Student Enthusiasm and Preferential Treatment in Centralized Admissions

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February 6, 2013

Abstract

Why is preferential treatment (PT) popular under centralized admissions? Basic intuition suggests that schools value student enthusiasm, but indeed how PT can serve this end remains an open question. We show that it can be beneficial for a single school to adopt PT so as to drive out the unenthusiastic students that would otherwise be admitted (driving-out effect), meanwhile, admitting more enthusiastic students is also feasible through using PT when another school also does so (internalization effect). The voluntary adoption of PT serves as a tool for schools to identify enthusiastic students, who are indeed rewarded for truthfully reporting their preferences. Understanding endogenous PT has high policy relevance as to whether the choice of PT should be permitted as under the old first-preference-first system in England.

"We want the students that want us most!" — A New York City educator (New York Times, November 29, 2004)

1 Introduction

This paper focuses on a real world practise in centralized admissions systems, which we call preferential treatment (PT), wherein a school gives priority to applicants that rank it as their first choice over those that only rank it as their second choice, etc.,

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†We thank Steve Ching, Frances Xu, Ira Horowitz and participants at the Asian Econometric Society Meeting (Seoul 2011) for comments. Any remaining errors are our own. Financial support from the Hong Kong Research Grants Council (HKU742407H, HKBU255112) is gratefully acknowledged.
even though the latter students may be academically stronger. One example is the
so-called Boston mechanism (see Chen and Sönmez, 2006; Ergin and Sönmez, 2006;
Abdulkadiroğlu, Pathak, Roth, and Sönmez, 2005; Pathak and Sönmez, 2008), in which
schools are required to abide by the rule of PT, which is thus exogenous. Our interest is
in situations in which schools adopt PT on their own volition, rather than by mandate.

Real-world examples abound. Hong Kong’s college admissions programme JUPAS
makes use of a deferred acceptance mechanism. Schools, however, are informed of their
applicants’ rank-ordering of the schools to which they have applied. While admissions
committees can just ignore this information, they can also choose to consider it as one
criterion, and weight it accordingly, in evaluating applicants.¹

A restricted form of the JUPAS mechanism is the "first preference first" (FPF, in
short) system that until 2007 was used to allocate high school places in England. In
this system, a school can choose to become either a non-preference school or an FPF
school. A non-preference school ignores the order of schools its applicants rank in the
preparation of its priority list; while an FPF school must first fill its places with the
applicants that rank it as their top choice, before filling its places with those that rank
it as their second choice, and so on (see Pathak and Sönmez 2012 for more detail on the
system).²

Unlike exogenous PT, endogenous PT drew little attention from economic theorists.
Chiu and Weng (2009) identified two strategic motives as to why schools may adopt PT:
notably, a less popular school can use it to attract good applicants that otherwise would
not rank it as their top choice (stealing motive), while a more popular school can use it
to deter the less popular schools from using it, and thus prevent its applicants from
being stolen (preemptive motive).³ These insights, however, are unlikely to explain all
instances of why schools adopt PT, nor to explain why a central mechanism with a PT

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¹The Joint University Programme Admissions System (JUPAS in short) has been in place for more
than two decades in all public tertiary institutions in Hong Kong. Its private tertiary education sector
is much smaller in terms of the number of full time undergraduate places and admits students in a
decentralized manner.

²Another example is the system used in New York City prior to its reform in 2003 (see Abdulkadiroğlu,
Pathak and Roth, 2005, 2009). The national college admissions exercise in mainland China is also
characterized by endogenous preferential treatment, see Chiu and Wong (2009) for detail. Chen and
Kesten (2011) study some innovations to the existing system.

³Liu and Chiu (2011) reexamine the stability issue in the JUPAS, in which endogenous preferential
treatment is the main feature. There a school is said to have reciprocating preferences if the school
intrinsically likes an applicant that ranks it as her top choice more than it does another applicant that
ranks it only as her second choice. Given that all schools have such reciprocating preferences, the authors
show that the resulting matching is stable. The reciprocating preferences are different from the school
preferences that we study. In our paper, schools recognize the possibility that applicants that rank them
as their top choice may not be really enthusiastic in heart and are not what they really want.
Building on our earlier work, we address the issue with an alternative preferences assumption on the part of schools: specifically, admissions committees have an innate concern about their applicants’ enthusiasm towards them. As recorded in the meeting minutes of the Bath and North East Somerset Council in England, "some heads have stated that they would prefer a first preference system so they only have children in their schools who really want to be at their schools; the feeling is that pupils and parents for whom the school is not first preference will be less committed to the school." Empirical work by Hastings, Neilson and Zimmerman (2012) confirms that getting admitted to a chosen school by winning the first-choice maximizing lottery indeed has a positive effect on both students’ intrinsic motivation and their academic attainment. Avery and Levin (2010) also use such school preferences, in a decentralized admissions setting, to explain why some U.S. colleges adopt early admissions, wherein students can signal their enthusiasm for the school, a signal that otherwise could not be known.

Because students’ enthusiasm is not observable, their rank-order lists are both a way for applicants to signal, and admissions committees to infer, their enthusiasm for a school. We maintain that this is the main reason behind the design of mechanisms that allow endogenous PT. The question we address is how well this purpose is served and whether PT should be made exogenous or endogenous.

Building on our earlier model (Chiu and Weng 2009), we characterize students by their academic ability and enthusiasm, which is private information, while schools have the aforementioned school preferences. A central clearinghouse is in place to assign school places according to the student-propose-to-school deferred-acceptance mechanism. In the compilation of its priority list, each school can completely ignore the order of schools applicants rank, or assign those that rank it as their first choice a higher priority than those that rank it as their second choice, and so forth, and use ability only in tie-breaking. This form of preferential treatment is essentially the FPF rule.

We consider two models that capture different real-world scenarios. In the demand-uncertainty model, students know their own academic ability, but not exactly the other students’ innate preferences. In the ranking-uncertainty model, students do not know

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4 The emphasis on student enthusiasm is also evidenced by a complaint made by a New York city educator after learning their school choice system had ceased to contain information on how each school was ranked by each applicant (see the quote on the first page).
their own academic ability/performance, either, before submitting their applications.

We identify two effects through which a school finds using PT beneficial. The driving out effect is realized when a school dissuades some unenthusiastic applicants that would otherwise be admitted, thus making room for the admissions of additional enthusiastic applicants. The internalization effect is realized when a school takes advantage of the other school’s PT – hence internalizing its external effect – by also using the PT policy. Also intended to dissuade unenthusiastic applicants, this effect is achieved only when some other school uses PT. We find that the former effect exists in both models, while the latter effect exists only under the demand-uncertainty model.

PT allows schools to value the enthusiasm that students show towards them as seen from their rank-order lists. If this results in separation, the school will be able to learn more about applicants’ enthusiasm and PT is thus beneficial. If this results in pooling by unenthusiastic students, however, the rank-order lists are too noisy to reflect applicants’ enthusiasm and using PT may be self-defeating. The two models studied differ in the possibility of such an outcome. In general, a less popular school is more likely to benefit from the adoption of PT, and students enthusiastic about a weaker school are more likely to benefit from the policy. This benefit is even more prominent under the ranking-uncertainty model.

The paper is related to research on the Boston mechanism, whose main feature is exogenous PT. Abdulkadiroğlu and Sönmez (2003) were the first to introduce the Boston mechanism to the literature. Earlier work on the Boston mechanism argues that it is not desirable. Because it is not a weakly dominant strategy for applicants to report their true preferences, the mechanism not only inconveniences applicants, but also creates an unequal playing field that favors sophisticated applicants at the expense of their naive counterparts (Erin and Sönmez 2006 and Pathak and Sönmez 2008). More recently, however, Abdulkadiroğlu, Che, and Yasuda (2010) show that under some conditions, applicants’ ex ante utility is greater under the Boston mechanism than under the deferred-acceptance mechanism (see also Miralles 2008 and Featherstone and Niederle 2008). Chen and Kesten (2011) investigate a family of proposal-refusal mechanisms with the deferred-acceptance mechanism and the Boston mechanism as the two extreme members. The intermediate member, the Shanghai mechanism, is suggested as a remedy for the incentive problem that exists under the Boston mechanism, at no significant welfare cost. Moreover, the common presence of constrained choice – applicants can

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6 See Abdulkadiroğlu, Pathak, Sönmez and Roth (2005) for more details about the Boston mechanism and the considerations for its reformation.

7 The Shanghai mechanism is defined as the simplest version of the Chinese parallel mechanism, first
only submit a list including a limited number of schools out of the pool from which they are allowed to choose – also imposes a significant negative effect on the performance of the deferred-acceptance mechanism, under which truth-telling is no longer a weakly dominant strategy for most applicants (see Haeringer and Klijn (2009) and Calsamiglia, Haeringer and Klijn (2010)). This literature has not taken into account the role played by school preferences that value enthusiasm.

Using the same kind of school preferences, Avery and Levin (2010) study schools’ incentives to adopt early admissions or early decisions in a decentralized setting. Because applicants can apply only to one school, early admissions provide applicants with an opportunity to signal their enthusiasm. In equilibrium, the admissions criterion in early admissions or early decisions is more lenient than that under the regular round. This phenomenon is similar to what is observed in centralized admissions when schools adopt PT. We will discuss its relationship with our paper in more detail later.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 examines a benchmark for future comparison. Section 4 examines the incentive for an individual school to adopt PT. Section 5 examines the interaction when schools have to simultaneously make the policy choices. Section 6 examines the insight provided by another model, which complements the analysis in the previous sections. Section 7 discusses and Section 8 concludes.

2 The Model

A mass of applicants of size $N$ are competing to enter school 1 and school 2, with places $s_1$ and $s_2$, respectively, where $s_1 + s_2 < N$. Each applicant is indexed by an attribute pair $(y, \theta)$. The first argument $y$ denotes her academic quality (the lower $y$ is, the better) and is distributed over $[0, 1]$ with a cumulative function $F$, which is continuous and has no mass point. The second argument $\theta \in \{1, 2\}$ denotes the applicant’s enthusiasm – she is enthusiastic about school $\theta$ and strictly prefers it to the other school. While the applicant’s $y$ is publicly observable, her $\theta$ is her private information and is referred to as her type.

We assume that, for any given $y$, a fraction of applicants, $\mu_\theta \in (0, 1)$, are of type-$\theta$, where $\mu_1 + \mu_2 = 1$. A type-1 applicant obtains a utility of $u_i > 0$ from attending school $i$, while a type-2 applicant obtains a utility of $v_i > 0$ from attending school $i$, $i = 1, 2$.

introduced to China’s college admissions system in 2008 and since then, in some other provinces. Under this mechanism, students can put down several parallel choices of college in each subcategory on their rank-order list. See Chen and Kesten (2011) for more details.

We use feminine pronouns to refer to applicants in this paper.
We assume that, for any applicant, not attending school results in zero utility, and thus it is the worst scenario an applicant could possibly face. We therefore have $u_1 > u_2 > 0$ and $v_2 > v_1 > 0$.

Schools value both ability and enthusiasm. For any applicant $i$, characterized by $(y_i, \theta_i)$, school $j$ computes for her a quality score

$$Q^j_i = \alpha_j I_j(\theta_i) + (1 - y_i),$$

where $I_j(\theta_i) = 1$ if $j = \theta_i$ and $= 0$ otherwise and $\alpha_j > 0$ is the weight given to enthusiasm. Given any two applicants, $i$ and $i'$, school $j$ strictly prefers $i$ to $i'$ if and only if $Q^j_i > Q^j_{i'}$. The overall quality score of a set of applicants is just the average of the individual quality scores of all the applicants in the set.\(^9\)

We focus on the case in which $\alpha_j > 1$ so that, given our assumption of $y \in [0, 1]$, the school will always strictly prefer an enthusiastic applicant over an unenthusiastic applicant. This assumption implies that, given any two same-sized groups of applicants, the one with a higher fraction of enthusiastic applicants is strictly preferred. The schools will take applicants' ability into account only when the two groups have the same fraction of enthusiastic applicants.

There is uncertainty about the proportions of the two types of applicants, capturing environments in which applicants may not be certain of the latest trends regarding whether or not one school is more popular than the other, or by how much one school is more popular than the other school. More specifically, there are two states of the world, state 1 and state 2, occurring with probability $\pi_1$ and $\pi_2 = 1 - \pi_1$. In state $k$, $\mu_i = \mu^k_i$, where $i = 1, 2$ and $k = 1, 2$. Without loss of generality, we assume that $\mu^1_i > \mu^2_i$. Given that $\mu^1_2 = 1 - \mu^1_1$, it is equivalent to

$$\frac{\mu^1_1}{\mu^1_2} > \frac{\mu^2_1}{\mu^2_2}$$

We call $\mu^k_j/s_j$ school $j$'s popularity in state $k$. The reason is that $\mu^k_j N/s_j$ measures the "oversubscription rate" that school $j$ has in state $k$ when its only students are those that are enthusiastic about it. School $j$ is said to be more popular than school $l$ in state $i$ if $\mu^1_j/s_j > \mu^1_l/s_l$. A school is said to have absolute popularity if it is more popular than the other school in both states. Such a scenario is said to be an absolute-popularity scenario. W.l.o.g., in this scenario, we assume school 1 has the absolute popularity, i.e.,

\(^9\)We assume that, for each school, admitting some applicant is always better than leaving the seat unfilled.
\[
\frac{\mu_1^1}{\mu_1^2} > \frac{\mu_2^2}{\mu_2^1} > \frac{s_1}{s_2}.
\] (3)

If one school is more popular in one state and another school is more popular in another state, it is said to be a relative-popularity scenario. W.l.o.g., in this scenario, we assume school 1 is more popular in state 1 and school 2 is more popular in state 2, i.e.,

\[
\frac{\mu_1^1}{\mu_2^1} > \frac{s_1}{s_2} > \frac{\mu_1^2}{\mu_2^2}.
\] (4)

A central clearinghouse is in place to assign school places according to the student-propose-to-school deferred-acceptance mechanism, based on applicants’ rank-order lists for schools and the schools’ priority lists for applicants. There are two available policies whereby schools compile their priority lists. In the first, the school completely disregards the order of schools that applicants rank, and base solely on ability to make admissions decisions — the lower \( y \) the higher the priority. We call this a non-PT policy (NPT), as opposed to PT, in which a school assigns those that rank it as their top choice a higher priority than those that rank it only as their second choice (and likewise assigns those that rank it as their second choice a higher priority than to those that fail to rank the school at all). Academic ability is used only in tie-breaking.

PT as defined here is the most extreme form of preferential treatment that one can think of. In a less extreme form, an applicant with sufficiently better ability, who ranks the school as her second choice, may still have a higher priority than one that ranks it as her first choice. In addition to expositional convenience, we adopt the restriction for two reasons. First, given our earlier assumption of \( \alpha_j > 1 \) and provided that applicants reveal their preferences truthfully, PT is indeed optimal for the school. Second, PT of this extreme form is indeed seen in real-world applications, as in the FPF mechanism mentioned in the introduction.\(^\text{10}\)

There are two arrangements through which PT is conducted. In one arrangement, a school is indeed informed of the order of schools that each applicant has ranked, as well as the applicant’s particulars. In an alternative arrangement, an uninformed school is asked to report a formula that instructs the central clearinghouse to compile a priority list on its behalf. For expositional purposes we assume the former scenario is the case.

\(^{10}\)In the FPF system, each school is allowed to choose to become an equal-preference school or a FPF school. An equal-preference school is one that adopts NPT and an FPF school is one that adopts PT, as defined in this paper. An equal-preference school is not allowed to use a more complicated policy that makes use of the order of schools that applicants rank. See Pathak and Summet (2012) for more details about the system.
The time line of the game is as follows. First, the state of the world is determined. Second, the two schools announce their admissions policies, NPT or PT. Third, applicants submit their rank-order lists. Fourth, each school is informed as to which applicants ranked it as their first or second choice. Fifth, each school compiles a priority list in accordance with their announced policy, which we assume the school is faithfully implemented. Finally, given applicants’ rank-order lists and the schools’ priority lists, the central clearinghouse determines the matching outcomes.

Our model differs from the one in our earlier paper in two major aspects. First, although our earlier analysis of students’ rank-ordering decisions and subsequent place allocations remains valid, enthusiasm preference will have different implications as to school welfare, and hence the adoption of a PT versus an NPT policy. Second, our earlier paper restricts itself to the absolute-popularity scenario (AP, in short), and the relative-popularity scenario (RP, in short) is not considered. Under the RP scenario, however, we find an important additional result: namely, the internalization effect.\textsuperscript{11}

3 Benchmark

We first characterize the equilibrium outcome in which both schools use NPT.\textsuperscript{12} Their strategies are simple: applicants rank order the schools in accordance with their true preferences, and schools generate their priority lists solely according to their applicants’ academic ability. Figure 1 shows the admissions outcome under the AP scenario; the number in each region indicates which school the applicants in that region enter (a zero means not entering any school). We use \( c^1_i \) to denote the cutoff set by school \( i \) in state \( j \). School 1 admits only type-1 applicants (with \( y \leq c^1_2 \)). School 2 admits everybody else with academic ability \( y \leq c_2 \). That is, school 2 admits all enthusiastic (type-2) applicants with \( y \leq c^2_2 \) and all unenthusiastic (type-1) applicants with \( y \in (c^1_1, c^2_2) \), where \( c^1_2 = c^2_2 = c_2 \). At the end, in both states, only applicants with sufficiently good academic scores (\( y \leq c_2 \)) get admitted.\textsuperscript{13}

Figure 2 shows the admissions outcome under the RP scenario. Now each school admits both types of applicants. School \( i \) has cutoffs \( c^1_i \) and \( c^1_j > c^1_i \), where \( j \neq i \). In state \( i \), school \( i \) admits only enthusiastic type-\( i \) applicants, those with \( y \leq c^1_i \); in the

\textsuperscript{11} PT and NPT are known in the work of Chiu and Weng (2009) as the immediate acceptance policy (IA) and deferred-acceptance policy (DA), respectively.

\textsuperscript{12} In this paper, we focus on equilibria in which protagonists choose their weakly-dominant strategies whenever existent. For more details about the equilibrium, see Chiu and Weng (2009).

\textsuperscript{13} The two cutoffs are defined as follows: \( c_2 \equiv F^{-1} \left( (s_1 + s_2) / N \right) \) and \( c^1_i \equiv F^{-1} \left( s_i / (\mu^1_i N) \right) \). Despite uniform distribution of \( y \) used in the figure, as well as all subsequent figures, the results in the paper do not hinge on such a restriction.
Figure 1: Both schools use NPT in the benchmark. With absolute popularity, school 1 admits only enthusiastic applicants (type-1) in both states of world, in contrast with school 2, who surely admits both types of applicants.
Figure 2: Both schools use NPT in the benchmark. With relative popularity, each school gets to admit only enthusiastic applicants in its favorable state of world, while has to admit both types of applicants in its unfavorable state of world.

other state, it admits both types of applicants — enthusiastic (type-i) applicants with \( y \leq c_i^1 \) and unenthusiastic (type-j) applicants with \( y \in (c_j^1, c_j^2) \).\(^{14}\) At the end, only applicants with sufficiently good academic scores \( (y \leq c_2) \) get admitted in each state.

We call the aforementioned strategies used by applicants the benchmark strategies, the equilibrium the benchmark equilibrium, and the admissions outcome the benchmark outcome.

There is a group of type-1 applicants (with \( y \in (c_1^1, c_1^2) \)) that enter school 2 under state 1 despite their lack of enthusiasm for the school. These non-enthusiastic applicants

\(^{14}\)Notice that \( c_i^j \equiv F^{-1}\left( s_i / (\mu_i N) \right) \) and \( c_i^j \equiv F^{-1}\left( (s_1 + s_2) / N \right) \).
are precisely the ones to whom school 2 wants to deny admission, and the use of PT may be a means to this end. Notice that the presence of this group of applicants is predicated on the fact that $c_1^1 < c_2^1$, which results from the possibility of different states of nature. PT would have no bite if there were just one state of nature.

For ease of exposition, we have the following definition.

**Definition 1** Assume that $c_i^1 \neq c_j^1$. School $i$ is the leading school if $c_i^1 < c_j^1$. School $j$ is the non-leading school if it is not the leading school.

The leading school is determined through a comparison between the two schools’ admissions cutoffs in their respectively more selective states. According to this definition, in the AP scenario, school 1 is the leading school. In the RP scenario, school 1 is the leading school when $c_1^1 < c_2^1$ and school 2 is the leading school when $c_1^1 > c_2^1$. We assume, w.l.o.g., the former is the case and hence school 1 is also the leading school in the RP scenario.\(^{15}\)

### 4 Unilateral Incentive

Given the benchmark in which both schools use an NPT policy, we now study under what circumstances schools want to use PT. In this section, then, we consider whether one school can benefit from using PT, given that the other uses NPT, while relegating the question of equilibrium policy choices to the next section.

#### 4.1 When the non-leading school benefits from using PT

We first study school 2’s incentive for using to use PT, when school 1 does not use PT.

##### 4.1.1 Driving out effect

*Our first message is that even though school 2 values applicants’ enthusiasm, it may not be a good idea for it to use PT.* The reason is that non-enthusiastic applicants may be induced to mimic enthusiastic applicants through untruthful rank-ordering listings, so that the school would end up admitting more non-enthusiastic applicants than otherwise. Consider the AP scenario. Recall school 2 always admits some type-1 applicants (refer to Figure 1). In particular, type-1 applicants with $y \in (c_1^1, c_2^1]$ enter school 2 in state 1 and school 1 in state 2. Under the policy pair (NPT,PT), however, each such applicant faces a trade-off. By ranking school 1 as her top choice, the applicant will be admitted.\(^{15}\) Here, $c_1^1 < c_2^1$ if and only if $s_1/\mu_1^2 < s_2/\mu_2^2$.
by school 1 in state 2 and by no school in state 1, obtaining an expected utility of \( \pi_2 u_1 \); by ranking school 2 as her top choice, however, she will be admitted by school 2 in both states, obtaining an expected utility of \( u_2 \). The latter strategy is optimal if \( u_2 > \pi_2 u_1 \).

In this case, relative to the benchmark outcome, school 2 ends up admitting more non-enthusiastic applicants in state 2 (with admissions in state 1 unchanged) and is worse off.

When \( u_2 < \pi_2 u_1 \), by contrast, the type-1 applicants with \( y \in (c_1^1, c_1^2) \) will rank school 1 as their top choice, resulting in their rejections by both schools in state 1 and admissions by school 1 in state 2. As a result, in state 1, school 2 admits a group of applicants of both types (see the shaded region in the upper panel of Figure 3) in lieu of the aforementioned type-1 applicants. School 2 thus admits a larger fraction of enthusiastic applicants. Given that its admissions are unchanged in state 2, school 2 is better off because the PT succeeds in eliminating a set of non-enthusiastic applicants that would have been admitted under (NPT,NPT). We call this effect the driving-out effect.

### 4.1.2 Relative popularity

A similar insight prevails in the RP scenario (refer to Figure 2). School 2 admits some non-enthusiastic (type-1) applicants (with \( y \in (c_1^1, c_2^2) \) in the upper panel) in state 1 that it does not admit in state 2. Ideally, school 2 wants to eliminate these applicants, who can be further divided into two subgroups, one with \( y \in (c_1^1, c_2^2) \) and another with \( y \in (c_2^2, c_2^2) \). The first subgroup of applicants face exactly the trade-off described earlier in the AP scenario. If \( u_2 < \pi_1 u_1 \), these applicants will rank school 1 as their top choice; hence, they will not be admitted by school 2, by contrast with the benchmark solution.

The second subgroup, however, cannot be driven out as they do not really face such a trade-off. By ranking school 1 as their top choice, they will be admitted by school 1 in state 2 but not admitted by any school under state 1. By ranking school 2 as their top choice, they are guaranteed to be admitted by school 2 under state 1, as under the benchmark. Because of their weaker \( y \)'s, they will not be admitted by school 2 in state 2. Instead, they will be admitted by school 1 after being turned down by school 2 thanks to school 1’s NPT policy. Therefore, top-ranking school 2 is their dominant strategy, and the use of PT cannot drive out this subgroup of unenthusiastic students for school 2.

The following summarizes our discussion (unless otherwise stated, all proofs of Propositions will be relegated to the appendix).

**Proposition 1** Suppose school 1 is the leading school and school 2 is the non-leading
Figure 3: Given school 1’s use of NPT and $u_2 < \pi_2 u_1$, school 2 has the incentive to use PT unilaterally, so as to drive out type-1 applicants with $y \in (c_1^1, c_1^2)$ for more admissions of type-2 applicants in state 1.
Figure 4: Unlike school 2, school 1 doesn’t gain from using PT unilaterally. Even in this case with relative popularity, where school 1 wishes to drive out type-2 applicants with $y \in (c_2^2, c_2)$, the effect of PT is neutralized by those applicants’ strategic reporting of school 1 as their top choice.
1. (The driving-out effect) If $u_2 < \pi_2 u_1$, school 2 admits a larger fraction of enthusiastic applicants under $(NPT, PT)$ than under $(NPT, NPT)$.

2. (Self-defeating PT) If $u_2 > \pi_2 u_1$, school 2 admits a smaller fraction of enthusiastic applicants under $(NPT, PT)$ than under $(NPT, NPT)$.

The driving-out effect occurs exactly under the conditions in which the stealing effect is absent in our earlier paper. In that paper, school 2 seeks to attract students with a higher academic ability notwithstanding their lower enthusiasm, and benefits from using PT when succeeding in doing so. This is the stealing effect identified there. Because of a difference in preferences, in the current paper, school 2 is not interested in such students at all and wishes to drive them out.

### 4.2 The leading school is indifferent between using PT or not

We next turn to school 1’s incentive of using PT. We argue that school 1 is indifferent between using PT and NPT, given that the other school uses NPT. This result is easy to understand for the AP scenario. Since school 1 admits only type-1 applicants under (NPT,NPT), there are no unenthusiastic students for it to drive out. Using PT or NPT will bring in the same intakes. In the RP scenario, school 1 admits some type-2 applicants (those with $y \in (c_2^2, c_2]$) in state 2 that it wishes to turn down if possible (refer to figure 4). It is impossible to drive out this group under (PT,NPT), however, as this group faces no trade-off. They can easily circumvent school 1’s driving-out intention by ranking school 1 as their top choice. By so doing, they will be admitted by school 1 in state 2, as under the benchmark. They will also be admitted by school 2 in state 1 after being turned down by school 1.

**Proposition 2** Suppose that school 1 is the leading school and school 2 is the non-leading school. For both absolute-popularity and relative-popularity scenarios, school 1’s admitted applicants under (PT,NPT) are the same as under (NPT,NPT).

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16 Under (PT,NPT), all type-1 applicants will continue to rank school 1 as their top choice. Any type-2 applicant that is strong enough to be admitted by school 1 under (PT,NPT) could easily get admitted by school 2 and hence will not enter school 1 under (PT,NPT). It then follows that school 1’s intakes are all of type-1, and they are also the best among this type.

17 In equilibrium, all of these applicants will rank school 1 as their top choice. By doing so, they are certain to be admitted by school 1 under state 2. In addition, they will still be admitted by school 2 in state 1.
4.3 The effects of using PT on the other school

Thus far, we have examined the incentive of a school for using PT when the other school continues to use NPT. A related question is how the other school (that uses NPT) is affected by this school’s switching to PT. We obtain the following proposition.

**Proposition 3**

1. Consider a switch from (NPT,NPT) to (NPT,PT). If school 2 is not worse off, school 1 is not worse off either.

2. Consider a switch from (NPT,NPT) to (PT,NPT). School 2 is not worse off (it admits exactly the same applicants).

The intuition of Result 1 is as follows. According to Proposition 1, given school 1 adheres to NPT, whenever school 2 benefits from switching to PT, it benefits it through driving out non-enthusiastic (type-1) applicants in some state. Because of this, the set of type-1 applicants available for school 1’s admissions is enlarged and hence school 1 cannot be made worse off.\(^\text{18}\) The intuition of Result 2 is similar. According to Proposition 2, school 1’s use of PT has no effect on its admissions: it does not take away any students from school 2’s benchmark admissions. School 2 is thus not worse off.

The bottom line is that, the use of PT by a school is generally benign to the other school. Because of heterogeneous school preferences, any successful attempt by one school to improve its intakes means driving out unenthusiastic applicants, who will then join the pool of enthusiastic applicants that the other school can now admit.

4.4 The effects of using PT on applicants

When PT is used, some enthusiastic applicants who are rejected by their preferred school under the benchmark may now succeed in getting admitted by that school. This is the main merit of using PT. That said, there are also drawbacks. First, students’ ranking of schools now becomes a strategic choice, and not every student is well informed as to how the system is run, and advising students is generally difficult. Moreover, applicant welfare may be non-monotonic as a function of \(y\) – students with weaker \(y\) may end up having a greater expected utility than students of the same type that have stronger \(y\). This raises fairness concerns.\(^\text{19}\)

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\(^{18}\)Whenever it is detrimental for school 2 to switch to PT, it hurts through driving in non-enthusiastic (type-1) applicants in some state, hence harming school 1 by taking away its enthusiastic applicants.

\(^{19}\)To see this, consider the policy pair (NPT,PT) under the RP scenario. Suppose \(u_2 < \pi_2u_1\) so that school 2 indeed benefits from adopting PT given school 1 uses NPT. Recall that in equilibrium, the type-1 applicants with \(y \in [c_1^*, c_2]\) will not enter any school in state 1 and will enter school 1 in state 2, obtaining an expected utility of \(\pi_2u_1\), while the type-1 applicants with \(y \in [c_2^*, c_2]\) will enter school 2 in state 1 and school 1 in state 2, obtaining a greater expected utility of \(\pi_1u_2 + \pi_2u_1\).
5  Policy-Choice Game

In this section, we study equilibrium behavior when the two schools choose between NPT and PT non-cooperatively. To this end, we first examine the incentive of a school to also use PT when the other school uses PT.

5.1  The best response to PT

Suppose $u_2 < \pi_2u_1$ and school 2 uses PT.\(^{20}\) We want to understand school 1’s incentive to use PT. In the AP scenario, under (NPT,PT), school 1 will admit only type-1 applicants in both states and these applicants are the best of the type. Because of this, school 1 cannot improve further by switching to PT, although it will not worsen by doing so either. In other words, school 1 is indeed indifferent between using or not using PT.

The internalization effect  The RP scenario is more interesting. As explained in Section 4.2, school 1’s use of PT under (PT,NPT) is rendered ineffective because the non-enthusiastic (type-2) applicants targeted for driving out (the targeted group in short) will report school 1 as their top choice as they do not face a trade-off. Under (PT,PT), however, they do face a trade-off. By reporting school 2 as their top choice, each such applicant will be admitted by school 2 in state 1 and no school in state 2, obtaining a utility of $\pi_1v_2$. By reporting school 1 as her top choice, each such student will be admitted by no school in state 1 and school 1 in state 2, obtaining a utility of $\pi_2v_1$. The latter strategy is optimal if $\pi_1v_2 < \pi_2v_1$. In this case, the targeted group will strategically rank school 1 as their top choice, rendering school 1’s PT irrelevant – school 1 will have the same intakes as under (NPT,PT). If $\pi_1v_2 > \pi_2v_1$, however, the targeted group will report school 2 as their top choice, allowing school 1 to drive them out. School 2’s use of PT is out of self-interest, but it turns out to be beneficial for school 1, too. School 1 can benefit from using PT through internalizing the effect from school 2’s use of PT. We call this effect the internalization effect, and note in passing that there is no counterpart of this effect in our earlier paper.

Proposition 4 Suppose $u_2 < \pi_2u_1$.

1. Under the absolute-popularity scenario, school 1 is indifferent between (NPT,PT) and (PT,PT), admitting exactly the same applicants.

\(^{20}\)The case where $u_2 > \pi_2u_1$ is uninteresting because, as Proposition 5 will show, in this case school 2’s strictly dominant strategy is to use NPT, and the use of PT by school 2 will not occur in equilibrium.
2. Under the relative-popularity scenario, school 1 admits a greater fraction of enthusiastic applicants under (PT,PT) than under (NPT,PT) if $\pi_1 v_2 > \pi_2 v_1$ and admits exactly the same applicants if $\pi_1 v_2 < \pi_2 v_1$.

School 2’s best response given school 1’s use of PT is more straightforward, as summarized in the following proposition.

**Proposition 5**

1. Suppose $u_2 < \pi_2 u_1$. School 2 admits a larger fraction of enthusiastic applicants under (PT,PT) than under (PT,NPT).

2. Suppose $u_2 > \pi_2 u_1$. School 2 admits a smaller fraction of enthusiastic applicants under (PT,PT) than under (PT,NPT).

The proposition states that, given school 1’s use of PT, it is optimal for school 2 to use PT if $u_2 < \pi_2 u_1$ and to use NPT if $u_2 > \pi_2 u_1$. Together with Proposition 1, it suggests that school 2’s incentive to use PT is independent of school 1’s policy.

### 5.2 Equilibrium

We are now ready to state our results on the equilibrium of the policy-choice game. The AP scenario is straightforward. School 1’s role is passive, as it is indifferent between using PT or not, regardless of school 2’s policy choice. Thus, the equilibrium is solely determined by school 2’s incentive, and it will choose PT if and only if $u_2 < \pi_2 u_1$. The results are summarized in the following proposition.

**Proposition 6** Suppose absolute popularity.

1. Suppose $u_2 < \pi_2 u_1$. Then there exist two (pure-strategy) equilibria, (NPT,PT) and (PT,PT), which are admissions outcome equivalent.

2. Suppose $u_2 > \pi_2 u_1$. Then there exist two (pure-strategy) equilibria, (NPT,NPT) and (PT,NPT), which are admissions outcome equivalent.

The RP scenario is more complicated. When $u_2 > \pi_2 u_1$, school 2’s dominant strategy is to use NPT regardless of whether school 1 uses NPT or PT (Propositions 1.2 and 5.2, respectively). Hence, the equilibrium must entail school 2 using NPT. Taken into account that school 1 is indifferent between using PT or NPT (Proposition 2) in this case, there are two outcome-equivalent equilibria (NPT,NPT) and (PT,NPT).

When $u_2 < \pi_2 u_1$, any pure-strategy equilibrium, if existent, must entail school 2 choosing PT (Propositions 1.1 and 5.1 together imply that using NPT is a strictly
dominated strategy for school 2). We argue that such an equilibrium does indeed exist. We further clarify that, when $\pi_1 v_2 > \pi_2 v_1$ school 1 also adopts PT so that there exists a unique equilibrium (PT,PT), in which both schools are strictly better off compared with under (NPT,NPT). When $\pi_1 v_2 < \pi_2 v_1$, school 1 is indifferent between using PT or NPT and as a result there exists two outcome equivalent equilibria (NPT,PT) and (PT,PT). We summarize our results as follows.

**Proposition 7** Suppose relative popularity.

1. Suppose $u_2 < \pi_2 u_1$. If $\pi_1 v_2 > \pi_2 v_1$, there exists a unique (pure-strategy) equilibrium (PT,PT), which is better than (NPT,NPT) for both schools.

2. Suppose $u_2 > \pi_2 u_1$. Then there exist two (pure-strategy) equilibria, (NPT,NPT) and (PT,NPT), which are admissions outcome equivalent.

There are a few lessons to be drawn. First, the main merit of the mechanism studied is that it allows schools to use applicants’ rank-order lists to ascertain their enthusiasm. As a result, some enthusiastic students that otherwise would not be admitted by their preferred school are admitted when the use of PT is permitted.

Second, in general it is the weaker school that benefits from the use of PT. The reason is that, without PT, some enthusiastic yet less able applicants will be crowded out by unenthusiastic yet more able applicants. This possibility rarely occurs for the leading school, because the admitted applicants must be sufficiently able academically (as denoted by $y$).

The third lesson is that, the leading school may benefit from the use of PT by another school. This suggests that school competition when student enthusiasm is valued is less severe than in the case when it is not.

Our fourth and last lesson is that the use of PT also has its drawbacks. Despite all its merits, there is still a possibility that it will lead to pooling by unenthusiastic students. Such a case is more likely when the utility from attending the less-preferred school is not too low ($u_2/u_1$ is not too low) or the probability of getting admitted to that school in the benchmark case is large ($\pi_1$ is large). In such cases, compelling schools to use PT is harmful and a mechanism like the Boston mechanism is unwarranted, even if schools care about enthusiasm.
6 An Alternative Formulation

In this section, we explore another model that captures a different aspect of uncertainty. This alternative model is the same as the baseline model, except for the following differences. The fraction of type-1 (type-2) applicants in the pool is $\mu_1 \in (0, 1)$ ($\mu_2 \equiv 1 - \mu_1$) and is commonly known; thus there is no uncertainty about the intrinsic popularity of the two schools. We still assume that school 1 is more popular: $\mu_1/s_1 > \mu_2/s_2$. As before, schools continue to observe applicants’ abilities but not their enthusiasm. Each applicant, however, knows her own enthusiasm but not her own ability.\(^{21}\) Therefore, she is uncertain of her percentile in ability. This formulation is known as the ranking-uncertainty model as in our earlier paper.\(^{22}\)

The benchmark outcome is characterized as follows. Under the policy pair (NPT,NPT), applicants simply submit truthful rank-order lists. School 1 admits all type-1 applicants with $y < c_1$. School 2 admits all type-2 applicants with $y < c_2$ and all type-1 applicants with $y \in (c_1, c_2]$ (refer to Figure 5). School 2, which values applicants’ enthusiasm, thus has an incentive to drive out the non-enthusiastic (type-1) applicants that it would have to admit in this benchmark.

The equilibrium under (NPT,PT) is easy to characterize. When $u_2/u_1$ is sufficiently low (refer to Figure 6), no type-1 applicant bothers to rank school 2 as her top choice despite the latter’s PT policy. As a result, school 2 succeeds in driving out all unenthusiastic applicants, while school 1 has the same admissions as under (NPT,NPT). When $u_2/u_1$ is high enough (see Figure 7), some type-1 applicants will rank school 2 as their top choice. School 2 will fill its capacity with those that ranked it as their top choice, setting a cutoff at $y = c_2'$. Still admitting some type-1 applicants, school 2 will admit a greater fraction of type-2 applicants and is better off relative to the benchmark. School 1 still admits only type-1 applicants, but because some better type-1 applicants are lost to school 2, school 1 is worse off (evidenced by a less stringent admissions cutoff at $y = c_1'$ in contrast to $c_1$).

Proposition 8 Define $r \equiv \frac{\mu_2 s_1}{\mu_1 s_2}$. Consider a switch from (NPT,NPT) to (NPT,PT).

1. If $u_2/u_1 < r$, school 2 is better off, admitting only enthusiastic applicants; and school 1’s admissions remain unchanged.

\(^{21}\) A more realistic model is one in which the applicant has some noisy, informative signal about her own ability.

\(^{22}\) The model here differs from that model in two aspects. The first is that schools here value enthusiasm while schools there do not; the second is that there are two types of applicants here, while only one type of applicant is considered there.
Figure 5: Both schools use NPT in the benchmark. This figure shows the admissions outcome under ranking uncertainty. School 1 admits only type-1 applicants, while school 2 admits applicants of both types.
Figure 6: Assuming $u_2/u_1 < r \equiv \frac{\mu_2}{\mu_1} \frac{\alpha_2}{\alpha_1}$, this figure shows the admissions outcome under (NPT, PT). Each school admits only applicants who are enthusiastic towards it.
Figure 7: Assuming \( u_2/u_1 > r \equiv \frac{\mu_2}{\mu_1} \frac{\sigma_1}{\sigma_2} \), this figure shows the admissions outcome under \((NPT, PT)\). Some type-1 applicants choose to top rank school 2, who thus admits applicants of both types. Still admitting type-1 applicants only, school 1’s average quality of admissions is worse off.
2. If \( u_2/u_1 > r \), school 2 is better off, admitting a greater fraction of enthusiastic applicants; and school 1, while still admitting only enthusiastic applicants, is worse off because of the weaker average ability of applicants admitted.

The cutoff of \( u_2/u_1 \), denoted by \( r \) here, is characterized by an indifference condition \((s_1/\mu_1 N)u_1 = (s_2/\mu_2 N)u_2\), where the left- (right-) hand side is the expected utility that a type-1 applicant will receive by ranking school 1 (2) as her top choice, provided that all other applicants truthfully report their preferences. The condition can then be rewritten as \( \frac{u_2}{u_1} = r \equiv \frac{s_1}{s_2} \frac{\mu_2}{\mu_1} \).

Next we turn to school 1’s incentive for using PT. Under (NPT,NPT), school 1 already admits only type-1 applicants and they are the best of this type. As a result, switching to PT will not further improve the quality of its intakes.

Proposition 9 The admissions outcome under (NPT,NPT) is the same as that under (PT,NPT).

Now we turn to the equilibrium policy choices.

Proposition 10 Consider the policy-choice game in which both schools choose between NPT and PT independently and simultaneously. There exist two (pure-strategy) equilibria (PT,PT) and (NPT,PT), which are outcome equivalent.

The intuition is as follows. School 1, which has a higher intrinsic popularity, admits only those applicants that rank it as their top choice in both equilibria. Therefore, there is no difference whether school 1 uses PT or not, and this explains why the two equilibria are outcome equivalent.

In summary, in this formulation, we have found similar results as under the baseline, demand-uncertainty model. We still find the main merit that, enthusiastic students that would not be admitted by their preferred school under the benchmark (NPT,NPT) outcome are admitted when PT is permitted. Moreover, as in the baseline model, the non-leading school has a stronger incentive to contemplate using PT, and the leading school has no such incentive on its own.

We have also found different results. First, the non-leading school has a stronger incentive to use PT in this formulation than in the baseline formulation. Under the baseline formulation, choosing PT may be harmful to it. It is never the case here. Second, it is more likely for the leading school to be hurt by school 2’s use of PT. This happens in the baseline model under a very unlikely situation – when school 2’s use of PT also hurts itself. Third, in the baseline model, there is some hope that school 1 may
actually benefit from school 2’s use of PT (through the internalization effect). But such a possibility does not exist in the alternative model.

There is another interpretation of the applicant’s "ability". Instead of ability, it is indeed a random number generated by a mechanism that schools use in tie-breaking and that the same random number is assigned to a student throughout the admissions exercise.\textsuperscript{23} With this interpretation, all students are grouped into one indifference class and a student with lower $y$ does not necessarily mean that it is academically more qualified. Because of this, the harmful effect that school 2’s use of PT will have on school 1 does not exist (Proposition 8.2).

We end this section with some discussion of Avery and Levin (2011), in which they study early admissions\textsuperscript{24} by two schools in a decentralized admissions setting. As in the present paper, schools value both student ability and enthusiasm. Because each student is allowed to apply to one school through early admissions, the early admissions option allows students to signal their enthusiasm. Enthusiasm is a continuous variable, and through early decisions the leading school trades some high-ability low-enthusiasm students for low-ability high-enthusiasm students. This leads to unclear trade-offs. For us, enthusiasm is a discrete variable. Through preferential treatment the leading school trades high-ability students for low-ability students of the same level of enthusiasm and it thus must be worse off, in the presence of ranking uncertainty.

7 Discussions

7.1 Multiple schools

Our baseline model comprises only two schools. We want to explore whether the insights obtained can be generalized to more than two schools.

Multiple driving-outs Our baseline model identified the beneficial driving-out effect whereby a weaker school benefits from using PT. We also identified a scenario in which the use of PT is self-defeating (we can call the latter the driving-in effect). In that model, the school experiences either effect, but not both. We can easily envision the interplay

\textsuperscript{23}When randomization is used, there is a question as to whether an applicant’s random number should be fixed for the whole exercise, or should be redrawn for each school. Erdil and Ergin (2008) show that, in an expected sense, the fixed number scheme is more beneficial to students.

\textsuperscript{24}There are two types of early admissions: early action and early decision. The former refers to the case where schools are committed to enrolling the students that they have accepted although students have no commitment to enroll. The latter refers to the case where students are committed to enrolling once accepted.
of both of effects in a multiple-school setting.

Consider a model with three schools, $q_1$, $q_2$, and $q_3$. Suppose there are three types of applicants (type-1, 2, and 3) with the following strict preferences: $q_1 \succ_1 q_2 \succ_1 q_3$, $q_2 \succ_2 q_1 \succ_2 q_3$, and $q_3 \succ_3 q_1 \succ_3 q_2$. We assume that each school seeks to admit only those applicants that see it as their most preferred school, but does not distinguish the other two types of students in terms of enthusiasm.

There are two states of the world with probabilities $\pi_1$ and $\pi_2 \equiv 1 - \pi_1$ so that the benchmark outcome under policy profile (NPT,NPT,NPT) is depicted in Figure 8. As depicted, under the benchmark outcome, type-1 applicants with $y \in [c_1^1, c_1^2]$ enter school 1 in state 2 and school 2, a less-preferred school, in state 1; type-3 applicants with $y \in [c_3^2, c_3^3]$ enter school 3 in state 1 and school 2, a less-preferred school, in state 2. These applicants are thus concerned about the use of PT by school 2. School 2 is able to drive out the aforementioned type-1 applicants in state 2 and the aforementioned type-3 applicants in state 1, provided that:

\begin{align*}
  u_2(1) &< \pi_2 u_1(1) & \text{(for type-1 applicants)} \\
  u_2(3) &< \pi_1 u_3(3) & \text{(for type-3 applicants)}
\end{align*}

and that no other schools use PT. As a numerical example, the two conditions are indeed satisfied for all $\pi_1 \in (0.2, 0.5)$ if $u_1(1) = u_3(3) = 10$ (utility from the most-preferred schools), $u_2(1) = 5$ (utility from the moderately-preferred schools) and $u_2(3) = 2$ (utility from the least-preferred schools).

**Simultaneous driving-in and driving-out** The above parameterization supports the driving out of more than one type of unenthusiastic applicant. The next parameterization suggests the presence of both driving-in and driving-out effects, when PT is used. Consider a change such that $\pi_1 \in (0.2, 0.5)$. Then, while (6) still holds true, (5) is violated so that

\[ u_2(1) > \pi_2 u_1(1) \quad \text{(for type-1 applicants)} \]

In other words, under (NPT,PT,NPT), while still driving out type-3 applicants under state 1, school 2 will also drive in the type-1 applicants under state 2. Hence, whether school 2 benefits from using PT depends on which effect is more dominant. It is easy to verify that either is possible. The more important message is that the driving-out and driving-in effects can be triggered by one act of PT.
Figure 8: In this multiple-school case, under some condition, school 2’s use of PT is expected to drive out type-3 applicants with $y \in (c_3^2, c_3^1)$ in state 1 and type-1 applicants with $y \in (c_1^1, c_1^2)$ in state 2.
7.2 First preference first versus Equal preference (in U.K.)

Our results rationalize those admissions committees’ preference for using the FPF system to admit the more-committed students (as mentioned in Section 1). This paper reconciles the competing arguments on FPF. The main argument against the use of the FPF system is that it causes some applicants to unnecessarily lose their second- or third-choice schools, which are filled up by those that only strategically report that school as their first choice. We have shown that this only happens when all schools are similarly desirable to applicants. When the desirability of the preferred school is significantly higher than that of the less-preferred school, FPF may actually allocate more applicants to their first-choice schools by replacing those who only like the school less, but get admitted under the equal-preference system. In this sense, mandatory use of non-preferential treatment systems (i.e., equal-preference system) comes at a cost.

In practice, giving schools the choice of PT also means giving applicants choice to overcome some imperfectness of the existing (non-preference) over-subscription criteria\(^{25}\), in addition to signaling their high preference intensities towards a school. As has been emphasized in both policy debate and the literature, allowing for school choice is important so that students’ are not restricted, for example, to the choices of their neighborhood schools only. To some extent, this helps enhance social justice, which is an essential concern of policy discussions on the transition from FPF to the equal-preference system in England.\(^{26}\) The equal-preference system, that heavily relies on the exogenously given over-subscription criteria to allocate seats among students,\(^{27}\) thus provokes complaints by parents that are disadvantaged by those criteria.\(^{28}\) Moreover, the very constrained number of school choice (mainly from three to six choices allowed) also imposes a challenge for policy makers, who firmly believe that switching to the mandatory equal-preference system relieves parents from the burden of playing the school-choice game (see Haeringer and Klijn (2009) and Calsamiglia, Haeringer and Klijn (2010)).

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\(^{25}\)Over-subscription criteria are like "priorities" in the one-sided school choice problem, which are counterpart of school preferences towards students in the two-sided college admissions problem. See Gale and Shapley (1962).

\(^{26}\)See page 32 of the Research Report DCSF-RR020 by Coldron et.al. (2008).

\(^{27}\)Under the equal-preference system, all applicants are considered equally by each school they list as a choice. Over-subscribed schools allocate seats according to applicants’ priorities. Only when an applicant gets short-listed by more than one school will their own preference list comes into play to decide which school she will attend.

\(^{28}\)A parent complained that: "...the shameful 'equal preference' proposal which would allow the council to claim first choices have been offered to all parents, because all choices are now to be first choices – compound the injustice of the proposed changes." This can be found at http://www.theargus.co.uk/archive/2005/11/%2024/The+Argus+Archive/6807149.Letter_—_%20Don_t_just_shift_schools_problem_to_other_areas/
Given that most applicants still have to play strategically under constrained choice, FPF may outperform the equal-preference system in rewarding parents' choices, which better reflect individuals' various concerns and not just the factors captured by over-subscription criteria and the fixed weights placed on those factors.

Under the equal-preference system, every selective school faces many more "first-choice" applicants than under the FPF system, lowering the effectiveness of the existing over-subscription criteria and increasing the use of random tie-breaking rules. This problem is well recognized by parents, as one of them pointed out "...it fills me with dismay that Brighton and Hove City Council is putting its trust for sorting out children's futures in a computer system"\textsuperscript{29} under the equal-preference system in contrast with the FPF system, where they're allowed to have a say. The powerful effect of exogenously given priorities on the matching outcome is also verified in theory by Calsamiglia and Miralles (2012). They show that, when either the deferred-acceptance mechanism or the Boston mechanism is used, the resulting allocation may fully respond to the priorities regardless of parents' listed preferences, if all individuals agree on which schools are the worse.

In a word, both the relative desirability of schools among applicants and the school popularity distribution\textsuperscript{30} are worth more consideration in choosing between FPF and the equal-preference system. When schools indeed concern about student enthusiasm and have different comparative advantages, allowing scope for schools to admit more committed students seems sensible.\textsuperscript{31} It is not intended by this paper to thoroughly compare these two competing systems. What we suggest is more consideration should be made before we take away from schools, and parents in effect, the choice between these two admissions policies.

8 Concluding Remarks

This paper concerns the use of endogenous preferential treatment in many centralized admissions systems. Unlike exogenous preferential treatment, endogenous preferential treatment has drawn little attention among economic theorists. In particular, it has not been shown why a central mechanism with the option of preferential treatment

\textsuperscript{29}This can be found at http://www.theargus.co.uk/opinion/letters/1123694.Substance_abuse/.

\textsuperscript{30}Desirability of a school means the preference intensity an applicant has towards a school, i.e., how much utility an applicant can draw from entering the school.

\textsuperscript{31}The AP and RP scenarios can be seen as two cases with different school popularity distributions.

The New York City high school matching system remains a two-sided one, where schools are allowed to have a say of their preferences over students, even after the reformation in year 2003. See Abdulkadir{	extendash}oglu, Pathak and Roth (2005, 2009) for more details about the NYC high school match system.
would exist in the first place. In this paper, we argue that a most natural reason is that schools are concerned about student enthusiasm. The use of preferential treatment gives enthusiastic students, who truthfully report their preferences, higher priority than to non-enthusiastic students. If the use of preferential treatment results in separation, the school will be able to learn more about applicants’ enthusiasm and preferential treatment is thus beneficial. We have conditions under which some enthusiastic students that otherwise would not be admitted are admitted by their preferred school when preferential treatment is adopted.

If the use of preferential treatment results in pooling by unenthusiastic students, however, the rank-order lists are too noisy to reflect applicants’ enthusiasm and using preferential treatment may be self-defeating. This suggests that mandatory preferential treatment, as in the Boston mechanism, is not beneficial to schools even if they cherish student enthusiasm. Instead, an admissions system in which schools are left to decide whether to adopt preferential treatment on their volition is a better system. In this sense, we justify the use of centralized admissions that permits endogenous preferential treatment.
References


Appendix: Proofs

Proof of Proposition 1

Proof. For the AP scenario, the equilibrium admissions outcome under (NPT,PT) is the same as its counterpart under (DA,IA) in Chiu and Weng (2009) and is fully described in the proof of Proposition 1 in that paper. As a result we focus in what follows on the RP scenario, which is new.

Case 1: \( u_2 < \pi_2 u_1 \). We first describe the applicants’ equilibrium strategies. Type-2 applicants always rank school 2 as their top choice and school 1 as their second choice. Type-1 applicants’ strategies are as follows: if \( y \leq c_1^2 \), rank school 1 as top choice; if \( y \in (c_2^2, \bar{c}] \), rank school 2 as top choice; if \( y > \bar{c} \), strategies are irrelevant, where \( \bar{c} \) is solved from

\[
\mu_1^1 \left( F \left( c_2^2 \right) - F \left( c_1^1 \right) \right) = F \left( \bar{c} \right) - F \left( c_2 \right). \tag{7}
\]

The equilibrium admissions outcome is as follows. In state 2, it is exactly as under (NPT,NPT), as depicted in the second panel in Figure 4. In state 1, it is the same as its counterpart under (NPT,NPT) (which is depicted in the first panel) except for the following changes: the type-1 applicants highlighted there are not admitted by any school, and school 2 now also admits all students with \( y \in (c_2^2, \bar{c}] \) in lieu of them. This shows result 1.

Case 2: \( u_2 > \pi_2 u_1 \). We first describe the equilibrium strategies. Type-2 applicants always rank school 2 as their top choice and school 1 as their second choice. Type-1 applicants’ strategies are as follows: if \( y \leq c_1^1 \), rank school 1 as top choice; if \( y \in (c_1^1, c_2] \), rank school 2 as top choice; if \( y > c_2 \), strategies are irrelevant. Under state 1, the admission outcome is exactly the same as under (NPT,NPT). Under state 2, school 1 admits all type-1 applicants with \( y \leq c_1^1 \), as well as all applicants with \( y \in (c^*, c^{**}] \), where \( c^* < c_2^2 \) and \( c^{**} < c_2 \) satisfying

\[
\mu_1^2 \left( F \left( c^* \right) - F \left( c_1^1 \right) \right) = \mu_2^2 \left( F \left( c_2^2 \right) - F \left( c^* \right) \right) = F \left( c_2 \right) - F \left( c^{**} \right); \tag{8}
\]

and school 2 admits all type-2 applicants with \( y \leq c^* \), as well as all type-1 applicants with \( y \in (c_1^1, c^*] \). Relative to (NPT,NPT), school 2 admits the same group of type-2 students in state 1 but a smaller fraction of type-2 students in state 2. This shows result 2. \( \blacksquare \)
Proof of Proposition 2

**Proof.** The scenario of AP is straightforward and omitted. For the RP scenario, referring to Figure 4, the equilibrium under (PT,NPT) is as follows. (There is no need to distinguish between the case where \( u_2 < \pi_2 u_1 \) and the other case where \( u_2 > \pi_2 u_1 \).) All type-1 applicants rank school 1 as their first choice (this being their weakly dominant strategy). All type-2 applicants with \( y \leq c_2^2 \) rank school 2 as their first choice. Type-2 applicants with \( y \in (c_2^2, c_2) \) rank school 1 as their top choice. All other type-2 applicants’ strategies are inconsequential and need not be specified. This leads to the following admissions outcome. School 1 admits all type-1 applicants with \( y \leq c_1^1 \) in state 1 and all type-1 applicants with \( y \leq c_2 \), as well as all type-2 applicants with \( y \in (c_2^2, c_2) \), in state 2. School 2 admits all type-2 applicants with \( y \leq c_2 \) as well as all type-1 applicants with \( y \in (c_1^1, c_2) \) in state 1, and admits all type-2 applicants with \( y \leq c_2^2 \) in state 2. Relative to (NPT,NPT), school 1 has exactly the same intakes under either state. ■

Proof of Proposition 3

**Proof.** Result 1. According to Proposition 1, a switch from (NPT,NPT) to (NPT,PT) will benefit school 2 if and only if \( u_2 < \pi_2 u_1 \). The equilibrium admissions outcome under (NPT,PT) when this condition holds is fully described in the proof of Proposition 1. As presented there, school 1 admits exactly the same as under (NPT,NPT), whether it is AP or RP.

Result 2. The scenario of AP is straightforward and thus omitted. For the scenario of RP, the equilibrium admissions outcome under (PT,NPT) is fully described in the proof of Proposition 2. As presented there, school 1 admits exactly the same as under (NPT,NPT), whether it is absolute popularity or relative popularity. ■

Proof of Proposition 4

**Proof.** Suppose \( u_2 < \pi_2 u_1 \). Result 1. Given AP, we argue that the admissions outcome under (PT,PT) is exactly the same as that under (NPT,PT). The equilibrium strategies under (PT,PT) are as follows. All type-1 applicants with \( y \leq c_1^2 \) rank school 1 as their top choice. All type-1 applicants with \( y \in (c_1^2, c') \) rank school 2 as their top choice, where \( c' \) is solved from

\[
\mu_1^1 \left( F\left( c_1^2 \right) - F\left( c_1^1 \right) \right) = F\left( c' \right) - F\left( c_2 \right) .
\]
All type-2 applicants rank school 2 as their top choice. It is easy to verify these constitute an equilibrium and the corresponding admissions outcome identical to that under (NPT,PT).

Result 2. Given RP, there are two cases to consider. Suppose $\pi_1v_2 > \pi_2v_1$. Under (PT,PT), the applicants’ equilibrium strategies for those with $y \leq c_2$ are simply truth-telling: type-1 (2) applicants with $y \leq c_2$ ranks school 1 (2) as top choice. (We do not need to specify the strategies for those with $y > c_2$.) School 1 now admits the same intakes – all type-1 applicants with $y \leq c_1$ – in state 1 as under (NPT,PT) (which is the same as under the benchmark). In state 2, its intakes include all type-1 applicants with $y \leq c_2$ and do not include any type-2 applicants with $y \in (c_2^2, c_2)$, and the vacancies are now open for admissions of other applicants, which must include some type-1 applicants. As a result, the set of type-1 applicants in its intakes must be a superset of its counterpart under the benchmark. In a whole, school 1 is better off. (We also consider school 2 for completeness. In state 1, its intakes include all type-2 applicants with $y \leq c_2$ and does not include any type-1 applicants with $y \in (c_1^1, c_2]$, and the vacancies are now open for admissions of other applicants, which must include some type-2 applicants. In state 2, its intakes are exactly the same as under the benchmark. In the whole, school 2 is also better off.) (See Figure 9 for the admissions outcome under (PT,PT).)

Consider another case where $\pi_1v_2 < \pi_2v_1$. Under (PT,PT), the applicants’ equilibrium strategies are as follows. Type-1 applicants with $y \leq c_2$ rank school 1 as top choice. Type-2 applicants with $y \leq c_2$ rank school 2 as top choice. Type-2 applicants with $y \in (c_2^2, c_2]$ rank school 1 as top choice. All applicants with $y \in (c_2, \bar{c}]$ rank school 2 as their top choice, where $\bar{c} > c_2$ and is solved from

$$F(\bar{c}) - F(c_2) = \mu_1^1 (F(c_2) - F(c_1)) + \mu_2^1 (F(c_2) - F(c_2^2)).$$

As a result, in state 1, school 1 admits the same intakes as the state under the benchmark; in state 2, it also admits the same intakes as in the state under the benchmark. (School 2 has the same intakes in state 2 as under the benchmark. Its intakes in state 1 change, however. More specifically, it admits all type-2 applicants with $y \leq c_2^2$ and all applicants (of both types) with $y \in (c_2, \bar{c}]$. Compared with under (NPT,PT), its fraction of enthusiastic students is smaller.)

Proof of Proposition 5

Proof. Result 1. Suppose $u_2 < \pi_2u_1$. As long as school 2 uses PT, those swinging type-1 applicants (with $y \in (c_1^1, c_2^2]$ in the RP case and $y \in (c_1^1, c_1^2]$ in the AP case,
Figure 9: Suppose $u_2 < \pi_2 v_1$ and $\pi_1 v_2 > \pi_2 v_1$. The admissions outcome under (PT,PT).
respectively) will insist reporting school 1 as their top choice to avoid being admitted to school 2 for certain. This leads to no difference between using PT or not for school 1. Hence, given school 1’s use of PT, school 2 would definitely prefer using PT to NPT, which allows those swinging type-1 applicants to be admitted as second-choice students, namely, (PT,PT) is preferred to (PT, NPT) by school 2.

Result 2. Suppose $u_2 > \pi_2u_1$. Those swinging type-1 applicants (as mentioned above) now find it optimal to save themselves each a seat at school 2 for certain. Even with their preferred school 1’s use of PT, with only $y < c_1^1$, ranking it as their top school doesn’t incur higher chance of admissions to school 1. Hence, if school 2 uses PT as well, this only makes it inevitable for school 2 to admit those swinging type-1 applicants.  

Proof of Proposition 6

Proof. There are four possible equilibrium combinations: (NPT,NPT), (NPT,PT), (PT,NPT), and (PT,PT).

Part 1. Suppose $u_2 < \pi_2u_1$. School 2 has the following preferences

\[
(PT, PT) \sim (NPT, PT) \quad \text{(because of prop 4)}
\]
\[
\succ (NPT, NPT) \quad \text{(because of prop 1)}
\]
\[
\sim (PT, NPT) \quad \text{(because of prop 2)}
\]

School 1 has the following preferences

\[
(PT, PT) \sim (NPT, PT) \quad \text{(because of prop 4)}
\]
\[
\sim (NPT, NPT) \quad \text{(because of prop 1)}
\]
\[
\sim (PT, NPT) \quad \text{(because of prop 2)}
\]

This completes the proof for part 1.

Part 2. Suppose $u_2 > \pi_2u_1$. For school 2 has the following preferences

\[
(PT, PT) \sim (NPT, PT) \quad \text{(because of prop 4)}
\]
\[
\prec (NPT, NPT) \quad \text{(because of prop 1)}
\]
\[
\sim (PT, NPT) \quad \text{(because of prop 2)}
\]

School 1 has the following preferences

\[
(PT, PT) \sim (NPT, PT) \quad \text{(because of prop 4)}
\]
\[
\sim (NPT, NPT) \quad \text{(because of prop 1)}
\]
\[
\sim (PT, NPT) \quad \text{(because of prop 2)}
\]
This completes the proof for part 2. ■

Proof of Proposition 8

Proof. As assumed in this paper, we focus on equilibrium in which agents choose their weakly dominant strategies, if ever existent. Under (NPT,PT), it is the weakly dominant strategy for type-2 students to report school 2 as their top choice. So they will do so. Now consider the case where \( u_2/u_1 < r \equiv (\mu_2/\mu_1)(s_1/s_2) \). Suppose all type-1 students report school 1 as their top choice. Then school 1 will admit only from type-1 students and school 2 will admit only from type-2 students. A type-1 student will receive an expected utility of \((s_1/\mu_1) \times u_1\), instead of \((s_2/\mu_2) \times u_2\) by reporting school 2 as his top choice. The former is greater when \( u_2/u_1 < r \equiv (\mu_2/\mu_1)(s_1/s_2) \). It is easier to show that this is the unique equilibrium, given that type-2 students use their weakly dominant strategy. Now that school 1 admits the best \( s_1 \) students among type-1 students. It is not worse off or better off. School 2 admits only type-2 students (they are also the best among type-2 students) so school 2 is strictly better off. This completes the proof for result 1. When \( u_2/u_1 > (\mu_2/\mu_1)(s_1/s_2) \), it cannot be an equilibrium where all type-1 students report school 1 as their top choice. So a fraction \( r \) of them will report school 2 as their top choice, and \( r \) is determined such that each type-1 student is indifferent between top ranking school 1 and school 2. The result is depicted as in Figure 7 and result 2 is immediate. ■

Proof of Proposition 10

Proof. We first show that (PT,PT) is an equilibrium. Suppose school 2 uses PT, we can verify that school 1 is indifferent between using NPT and PT. In either case, it admits the same group of students and fills up all its seats in the first round. (If \( u_2/u_1 > r \equiv (\mu_2/\mu_1)(s_1/s_2) \), all type-1 students rank school 1 as their top choice. If \( u_2/u_1 \leq r \), only a fraction of type-1 students rank school 1 as their top choice. But this variance over the parameters does not change the comparison between NPT and PT.) Given that school 1 uses PT, we can verify school 2 strictly prefer using PT to NPT. With the former, school 2 always fills up its seats in the first round. With the latter, it always admits some students who rank it as their second choice. The fraction of type-2 students in the intakes is higher under using PT is higher than under using NPT.

To show that (NPT,PT) is an equilibrium. We already argued that given school 2 uses PT, school 1 is indifferent between using PT or NPT. Hence, using NPT against school 2’s PT is indeed a best response. According to Proposition 7, given that school 1
uses NPT, school 2’s best response is PT. ■