.e., positive externality), further enhancement of quality or alternation of usage would be more difficult to achieve. However, the two tasks may be interdependent, sharing complementaries, such that making more of one investment will increase the returns of making more of another investment. For instance, a school may be built with better-quality and more-expensive-glass windows so that, whereas the subsequent operational cost is generally lower (i.e., positive externality), an increase in guard services during the operating stage may be more valuable as it prevents a greater loss from pupils’ vandalism. In this paper, we examine the implications of task interdependence that allows for substitutability and complementarity; another novelty is the way we model investment contractibility, which will be clear in a moment.

To briefly illustrate how task interdependence matters, let us revisit the contractual framework in Bennett and Iossa. Consider the case of task complementarity. In case of unbundling, the builder could bargain with the manager or the government. After the bargaining, the former party could share the benefits generated by the manager’s investment, while not bearing any cost incurred by such investment. Because of complementarity, a higher building investment leads to a higher operating investment, yielding a greater net surplus to be split. Anticipating more rents to
be extracted from the manager’s investment, the builder has a greater incentive to invest. As a result, investment complementarity helps mitigate the underinvestment problem of the builder under unbundling. In the case of bundling, on the contrary, when investing in the building stage, the consortium will internalize not only the benefits but also the cost of subsequent investment, resulting in a dampened investment incentive on his part. Thus, at the margin, task complementarity favors unbundling, relative to bundling. Notice that because task complementarity can be viewed a special kind of positive externality, this result sheds new, somewhat counter-intuitive, light to the issue on PPPs.

In the main body of this paper, we focus on a contractual framework somewhat different from Bennett and Iossa; we assume that the operation task becomes contractible subsequent to the building stage. Examples from construction sectors show that the contract on service provision is usually finalized until the infrastructure is in place. Moreover, even though a contract specifies the operating task in advance of the project, it may still be subject to adaptation and renegotiation after the construction is carried out. These observations are consistent with the idea that the requirements regarding the successive operation task become revealing as time goes by.\footnote{One fact, which is pointed out by Neher (1999), is that as the project matures, more human capital is converted into physical assets, making the alienable (contractible) elements of the project manifested.} We think that the framework of "interim contractibility" is worthwhile studying (see Iossa and Martimont 2008 on a discussion). To check the robustness of our results, we also examine the role of task interdependence in the incomplete contracting framework (as in Bennett and Iossa) and in complete contracting frameworks (as in Martimort and Pouyet and in Schmitz 2005).

In our framework of interim contractibility, if two separate firms are used, it would be more convenient for the government to set directly the level of operating investment in the middle of the game. On the contrary, if a consortium is used, while the underinvestment problem in the building stage may be mitigated, the consortium’s default choice of operating investment may deviate from the efficient level; in this case, mutually beneficial renegotiation between the government and the consortium is expected. Thus, the choice of optimal regime should trade off the flexibility embodied in the separation case and the incentive enhanced in the integration case. In this contractual framework, our findings suggest that under private ownership, task externality, as well as task interdependence, still plays an important role in shaping the trade-offs between integration and separation; however, under public ownership, the difference between integration and separation vanishes.
Besides the bundling versus unbundling problem, this paper also reexamines, in different contractual frameworks, whether the project should be privately owned or publicly owned. In general, our main results are consistent with those in Bennett and Iossa, where a larger residual value effect and a smaller social value effect favor private ownership, and public ownership is favored when the opposite is true.

This paper belongs to the strand of literature that investigates either desirable contracting schemes in the public-private partnership (i.e., Hart 2003, Martimort and Pouyet, Bentz, Grout and Halonen 2001, Iossa and Martimort 2008, Hoppe and Schmitz 2008) or optimal ownership structures in the public-private partnership (i.e., Hart, Shleifer and Vishny 1997, Francesconi and Muthoo 2006, Besley and Ghatak 2001, Bennett and Iossa). But none of these papers has addressed exactly the same questions we plan to address here.

Our research is also related to the papers that study the holdup problem of sequential specific investments. As pointed out by Smirnov and Wait (2004), if an initial investment is made, contracting on the subsequent investment becomes possible, since the resolution of the incompleteness is facilitated by the completion of some aspects of the project. They address a different question about whether the parties should make investments simultaneously or sequentially. Another relevant paper is De Fraja (1999), who find that if specific investments are made sequentially, ex ante contracting can solve the holdup problem even though there exist two-sided direct externalities across investments. Here, we study the interrelationship among three parties (namely, two investors and one principal), instead of between two, leading to different results.

The remainder of this paper is organized as follows. Section 2 presents the main model. Section 3 examines the optimal regime when the operation task becomes contractible after the building task is completed. Section 4 briefly discusses the issue in the framework used by Bennett and Iossa where all the tasks are non-contractible. Section 5 addresses the issue from a complete contract perspective. Section 6 concludes.

2 Model

We follow Bennett and Iossa’s model closely. A governmental agency (hereafter, the government) is contemplating a project that, upon its completion, will enhance

\footnote{Disagreeing with De Fraja, Che (2000) argues that the contract suggested by De Fraja provides almost no incentive for specific investments when they exhibit sufficiently large direct externalities. Che proposes an alternative contract. For other papers on the sequential specific investments, please see Lulfesmann (2004) etc.}
social benefit. The project consists of two sequential tasks, namely, "building" and "operating" a facility. The facility of a minimum standard can be built at a certain cost, and the completed facility can be operated at a certain cost as well. However, prior to the actual building, the builder can undertake an innovative activity that increases both the social benefit and the residual value of the facility and affects the costs and efficacy of the following operation task (the residual value refers to the value of the facility to the owner upon the expiration of the project). Likewise, subsequent to the completion of building and prior to the actual operation, the manager can undertake an innovative activity that reduces the operational cost and increases the social benefit. We use $a$ and $e$ to denote the level (also the cost) of innovative activity in the building stage and in the operating stage, respectively, and these two activities can be undertaken by two separate firms or by a consortium.  

Let $C(a, e)$ be the operational cost of the facility borne by the manager (or operator) in the operating stage. We assume that

$$C(a, e) = C_0 - d(a, e),$$

where $C_0$ is the positive default cost, and $d(a, e)$ is the reduction of operational cost caused by the investments of $a$ and $e$. The function $d(a, e)$ is three-order differentiable, satisfying the following properties:

(i) $d(0, 0) = 0$.
(ii) $d_2(a, e) > 0$, $d_2(a, 0) = \infty$, $d_2(a, \infty) = 0$, $d_{22}(a, e) < 0$.
(iii) If $d_1(a, e) > 0$, then $d_1(0, e) = \infty$, $d_1(\infty, e) = 0$, $d_{11}(a, e) < 0$; if $d_1(a, e) < 0$, then $d_1(0, e) = 0$, $d_1(\infty, e) = -\infty$, $d_{11}(a, e) < 0$.

This operational cost function generalizes that in Bennett and Iossa, since we allow not only the direct effect of $a$ on the operational cost but also the cross-terms between $a$ and $e$. In particular, if $d_1(a, e) > 0$, $a$ reduces the operational cost; if $d_1(a, e) < 0$, $a$ increases the operational cost. These positive and negative externalities are studied in Bennett and Iossa. Our focus, however, is the role of cross partials. If $d_{12}(a, e) > 0$, $a$ and $e$ are complementary investments; in this case, an increase in $a$ increases the marginal benefit of $e$. It is also conceivable that $d_{12}(a, e) < 0$, i.e., $a$ and $e$ are substitute investments. In this case, an increase in $a$ reduces the marginal benefit

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3 We assume that when the builder and the manager form a consortium, they act as one person. This assumption is also used in Bennett and Iossa, as well as in the main part of Martimort and Pouyet that we will contrast with in Section 5. This view of integration, however, is different from Grossman and Hart (1986). In general, integration and non-integration are different ownership structures, and there should still be incentive problems within an integrated firm. An alternative interpretation of our case is that we are indeed asking whether the two tasks should be assigned to one agent or to two separate agents.
of $e$. Note that in Bennett and Iossa, $d_{12}(a,e)$ is assumed to be zero. In our more generalized framework, there are four different possible relationships between $a$ and $e$: (i) $d_1(a,e) > 0$, $d_{12}(a,e) > 0$; (ii) $d_1(a,e) > 0$, $d_{12}(a,e) < 0$; (iii) $d_1(a,e) < 0$, $d_{12}(a,e) > 0$; and (iv) $d_1(a,e) < 0$, $d_{12}(a,e) < 0$ (the knife-edge cases with zero derivatives being omitted).

Three remarks are in order. First, we are agonistic as to which technology regime is more prevalent; in any case, this is an empirical issue and can only be settled by a careful examination of the public project we have at hand. Second, instead of studying the choice among these four technology regimes, we address the following question: given the technology regime, what kind of public-private relationship is most efficient. Third, to facilitate exposition, we will use the following nomenclature unless otherwise stated. *Task or investment externality* refers to the case that $d_1(a,e)$ is nonzero; in particular, positive (negative) externality refers to where $d_1(a,e) > 0$ ($d_1(a,e) < 0$). *Task or investment interdependence* refers to the case that $d_{12}(a,e)$ is nonzero; in particular, complementarity (substitutability) refers to where $d_{12}(a,e) > 0$ ($d_{12}(a,e) < 0$).

Next we assume that the project generates the following social benefits:

$$B(a,e) = B_0 + u(a) + v(e),$$

where $B_0$ is the positive default benefit; $u(0) = v(0) = 0$; $u'(a), v'(e) > 0$; $u''(a), v''(e) < 0$; $u'(0) = v'(0) = \infty$; $u'(\infty) = v'(\infty) = 0$.

The residual value of the facility is claimed and taken away by the owner when the project expires. For simplicity, we continue to assume that this value depends on $a$ but not on $e$; given the choice of $a$, the residual value equals

$$R(a) = R_0 + t(a),$$

where $R_0$ is the positive default residual value, $t(0) = 0$; $t'(a) > 0$, $t''(a) < 0$; $t'(0) = \infty$; $t'(\infty) = 0$. We make an assumption on the relationship between $t(a)$ and $d(a,e)$.

**A1** $t'(a) + d_1(a,e) > 0$.

This assumption implies that, although we entertain the possibility that $a$ increases the operational cost, this negative externality is only moderate. (Similar

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4 We maintain the functional form in Bennett and Iossa for simplicity. It is possible to consider a more general functional form of social benefit by allowing the cross-terms between $a$ and $e$. One can show that this assumption is not crucial in driving the main results.
assumption is used in Bennett and Iossa. As will be clear, this assumption ensures that if the consortium owns the facility it is always in his interest to adopt $a$.)

Notice that the first-best investments $(a^*, e^*)$ satisfy the following two equations:

$$u'(a^*) + t'(a^*) + d_1(a^*, e^*) = 1$$  \hfill (1)

and

$$v'(e^*) + d_2(a^*, e^*) = 1.$$  \hfill (2)

We assume that unique, interior solutions to these two equations exist.

Assume that both innovative activities $a$ and $e$ are observable so that, once chosen, there is no ambiguity about their values. These investments, however, could be non-verifiable so that no contracts can be made contingent on them. Whereas these assumptions are standard in the literature, what is novel here is that we assume that $e$ is \textit{interim contractible}; that is, $e$ is contractible if and only if $a$ has been chosen. The underlying reason is that once the facility is built, such fundamentals as operational costs or social benefits become describable.\footnote{Since we here do not assume any contracting frictions due to limited liability or risk aversion, a contract conditional on the level of operational cost is equivalent to that conditional on the level of $e$.}

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\textbf{Figure 1}

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The game can be summarized by a sequence of events by the time line in Figure 1. At time 0, the regime is determined. The various regimes include: separation with builder ownership ($SB$), separation with manager ownership ($SM$), separation with public ownership ($SP$), integration with consortium ownership ($IC$), and integration with public ownership ($IP$) (where separation means unbundling and integration means bundling). Moreover, the government can only sign contracts that specify the minimum standards and promise fixed payments based on such default values as $C_0$, $B_0$, and $R_0$.

At time 0.5, investment $a$ is chosen. At time 1, after the chosen $a$ becomes observable, the builder (or the consortium) bargains with the owner over the adoption of $a$. At time 2, the requirements on the operation task become revealed; consequently, the government steps in. In case the owner is the government or the manager (or the consortium), the government simply signs a complete contract with the latter on the choice and adoption of the operating effort $e$; in case the owner is the builder,
the government first signs a complete contract with the manager on the choice of \( e \), followed by a complete contract with the builder on the adoption of \( e \). At time 2.5, investment \( e \) is chosen. At time 3, in case no agreement on the adoption of \( e \) was reached at time 2, the manager (or the consortium) bargains with the owner over the adoption of \( e \). At time 4, the contractual relationship between the government and the agents ends, and all the payoffs are realized. As explained, we assume that bargaining is always bilateral, despite three stakeholders under some regimes.\(^6\) We also note that the costs of investing \( a \) or \( e \) are paid when the investment is chosen and becomes sunk.

Given \( a \), the efficient level of \( e \), denoted by \( \bar{e}(a) \), is given by

\[
v^I(\bar{e}(a)) + d_2(a, \bar{e}(a)) = 1. \tag{3}
\]

Given the interim contractibility of \( e \), the government could persuade the manager or the consortium to invest and implement \( \bar{e}(a) \) regardless of the ownership and organizational structure. The reason is that now that \( e \) is contractible, the presence of efficiency gain to share will make the involved parties reach a mutually beneficial agreement. For clarity of exposition, we use superscript \( O \) to denote the choice of operating investment in case the government failed to persuade the manager to choose \( \bar{e}(a) \).

We apply the Nash bargaining solution to calculate the ex post payoffs to any parties in a negotiation. In this case, the two involved parties are assumed to have equal ex post bargaining power. Two special features of the bargaining solution are worth mentioning. First, to best compare and contrast our results, we follow Bennett and Iossa in assuming that if the investor is the owner, he cannot commit not to implement his own investment (i.e., his investment will be implemented without going through any bargaining);\(^7\) on the other hand, whenever the investor and the owner are not the same, they will negotiate on the adoption of the investment. Second, we implicitly assume that, the owner cannot unilaterally use the innovation without the agreement of the investor.\(^8\)

\(^6\)In the builder ownership case, for instance, the stakeholders are the builder, the operator, and the government. One can imagine a scenario in which the three parties bargain among themselves at some point of time. Beyond the scope of this paper, such a setting is reasonable and deserves future studies. Because of this, our results should be read with care.

\(^7\)Without loss of generality, we use the pronoun "she" to represent the government, and use the pronoun "he" to represent the firms (or agents).

\(^8\)It should be noticed that this assumption, while also used in Bennett and Iossa, is in contrast with Aghion and Tirole (1994). An underlying reason is that the investment has some component of human capital, which makes it impossible to materialize without its investor’s final cooperation.
3 Interim contractibility

We use $g$ to denote the ex post payoff to the government; $f_b$ and $f_m$, respectively, to denote the ex post payoffs to the builder and the manager in the case of two separate agents; $f$ to denote the ex post payoff to the consortium; and $\Omega$ to denote the total surplus generated by the investments of $a$ and $e$. To simplify our presentation, we omit such default values as $C_0, B_0, R_0$ in the discussion of payoffs.

3.1 Investments under private ownership

In this subsection, ownership is assumed to be allocated to the investors. There are three possible regimes — separation with builder ownership, separation with manager ownership, and integration with consortium ownership. Here, we do not consider the manager-ownership case, since it is at least weakly dominated by some other regime. The intuition is simple. Given that $e$ is interim contractible, the manager is always provided with the right incentive to invest. While not improving this incentive further, conferring ownership to the manager complicates matters due to the likelihood that the builder’s investment incentive is reduced. Thus, so long as no confusion is caused, we refer interchangeably to the builder-ownership case and to the separation case under private ownership.

3.1.1 Ownership by the builder

Suppose that the tasks of building and operation are contracted out separately, and the builder owns the facility. We first notice that the builder, with his ownership, would unilaterally implement $a$, since he always gets a positive payoff (such as residual values $t(a)$) from doing so. Thus, negotiation does not happen on this issue.

Consider the operating stage. In the absence of any agreement made at the beginning of this stage (i.e., time 2 in Figure 1), the builder would extract one half of the net surplus generated by $e$ through the ex post negotiation (at time 3) over its adoption.\(^9\) In this case, anticipating such bargaining, the government would contract with a manager to choose $e = e_{SB}^O(a)$, which satisfies

\[
\frac{1}{2} \left[ v'(e_{SB}^O(a)) + d_2(a, e_{SB}^O(a)) \right] = 1. \tag{4}
\]

\(^9\)We assume that this ex post negotiation takes place between the builder and the manager. Alternatively, we can model the negotiation as one between the builder and the government. So long as the assumptions are consistent made over the whole game, the two modelings do not make a difference. The underlying reason is that the interests of the government and the manager can be aligned by complete contracting prior to this negotiation.
Notice that $e_{SB}^O(a)$ is necessarily smaller than $\bar{e}(a)$. Totally differentiating (4), we obtain

$$\frac{\partial e_{SB}^O}{\partial a} = -\frac{d_{21}(a, e_{SB}^O)}{v''(e_{SB}^O) + d_{22}(a, e_{SB}^O)}.$$  

(5)

It indicates that, an increase in $a$ leads to an increase in $e_{SB}^O$ in case of complementarity and to a reduction of $e_{SB}^O$ in case of substitutability.

However, given the interim contractibility of the operation task, the government could solicit the builder’s approval of adopting $e^O(a)$ prior to its choosing. As a result, the negotiation between the government and the builder should take place at the beginning of the operating stage (at time 2), instead of its end. In the bargaining, the threat point payoff of the government is

$$\Omega|_{e=\bar{e}} - \Omega|_{e=e_{SB}^O} = [v(\bar{e}) - v(e_{SB}^O)] + [d(a, \bar{e}) - d(a, e_{SB}^O)] - (\bar{e} - e_{SB}^O).$$

By using the Nash bargaining solution, the ex post payoff of the builder is,

$$f_b = t(a) + \frac{1}{2} [v(\bar{e}) + d(a, \bar{e}) - d(a, 0) + e_{SB}^O - \bar{e}] - a.$$

Thus, the builder maximizes $f_b$ subject to (3) and (4) by choosing $a = a_{SB}$, which satisfies

$$t'(a_{SB}) + \frac{1}{2} [d_1(a_{SB}, \bar{e}) - d_1(a_{SB}, 0)] + \frac{1}{2} \frac{\partial e_{SB}^O}{\partial a} = 1.$$  

(6)

Since $a_{SB}$, together with $\bar{e}$ and $e_{SB}$, is determined by (3), (4), and (6), there are two interesting observations. First, the function $u(.)$ does not appear in any of the three equations, and hence $a_{SB}$ is independent of $u(.)$. In other words, the builder fails to internalize the social benefit of his investment. Second, from (6), $a_{SB}$ is increased by task complementarity (when $\partial e_{SB}^O/\partial a > 0$ and $d_1(a_{SB}, \bar{e}) - d_1(a_{SB}, 0) > 0$) and is decreased by task substitutability (when $\partial e_{SB}^O/\partial a < 0$ and $d_1(a_{SB}, \bar{e}) - d_1(a_{SB}, 0) < 0$).

### 3.1.2 Ownership by the consortium

Suppose a consortium undertakes both building and operation and also owns the facility. In this case, as guaranteed by A1, the adoption of $a$ always yields a positive gain to the consortium, so bargaining does not happen on the issue. However, after the building task is completed, the government may negotiate (at time 2) with the consortium to recontract the operation task. The reason is that, in the absence of
such bargain, the consortium would ignore the social benefit and set $e = e_{IC}^O(a)$, which solves

$$d_2(a, e_{IC}^O(a)) = 1. \quad (7)$$

Totally differentiating (7), we obtain

$$\frac{\partial e_{IC}^O}{\partial a} = -\frac{d_{21}(a, e_{IC}^O)}{d_{22}(a, e_{IC}^O)}. \quad (8)$$

Clearly, $e_{IC}^O(a) \leq \overline{e}(a)$. Thus, signing a contract that adjusts the choice of $e$ to its efficient level, $\overline{e}(a)$, yields net gains to the government and the consortium. Foreseeing the corresponding negotiation (see the appendix for more detail), the consortium will choose $a = a_{IC}$, which satisfies

$$t'(a_{IC}) + \frac{1}{2} [d_1(a_{IC}, \overline{e}) + d_1(a_{IC}, e_{IC}^O)] - \frac{1}{2} v'(e_{IC}^O) \frac{\partial e_{IC}^O}{\partial a} = 1. \quad (9)$$

Notice that $a_{IC}$, together with $\overline{e}$ and $e_{IC}$, is determined by (3), (7), and (9). There are two interesting observations. First, the function $u(\cdot)$ does not appear in any of the three equations. Hence, $a_{IC}$ is independent of $u(\cdot)$, which means the consortium also fails to internalize the social benefit of $a$. Second, from (9), $a_{IC}$ is decreased by task complementarity (when $\partial e_{IC}^O / \partial a > 0$) and is increased by task substitutability (when $\partial e_{IC}^O / \partial a < 0$). Thus, in SB and IC regimes, while the social benefit of $a$ is equally ignored, the task interdependence plays contrasting roles on the determination of $a$.

### 3.2 Investment under public ownership

Under public ownership, there are two different regimes to consider — separation with the government’s ownership and integration with the government’s ownership.

#### 3.2.1 Separation and public ownership

Assume that the builder and the manager are two separate agents, and the government is the owner of the facility. First consider the operating stage. When the government contracts (at time 2) with the manager, she has already internalized all the benefits and costs generated by $e$. As a result, she chooses the efficient level, $\overline{e}(a)$, which satisfies (3); it is simply meaningless to adjust to any other level of $e$.

We then move back to the building stage. Subsequent to the choice of $a$, there is a bargaining between the builder and the government on its adoption. If $a$ is not adopted, the builder ends up with nothing except the default contract payment,
while the government has to set \( e = \bar{e}(0) \) and ends up with a continuation payoff
\[ [v(\bar{e}(0)) + d(0, \bar{e}(0)) - \bar{e}(0)]. \]
However, if the two parties reach an agreement on the adoption of \( a \), the total surplus generated is
\[ [u(a) + t(a) + v(\bar{e}(a)) + d(a, \bar{e}(a)) - \bar{e}(a)]. \]
Given Nash bargaining solution, the ex post payoff of the builder is reckoned as follows.

\[
f_b = \frac{1}{2} [u(a) + t(a) + v(\bar{e}(a)) + d(a, \bar{e}(a)) - \bar{e}(a) - v(\bar{e}(0)) - d(0, \bar{e}(0)) + \bar{e}(0)] - a.
\]

Anticipating such bargaining and its impact, the builder will maximize \( f_b \) subject to (3) by choosing \( a = a_{SP} \), which satisfies
\[
\frac{1}{2} [u'(a_{SP}) + t'(a_{SP}) + d_1(a_{SP}, \bar{e})] = 1. \tag{10}
\]

Altogether, \( a_{SP} \), together \( \bar{e} \), is determined by (3) and (10). Compared with the first-order condition for the first-best \( a^* \), we find that now the marginal benefit of increasing \( a \) is half as under the first-best case. As a consequence, \( a_{SP} < a^* \). In contrast with private ownership, now the social benefit of \( a \), represented by \( u(\cdot) \), is taken into account and task interdependence is irrelevant.

### 3.2.2 Integration and public ownership

Assume that a consortium is in charge of building and operation, but the government maintains the ownership. In this case, the government would approve the adoption of \( a \), and then ensure that \( \bar{e}(a) \) be chosen. Thus, bargaining happens sequentially on these two issues.

First, consider the operating stage. In case of no complete contract signed at time 2 with the government over the choice and subsequent adoption of \( e \), the consortium would at time 2.5 set \( e = \epsilon^O_{IP}(a) \), which solves\(^{10}\)
\[
\frac{1}{2} [v'(\epsilon^O_{IP}(a)) + d_2(a, \epsilon^O_{IP}(a))] = 1. \tag{11}
\]
Totally differentiating (11), we obtain
\[
\frac{\partial \epsilon^O_{IP}}{\partial a} = -\frac{d_2(a, \epsilon^O_{IP})}{v''(\epsilon^O_{IP}) + d_{22}(a, \epsilon^O_{IP})}. \tag{12}
\]

\(^{10}\)Through the bargaining with the government over the adoption of \( e \) at time 3, the consortium shares one half of the marginal benefit generated by \( e \). This explains the coefficient of one half in the left hand side of (11).
Obviously, $e_{IP}^O(a) < \bar{e}(a)$ and it is beneficial for the government and the consortium to sign a contract that sets $e = \bar{e}(a)$ at the beginning of the operating stage (i.e., at time 2). Negotiation takes place accordingly. Foreseeing such bargaining and the bargaining triggered by the approval of $a$ (see the appendix for more detail), the consortium would choose in the building stage $a = a_{IP}$, which satisfies

$$\frac{1}{2} \left[ u'(a_{IP}) + t'(a_{IP}) + d_1(a_{IP}, \bar{e}) \right] = 1. \quad (13)$$

Altogether, $a_{IP}$, together $\bar{e}$, is determined by (3) and (13). Notice that the marginal benefit from investing in $a$ under this regime is exactly as the same as under the $SP$ regime. As a result, $a_{IP} = a_{SP} < a^*$, and the two regimes are equivalent in terms of investment choices.

### 3.3 Optimal regime

Interim contractibility of $e$, which ensures that $e$ is always efficiently chosen, allows us to focus solely on $a$ in our comparison of alternative regimes. In case of under-investment of $a$ in all alternative regimes, the optimal regime is the one that yields the greatest $a$. We have already noted that, $a$ is obviously under-invested under the two public regimes considered and that the investments under the two private regimes considered are independent of social benefit effect of $a$, i.e., $u(a)$. However, the first-best level $a^*$ is increasing in $u'$. Therefore, whenever $u'$ is high enough, there is under-investment in all regimes.

To facilitate the discussion, we can parameterize $u$ and $t$ (the residual value), by writing $u(a) = \rho U(a)$ and $t(a) = \omega T(a)$, where $U$ and $T$ are the benchmark social benefit function and residual value function. An increase in $\rho$ (or $\omega$) means an increase in the importance of social benefit (or residual value) given $a$. We use $(\rho, \omega)$ to denote the problem under parameters $\rho$ and $\omega$.

**Lemma 1** (1) Given $\omega$, there exists $\rho^*$ such that there is underinvestment of $a$ in all regimes if and only if $\rho > \rho^*$.

(2) For all $\omega > 0$, if there is underinvestment of $a$ in all regimes under $(\rho, \omega)$, then there is also underinvestment of $a$ under $(\rho, \omega')$, for any $\omega' > 0$ and $\omega' \neq \omega$.

**Proof.** See the Appendix.

Part (1) suggests that whenever the social benefit of $a$ is sufficiently large (i.e., $\rho$ is sufficiently large), there always exists underinvestment problem, since such effect is not fully internalized by the investors. Part (2) suggests that the cutoff $\rho^*$ is independent of $\omega$; or the residual value function $t(.)$ does not play any role in determining
whether there is underinvestment of $a$ in all regimes. The role of part (2) will be made clearer soon.

The following proposition tells us whether bundling or unbundling of the two tasks is more desirable.

**Proposition 1** Suppose that $a$ is non-contractible, $e$ is interim contractible, and $\rho > \rho^*$. 

(1) Under private ownership, integration is better than separation for inducing both $a$ and $e$, if and only if, for all $a \in [\min \{a_{IC}, a_{SB}\}, \max \{a_{IC}, a_{SB}\}]$,

$$d_1(a, e_{IC}^O) + d_1(a, 0) > -\frac{d_{21}(a, e_{SB}^O)}{v'(e_{SB}^O) + d_{22}(a, e_{SB}^O)} - v'(e_{IC}^O) \frac{d_{21}(a, e_{IC}^O)}{d_{22}(a, e_{IC}^O)},$$

where $e_{IC}^O$ and $e_{SB}^O$ are defined by (7) and (4), respectively.

(2) Under public ownership, integration is equivalent to separation for inducing both $a$ and $e$.

**Proof.** See the Appendix. ■

We first consider part (1) of this proposition. Regarding (14), we notice that the left hand side, LHS, is positive if and only if investment $a$ has a cost-reduction effect on the operating stage. We can also verify that the right hand side, RHS, is positive (negative) if there is complementarity (substitutability) between $a$ and $e$. In other words, complementarity between investments makes it more difficult for integration to dominate separation under private ownership, while substitutability makes it easier. Thus, under private ownership, the desirability of bundling depends on both the externality measured by the partial derivative $d_1(a, e)$ and the task interdependence measured by cross derivatives $d_{21}(a, e)$.

The role of $a$’s cost-reduction effect can be understood as follows. Suppose $d_1(a, e) > 0$. When deciding how much to invest in the building stage, the consortium under integration is able to internalize the savings on operational costs, while the builder under separation is not able to.\footnote{Although the builder can bargain with the government in the "ownership by the builder" case, the threat points are defined by the payoffs after the adoption of $a$. The reason is that such bargaining is over contracting of $e$ at time 2, instead of adoption of $a$ at time 1. Thus, the builder cannot internalize the savings on operational costs through such bargaining.} Thus, a larger cost-reduction effect of $a$ strengthens the incentive of the consortium, but not that of the builder. This part of the result is consistent with that in Bennett and Iossa and also in Martimort and Pouyet.
The other part of the result is counter-intuitive, however. Despite a kind of positive externality, complementary effects between \(a\) and \(e\) disfavors integration, instead of favoring it. To understand it, we first assume that \(d_{21}(a,e) > 0\). Under consortium ownership, the government would negotiate with the consortium on the choice of \(e\). The consortium’s gains from the bargaining depend on his default choice of \(e\) in the event of bargaining breakdown (denoted by \(e_{IC}^O\)). A higher level of \(e_{IC}^O\) implies less improvement in social benefits through the bargaining, enhancing the relative bargaining position of the government. (Notice that the government is the direct beneficiary of the social benefits.) Also notice that given \(d_{21}(a,e) > 0\), \(e_{IC}^O(a)\) should be an increasing function of \(a\). As a result, to extract more subsidies from the government, the consortium would underinvest \(a\) to a greater extent.

Under builder ownership, ex ante contracting on the choice of \(e\) triggers bargaining between the builder and the government; through such bargaining, the builder shares the benefits generated by \(e\), while the cost of investing \(e\) is mostly paid by the government. Contrary to the integration case, if the default choice of \(e\) (denoted by \(e_{SB}^O\)) is higher in the event of bargaining breakdown, the relative bargaining position of the builder would be stronger. Thus, given investment complementarity, the solo builder is given additional incentive to invest \(a\) in order to induce the government to invest a higher level of \(e_{SB}^O\). This thus explains why investment complementarity favors separation over integration. The case of investment substitutability can be analogously analyzed.

Now turn to part (2) of Proposition 1, which states that under public ownership, neither task interdependence nor externality continues to shape the trade-offs between integration and separation. Three points are in order. Firstly, under public ownership, the adoption of \(a\) always triggers bargaining between its investor and the government. Through the bargaining, the investor (i.e., either the builder or the consortium, depending on the regime) would obtain one-half of the net surplus generated by the adoption of \(a\). Moreover, the threat point of this bargaining is defined by the payoffs when \(a = 0\); this means the level of \(a\) does not affect the relative bargaining positions of the two parties, and what the investor are concerned about is just the net surplus generated. Secondly, the net surplus generated by the adoption of \(a\) consists of the savings on operational costs due to \(a\), as well as the influence of \(a\) on the benefits and the costs of the subsequent investment \(e\). The former component explains that either the consortium or the solo builder internalizes one-half of the cost-reduction effect of \(a\); the latter component is relevant to the interaction between \(a\) and \(e\). Thirdly, whether the tasks are bundled or not, the equilibrium choice of \(e\) reacts to the investment of \(a\) in the same manner; anyway it is equal to its efficient
level given a. These three points, taken together, explain why the issue of integration or separation does not matter under public ownership in our interim contractible model.12

The following proposition answers whether or not the project should be privately owned.

**Proposition 2** Suppose that a is non-contractible, e is interim contractible, and ρ > ρ∗.

(1) Given the choice of integration, consortium ownership is better than public ownership for inducing both a and e, if and only if, for all a ∈ [min {aIC, aIP}, max {aIC, aIP}],

\[ t'(a) - u'(a) + d_1(a, e_{IC}^O) > -v'(e_{IC}^O) \frac{d_{21}(a, e_{IC}^O)}{d_{22}(a, e_{IC}^O)}, \]

where e_{IC}^O is defined by (7).

(2) Given the choice of separation, builder ownership is better than public ownership for inducing both a and e, if and only if, for all a ∈ [min {aSB, aSP}, max {aSB, aSP}],

\[ t'(a) - u'(a) - d_1(a, 0) > \frac{d_{21}(a, e_{SB}^O)}{v''(e_{SB}^O) + d_{22}(a, e_{SB}^O)}, \]

where e_{SB}^O is defined by (4).

**Proof.** See the Appendix.

This proposition implies that, in case of interim contractibility of e, when the government considers whether the project should be privately owned, she should consider not only the residual value effect, the social benefit effect, and the cost-reduction effect of the building investment, but also task interdependence.13 The following statements are true according to Proposition 2.

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12 As for the impacts of externality under the public ownership, there is a shape contrast between our result in this model and the corresponding part in Bennett and Iossa. In their model, since investment e is assumed to be non-contractible all the time, the solo manager indirectly shares certain part of benefits generated by the adoption of a under the separation-and-public-ownership regime. This explains why, in case of positive externality, the incentive of solo builder is diluted, compared with the consortium. However, in our model, the interim contractibility of e relieves the solo builder of such concern under the separation-and-public-ownership regime, since the manager would no longer be able to appropriate any surplus generated by the adoption of a due to complete contracting.

13 Result 2 of Lemma 1 ensures that the t(.) function, or the coefficient ω, does not play any role in determining whether a is underinvested in all regimes. Hence, given integration, we can always choose a high (low) enough ω so that (i) a is underinvested in all regimes and (ii) private-consortium ownership is better (worse) than public ownership. A similar claim holds when the private-consortium ownership is replaced by private-builder ownership.
(a) Whether the tasks are integrated or separated, a larger residual value effect favors private ownership, and a larger social benefit effect favors public ownership. This result is consistent with that in Bennett and Iossa.

(b) When the two tasks are integrated (separated), a larger cost-reduction effect of the building investment consolidates (reduces) the advantage of consortium (builder) ownership, relative to public ownership.

(c) When the two tasks are integrated, greater investment complementarity (substitutability) attenuates (enhances) the advantage of consortium ownership relative to public ownership.

(d) When the two tasks are separated, greater investment complementarity (substitutability) enhances (attenuates) the advantage of builder ownership relative to public ownership.

We end with a few more comments. Firstly, the model presented here indeed reflects the frequent adaptation of contracts in PPPs. We attribute the driving force of this phenomenon to interim contractibility, rather than falling into a conventional discussion of the contract length. Provided that the contract is incomplete at the outset, the resolved complexity is more likely to trigger renegotiation of the contract terms. This point is also contrasted with that in Bajari and Tadelis (2001) and Ellman (2006) who considered the renegotiation of contracts as a response to the changes in demand or discovery of new service innovations. Secondly, provided that $e$ is interim contractible, the equilibrium choice of operating investment matches with the efficient level, but the incentive for building investment varies by virtue of investment interdependence. In the next section, we will discuss to what extent the results obtained here carry to the environment in which $e$ is always non-contractible.

4 Non-contractibility

We have shown that task interdependence not only matters in the determination of the optimal regime but also plays a counter-intuitive role. As this paper is partially motivated by Bennett and Iossa, it is useful to clarify whether the insights obtained in our framework are generalizable to theirs, where the sequential investments, $a$ and $e$, are both non-contractible.

Analytically, one complication is that, without the assumption of interim contractibility, $e$ is commonly inefficiently chosen in any ownership regime. Then the determination of optimal regime requires comparison of both $a$ and $e$, and it is difficult
without further specification of the model. However, if we focus on how to alleviate the underinvestment problem of the building effort, there is a lot we can say about. In an earlier version of this paper (Chen and Chiu 2009), we show that, in presence of task complementarity (substitutability), it is more (less) likely that unbundling yields a greater \( a \) than bundling. If \( a \) is underinvested in every possible regime and, from the government’s point of view, \( a \) is of far more importance than \( e \), then it is indeed the case that separation is a more (less) desirable arrangement than integration under task complementarity (substitutability). In this sense, the counter-intuitive role of task interdependence holds true in Bennett and Iossa’s model, although the underlying reason is somewhat different. The intuition is as follows.

First consider the separation cases with the builder’s or the government’s ownership. In these cases, since \( e \) is non-contractible, its adoption predicates ex post bargaining, through which the builder can share partial benefits generated by the manager’s investment while not bearing any cost of it.\(^\text{14}\) If \( a \) and \( e \) are complementary, a higher level of \( a \) would lead to a more efficient level of \( e \), enlarging the net surplus to be split in the bargaining. Thus, by expecting more rents to (directly or indirectly) be extracted from the manager, the builder would like to invest a greater level of \( a \). However, under the integration case where a unified agent takes charge of building and operation, if the consortium invests a greater \( a \), he would bear the full cost of investing a greater \( e \) subsequently, while the benefit of such investment is shared by the government. Hence, at the margin, the consortium’s incentive to invest \( a \) is less enhanced by the complementarity than is the builder’s incentive.

To further explain our idea, it is interesting to revisit the example of Private Finance Initiative (PFI) schools mentioned in Bennett and Iossa. According to a report by the Audit Commission, the quality of traditionally procured schools is commonly better than that of PFI schools.\(^\text{15}\) Bennett and Iossa provide an explanation to this phenomenon by referring to the negative externality across the project phases. However, it is hard to believe that a well-built facility in the school would by itself increase the operational costs. Alternatively, more valuable assets may induce the school managers to exert more efforts to engage in maintenance, since failure to control vandalism now causes more to be damaged and costs more for repairs. If the school builder happens to be the manager (as under PFI), he may fear that more "managerial efforts" (instead of direct operational costs) would be put into the op-

\(^{14}\)In the separation with public ownership case, the builder can bargain with the government to indirectly share the benefits generated by the manager.

\(^{15}\)As stated in Bennett and Iossa, PFI refers to the "integration with consortium’s ownership" regime and traditional procurement refers to the "separation and public ownership" regime.
erating stage. On the contrary, a traditional procurement contract can relieve the school builder of such a concern. This alternative story, which seems plausible, shows that, when PFI is less favored, the tasks of construction and operation may exhibit certain degree of complementarity, instead of negative externality.

5 Complete contracting

Our analysis has found that under two variant models of incomplete contracting frameworks, bundling is disfavored by investment complementarity, it is natural to ask if the same insight still exists in a complete contracting framework. In a framework where both \( a \) and \( e \) are observable and contractible and no-renegotiation can be committed to, the government can simply enforce the first-best \( a \) and \( e \) using take-it-or-leave-it offers. In this sense, the organization structure becomes an irrelevant issue. To make the question more interesting, it would be useful to focus on models in which certain frictions, such as risk aversion and limited liability, are present in the contracting process. To this end, we first turn to Martimort and Pouyet, who show that, in a complete contracting framework, the cost reduction effect of the building effort favors bundling over unbundling. Extending this model with task interdependence, we argue that integration tends to be favored by task complementarity, in contrast with our earlier result.

5.1 Risk aversion

We follow Martimort and Pouyet closely. In their problem, a building investment, \( a \), and an operating investment, \( e \), are undertaken in a sequence, with costs \( a^2/2 \) and \( e^2/2 \), respectively. We assume that, in case of integration, a unified agent is in charge of investing both \( a \) and \( e \); in case of separation, two agents, namely, builder and manager, are in charge of investing \( a \) and \( e \) respectively. (Since there is no renegotiation, we do not consider the regime where one agent is the owner and hires the other agent.) Given \( a \) and \( e \), the following building quality (\( Q \)) and operational cost (\( C \)) are resulted:

\[
Q = a + \varepsilon, \tag{17}
\]
\[
C = \eta - e - \delta a - \theta ae, \tag{18}
\]

16 To better understand this point, it is also worthwhile distinguishing between the operational cost and the cost incurred by managerial effort. In the model of non-contractibility, the former cost is transferable and can be deferred for payment after the bargaining over the adoption of \( e \); the latter cost cannot be transferred and has already been sunk before that bargaining.
where $\varepsilon (\eta)$ are random shock normally distributed with mean zero ($\eta_0$) and variance $\sigma^2_\varepsilon (\sigma^2_\eta)$, and $\delta$ and $\theta$ are two coefficients. When $\theta = 0$, the model is exactly the same as Martimort and Pouyet; in this case, $\delta > 0$ ($\delta < 0$) means positive (negative) externality across stages: an increase in $a$ leads to a reduction (an increase) in operational cost. The task interdependence is brought in by a non-zero $\theta$: if $\theta > 0$ ($\theta < 0$), $a$ and $e$ are complementary (substitute) investments. The builder and the manager in the separation regime, as well as the builder-manager in the integration regime, have the same constant-absolute-risk-aversion utility function with coefficient $r$.\footnote{When $\theta = 0$, $a$ has cost reduction effect if and only if $\delta > 0$. Notice that $\partial C/\partial a = -\delta - \theta e$. Once $\theta \neq 0$, $a$ has cost reduction effect if and only if $\delta > -\theta e$; that means, whether $a$ has cost reduction effect does not depend solely on $\delta$.}

Two remarks are in order here. First, we assume that, despite the non-contractibility of $a$ and $e$, the building quality ($Q$) and operational cost ($C$) are contractible. Equally important is that the parties are able to commit to no re-negotiation. This framework is thus rightly called a complete contracting framework. Second, we assume that there is social benefit associated with project, denoted by $S \times Q$, where $S$ is a parameter reflecting the size of the social benefit.

Following Martimort and Pouyet, we restrict ourselves to linear contracts. Under the integration regime, the following linear contract is signed with the consortium (builder-manager):

$$\Psi(Q, C) = \xi + \beta Q - \alpha C, \quad (19)$$

where $\xi$ is interpreted as an aggregate fixed-fee payment and $\beta$ and $\alpha$ piece-rate parameters. Under the separation regime, a pair of linear contracts are signed with the builder ($b$) and manager ($m$), respectively,

$$\Psi_i(Q, C) = \xi_i + \beta_i Q - \alpha_i C, \quad (20)$$

where $\xi_i$ is interpreted as a fixed-fee payment and $\beta_i$ and $\alpha_i$ piece-rate parameters, where $i = b, m$. Obviously, $\beta_m = 0$, since the manager’s investment, $e$, does not impact the quality and any non-zero $\beta_m$ would expose unnecessary risk to the manager.

Let $a^I$ and $e^I$ denote the consortium’s choices under the integration regime; $a^S$ and $e^S$ denote the builder’s and manager’s choices under the separation regime.

**Lemma 2** Given contracts as stipulated in (19) and (20), the equilibrium choices are...
given by:

\[ a^I = \beta + \delta \alpha + \theta e^I \alpha; \quad (21) \]
\[ e^I = \alpha + \theta a^I \alpha. \quad (22) \]
\[ a^S = \beta_b + \delta \alpha_b + \theta e^S \alpha_b; \quad (23) \]
\[ e^S = \alpha_m + \theta a^S \alpha_m. \quad (24) \]

**Proof.** See the Appendix. ■

We can show the following proposition regarding the case of task complementarity.

**Proposition 3** Suppose \( \delta = 0 \) and \( \theta > 0 \) and linear contracts are used. Consider any pair of contracts under separation so that the builder’s and manager’s individual rationality (IR) constraints just bind. Then there always exists a contract under integration so that the builder-manager’s IR constraint is satisfied, the same risk premium under the separating contracts is paid, and the equilibrium choices satisfy: \( a^I > a^S \) and \( e^I > e^S \).

**Proof.** See the Appendix. ■

The proposition suggests that, if \( a \) and \( e \) are complementary investments, given any pair of unbundling contracts, we can always find a bundling contract that is more desirable in the sense that it generates the same level of distortion (i.e., risk premium) but induces a higher level of incentives. In other words, it gives an intuitive result that task complementarity favors integration over separation, in terms of alleviating the underinvestment problem. In other words, a unified agent is easier to be motivated since the agency cost is relatively lower in case of task complementarity.

The underlying rationale of the proposition is as follows. Notice that the risk premium under integration equals \( \beta^2 \delta^2_\epsilon + \alpha^2 \delta^2_\eta \), while the total risk premia under separation equals \( \beta^2_b \delta^2_\epsilon + (\alpha^2_b + \alpha^2_m) \delta^2_\eta \). Suppose \( \beta = \beta_b \). Then the equal-risk premium condition implies that \( \alpha \geq \alpha_b \) and \( \alpha \geq \alpha_m \), where the two equalities cannot hold at the same time. Using (21) to (24), we immediately conclude that \( a^S < a^I \) and \( e^S < e^I \). Therefore, despite the same risk premium and that the relevant IR constraints bind, the integration regime dominates the separating regime by better mitigating the underinvestment problem. The intuition is that, given the risk premium, under the separation regime, each agent is given a weaker incentive (\( \alpha_b \leq \alpha \) and \( \alpha_m \leq \alpha \)) to respond to the other agent’s investment.

While Proposition 3 is concerned about task complementarity, the corresponding result regarding task substitutability is more difficult to obtain. The reason is that
and $e$ normally change in opposite directions from one contractual regime to another, rendering the comparison difficult without further specification.

5.2 Limited liability

Despite the finding in the last subsection, the conventional understanding that task complementarity favors integration need not always hold even under complete contracting. Schmitz (2005) studies a sequential moral hazard model where control actions are contractible and limited liability is the source of agency problem. Assuming the effort in the first stage makes the effort in the second stage more effective (i.e., reduces the latter’s marginal cost or increases its marginal benefit), the author finds that, when she needs to induce high efforts, the principal might need to pay a greater agency cost to the integrated agent under bundling than to the two separate agents under unbundling.\footnote{According to Schmitz, when tasks are bundled, the agent might be tempted to shirk in the first stage, since by doing so he could increase the rents obtained in the second stage. The underlying reason is that the agency rent resulting from limited liability problem becomes larger when the effect of effort on the success probability is reduced.} In other words, this result predicts that task complementarity disfavors bundling.

Note that Schmitz’s counter-intuitive result is due to the fact that the effort in the first stage alters the agency rent generated by the effort in the second stage. To a certain extent, this underlying force is also existent in our interim contracting framework (as well as the incomplete contracting framework in Bennett and Iossa). It is nonetheless absent in Martimort and Pouyet, since when moral hazard problem arises from risk aversion, premium is paid only to cover the agents’ utility loss in bearing risks, and their utilities always match with reservation utility. In this sense, the agency rent in the following stage cannot be altered by the investment in the previous stage.

6 Conclusion

This paper has reexamined the proper scope of public-private partnerships in the context of a project consisting of two tasks, such as construction and operation of a facility. The focus of analysis has been the role of task or investment interdependence, i.e., the two investments are either complements or substitutes. Whether the operation investment is non-contractible or interim contractible, we have found that investment interdependence is a determinant of the optimal PPPs. In particular, favored by the cost-reduction effect of the building investment, integration is
nonetheless disfavored by investment complementarity. While the focus of this paper is on the interim contractibility framework, in which the second, operation task is contractible subsequent to the building stage, we have also argued that weaker but similar insights hold true in the incomplete contracting framework in the fashion of Bennett and Iossa.

These findings regarding the role of task interdependence, while counter-intuitive, contributes to the understanding of one empirical puzzle raised in the literature. In a framework of investment externality but not investment interdependence, unbundled projects are rationalized by negative externality. Despite a theoretical possibility, examples of negative externality are relatively rare (as pointed out by Iossa and Martimort). However, in our framework where investment interdependence is also allowed, unbundled projects can now be rationalized by weak positive externality together with strong complementarity. Our theory thus provides an alternative explanation to the phenomena. For example, the undesirable performance of PFI schools can be due to the fact that a better fitness might give rise to a higher value of guide service. In other cases involving high technology, such as IT projects, because more advanced and innovative system usually comes up with higher degree of complexity, more effort is required for the operator in learning how to handle it efficiently. The complementarity between building and operating thus weakens the advantage of bundling these tasks.

On the other hand, in such sectors as prison, waste disposal, transport and hospital, sequential investments of building and operating might exhibit certain level of substitutability. According to a report by National Audit Office (2003) in the United Kingdom, compared to traditional procurement, PFI prisons tend to improve performance and save costs. The reason is that innovative design solutions help reduce the level of staffing needed to ensure security in prison. The underlying reason may be the substitutability between building and operating, as much as the positive externality. In the waste disposal sector, a very well designed garbage-categorizing system may conceivably lessen the burden of garbage collectors and cleaners. According to our theory, bundling the tasks of designing and operating is favored in this case. In the transport sector, if an electronic eye system located in proper places along each highway is of very good quality, detection of speedsters may not demand much effort from the police or other rosters. Similarly, these tasks are better contracted out in a bundle. Because construction and management of many public projects often demonstrate interdependence, our model provides a greater latitude to understand and interpret real world phenomena.

We end with some comments on the role of the contracting framework. If project
quality and cost are available for contracting upon but the agents are risk averse, the conventional insight that task complementarity favors bundling holds true. On the other hand, if agency problem stems from limited liability, or if the contract is incomplete to a certain degree (non-contractible or interim contractible) and ex post bargaining is unavoidable, it is possible that task complementarity favors unbundling. The lesson is that when delegating public project to private sectors, the government should condition her decisions on the features of contracting environment and agency problem.
References


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Appendix A: Details about the first-order conditions derived in Section 3

A.1 Ownership by the consortium

Following the building stage, the government may negotiate with the consortium to recontract the operation task. After bargaining on such an issue, the ex post payoff of the consortium is

\[ f = t(a) + \frac{1}{2} [v(\bar{e}) - v(e_{IC}^0) + d(a, \bar{e}) - e_{IC}^0] - a. \]

(The method to derive ex post payoffs is similar to that used in analyzing the "builder-ownership" case.) Foreseeing all this, the consortium maximizes \( f \) by choosing \( a = a_{IC} \) subject to \( e \) satisfying (3) and \( e_{IC} \) satisfying (7). The first-order condition is (9) as stated in the main text.

A.2 Integration and public ownership

We first consider the bargaining over the adaptation of \( e \). The threat point of the government is \( \frac{1}{2} [v(e_{IP}^0) + d(a, e_{IP}^0) - d(a, 0)] \); the threat point payoff of the consortium is \( \{ \frac{1}{2} [v(e_{IP}^0) + d(a, e_{IP}^0) + d(a, 0)] - e_{IP}^0 \} \). The net surplus generated by adjusting \( e \) to \( \bar{e}(a) \) is

\[ \Omega|_{e=\bar{e}} - \Omega|_{e=e_{IP}^0} = [v(\bar{e}) - v(e_{IP}^0)] + [d(a, \bar{e}) - d(a, e_{IP}^0)] - (\bar{e} - e_{IP}^0). \]

After such bargaining, the government gets \( \frac{1}{2} [v(\bar{e}) + d(a, \bar{e}) - d(a, 0) - \bar{e} + e_{IP}^0] \) in the operating stage, while the consortium gains \( \frac{1}{2} [v(\bar{e}) + d(a, \bar{e}) + d(a, 0) - \bar{e} - e_{IP}^0] \) in this stage.

We then look at the building stage. Since the adoption of \( a \) requires the government’s approval, bargaining between the government and the consortium is triggered. The threat points for this bargaining are defined by the continuing payoffs of the two parties when \( a = 0 \); that is, in the event of bargaining breakdown, the government ends up with \( \frac{1}{2} [v(\bar{e}(0)) + d(0, \bar{e}(0)) - \bar{e}(0) + e_{IP}^0(0)] \), whereas the consortium ends up with \( \frac{1}{2} [v(\bar{e}(0)) + d(0, \bar{e}(0)) - \bar{e}(0) - e_{IP}^0(0)] \). However, if \( a \) is successfully adopted, the total surplus generated is

\[ [u(a) + t(a) + v(\bar{e}(a)) + d(a, \bar{e}(a)) - \bar{e}(a)]. \]
Thus, after the bargaining, the ex post payoff of the consortium is
\[
f = \frac{1}{2} \left[ u(a) + t(a) + v(\bar{e}(a)) + d(a, \bar{e}(a)) - \bar{e}(a) - e_{IP}^0(0) \right] - a.
\]

Foreseeing all this, the consortium maximizes \( f \) by choosing \( a = a_{IP} \) subject to \( \bar{e} \) satisfying (3) and \( e_{IP}^O \) satisfying (11). The first-order condition is (13) as stated in the main text.

Appendix B: Proofs

B.1 Proof of Lemma 1

**Proof.** Part (1). Compare (1), where \( a^* \) is solved, with (10) and (13), where \( a_{SP} \) and \( a_{IP} \) are solved. Notice that they are identical except for a one-half coefficient in the LHS of the latter two equations (as a function of \( a, e^* \) in (1) and \( \bar{e} \) in (10) and (13) are the same). As a result, \( a^* > a_{SP} = a_{IP} \), and no additional condition is needed to ensure the underinvestment of \( a \) under public ownership regimes. Next notice that under the "ownership by the builder" case, \( a_{SB} \), along with \( e_{SB}^O \) and \( \bar{e} \), is determined by (6) and (4) and (3). It is interesting to notice that \( u(.) \) does not appear in any of these three equations, and hence, \( a_{SB} \) is independent of \( u(.) \). On the other hand, (1), where \( a^* \) is solved, contains \( u'(.) = \rho U'(.) \) in the LHS, and hence \( a^* \) can be made to be arbitrarily large by increasing \( \rho \). Hence, there must exist a finite \( \rho^* \) so that for all \( \rho > \rho^* \), \( a_{SB} < a^* \). The case of "ownership by the consortium" regime is similar and omitted. The claim is thus proved.

Part (2). Note that the term \( t'(.) \) appears in the first order conditions of solving \( a^*, a_{SB} \) and \( a_{IC} \). This means function \( t(.) \) does not play any role in affecting relative magnitude of these choices of \( a \). Finally, as noticed earlier, the choices of \( a \) under the two public ownership regimes are always less than \( a^* \). Regarding this, \( t(.) \) also does not play any role as well. ■

B.2 Proof of Proposition 1

**Proof.** We start with the first result. Suppose that \( a_{IC} > a_{SB} \). Notice that all the second-order conditions are satisfied. Then, making use of the first-order condition for the choice of \( a_{IC} \) (see (9)), we know that, for \( a \in [a_{SB}, a_{IC}] \),
\[
t'(a) + \frac{1}{2} \left[ d_1(a, \bar{e}(a)) + d_1(a, e_{IC}^O(a)) \right] - \frac{1}{2} v'(e_{IC}^O(a)) \frac{\partial e_{IC}^O(a)}{\partial a} \geq 1,
\]

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where the equality holds only when $a = a_{IC}$; making use of the first-order condition for the choice of $a_{SB}$ (see (6)), we also know that, for $a \in [a_{SB}, a_{IC}]$,

$$t'(a) + \frac{1}{2} [d_1(a, \bar{e}(a)) - d_1(a, 0)] + \frac{1}{2} \frac{\partial e_{SB}^O(a)}{\partial a} \leq 1,$$

where the equality holds only when $a = a_{SB}$. Therefore, by subtracting the second inequality from the first, we obtain the following result:

$$a_{IC} > a_{SB} \Rightarrow d_1(a,e_{IC}^O(a)) + d_1(a,0) + \frac{e_{SB}^O(a)}{\partial a} > 1$$

for all $a \in [a_{SB}, a_{IC}]$.

(25)

Suppose that $a_{IC} < a_{SB}$. In the similar manner, we obtain the following result:

$$a_{IC} < a_{SB} \Rightarrow d_1(a,e_{IC}^O(a)) + d_1(a,0) - \frac{e_{SB}^O(a)}{\partial a} < 1$$

for all $a \in [a_{IC}, a_{SB}]$.

(26)

Finally, by combining (25) and (26) and making use of (8) and (5), we obtain the claimed result.

The second result, i.e., the equivalence result under public ownership, is obtained in a similar manner by comparing (13) and (10) and the proof is omitted. ■

B.3 Proof of Proposition 2

**Proof.** We start with the first result. Suppose that $a_{IC} > a_{IP}$. Notice that all the second-order conditions are satisfied. Then, making use of the first-order condition for the choice of $a_{IC}$ (see (9)), we know that, for $a \in [a_{IP}, a_{IC}]$,

$$t'(a) + \frac{1}{2} [d_1(a, \bar{e}(a)) + d_1(a,e_{IC}^O(a))] + \frac{1}{2} v'(e_{IC}^O(a)) \frac{\partial e_{IC}^O(a)}{\partial a} \geq 1,$$

where the equality holds only when $a = a_{IC}$; making use of the first-order condition for the choice of $a_{IP}$ (see (13)), we also know that, for all $a \in [a_{IP}, a_{IC}]$,

$$\frac{1}{2} [u'(a) + t'(a) + d_1(a, \bar{e})] \leq 1,$$

where the equality holds only when $a = a_{IP}$. Therefore, subtracting the second inequality from the first and rearranging, we obtain the following result:

$$a_{IC} > a_{IP} \Rightarrow t'(a) - u'(a) + d_1(a,e_{IC}^O(a)) - v'(e_{IC}^O(a)) \frac{\partial e_{IC}^O(a)}{\partial a} > 0$$

for all $a \in [a_{IP}, a_{IC}]$.

(27)
Suppose that $a_{IC} < a_{IP}$. In a similar manner, we could obtain the following result:

$$a_{IC} < a_{IP} \Rightarrow t'(a) - u'(a) + d_1(a, e^0_{IC}(a)) - v'(e^0_{IC}(a)) \frac{\partial e^0_{IC}(a)}{\partial a} < 0 \text{ for all } a \in [a_{IC}, a_{IP}].$$

(28)

By combining (27) and (28), and making use of (8) we show the claimed result.

As for the second result, i.e., (16), it is obtained by comparing (6) and (10). The proof is much similar and hence omitted.

B.4 Proof of Lemma 2

Proof. In the bundling case, the consortium chooses $a = a^l$ and $e = e^l$ to maximize the following payoff

$$f = \xi - \alpha \eta_0 + \beta a + \alpha (e + \delta a + \theta a e) - \frac{1}{2} r (\beta^2 \sigma_e^2 + \alpha^2 \sigma_\eta^2) - \frac{1}{2} a^2 - \frac{1}{2} e^2.$$

(Notice that the risk premium generated in this sort of contract is $\frac{1}{2} r (\beta^2 \sigma_e^2 + \alpha^2 \sigma_\eta^2).$) It can be verified that $a^l$ and $e^l$ as described in the Lemma are optimal. In the unbundling case, the builder chooses $a = a^S$ to maximize his payoff

$$f_b = \xi_b - \alpha_b \eta_0 + \beta_b a + \alpha_b (e + \delta a + \theta a e) - \frac{1}{2} r (\beta_b^2 \sigma_e^2 + \alpha_b^2 \sigma_\eta^2) - \frac{1}{2} a^2;$$

the manager chooses $e = e^S$ to maximize his payoff

$$f_m = \xi_m - \alpha_m \eta_0 + \alpha_m (e + \delta a + \theta a e) - \frac{1}{2} r (\alpha_m^2 \sigma_\eta^2) - \frac{1}{2} e^2.$$

It can be verified that $a^S$ and $e^S$ as described in the Lemma are optimal.

B.5 Proof of Proposition 3

Proof. Let $C_b(\beta_b, \alpha_b)$ and $C_m(\beta_m = 0, \alpha_m)$ be the contracts under separation that just satisfy the builder and manager’s IR constraints. (The IR constraints are satisfied as long as the risk premia, which are independent of the actual investment choice, are included.) Suppose $\delta = 0$. It is straightforward to show that the equilibrium choices can be represented by

$$a^S = \phi(\beta_b, \alpha_b, \alpha_m) \equiv \frac{\beta_b + \theta \alpha_b \alpha_m}{1 - \theta^2 \alpha_b \alpha_m},$$

$$e^S = \omega(\beta_b, \alpha_b, \alpha_m) \equiv \frac{\alpha_m (1 + \theta \beta_b)}{1 - \theta^2 \alpha_b \alpha_m}.$$
Under integration, we can construct a grand contract $C(\beta, \alpha)$ such that $\beta = \beta_b$ and $\alpha = \sqrt{\alpha_b^2 + \alpha_m^2} > \max \{|\alpha_b|, |\alpha_m|\}$. One can verify that such a bundling contract $C(\beta, \alpha)$ generates the same level of risk premium as the pair of unbundling contracts $C_b(\beta_b, \alpha_b)$ and $C_m(\beta_m = 0, \alpha_m)$ in total, that is,

$$\beta^2 \delta^2 + \alpha^2 \delta^2 = \beta_b^2 \delta^2_b + (\alpha_b^2 + \alpha_m^2) \delta^2_m. \tag{29}$$

However, if $\theta > 0$, the equilibrium choices of the consortium with contract $C(\beta, \alpha)$ can be represented by $a = \phi(\beta, \alpha, \alpha) > \phi(\beta_b, \alpha_b, \alpha_m) = a^S$, and $e = \omega(\beta, \alpha, \alpha) > \omega(\beta_b, \alpha_b, \alpha_m) = e^S$. In other words, a grand contract $C(\beta, \alpha)$, with $\xi$ equal to sum of investment costs, $(a^I)^2 / 2 + (e^I)^2 / 2$, plus the equivalent risk premium, will be acceptable for the consortium, and will implement $a^I > a^S$ and $e^I > e^S$. $\blacksquare$
Figure 1: Time line of the game

- regime is given
- investing
- payoffs are realized
- possible bargaining on the adoption of e
- re-contracting on e
- contracting on e
- possible bargaining on the adoption of e
- investing e

Building stage

Operating stage

Figure 1: Time line of the game

- regime is given
- investing
- payoffs are realized
- possible bargaining on the adoption of e
- re-contracting on e
- contracting on e
- possible bargaining on the adoption of e
- investing e

Building stage

Operating stage