Public-Private Partnerships and Task Interdependence
Revisited

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Abstract

This paper is concerned with the issue of contract bundling when a public project involves both a building stage and an operation stage. In a widely cited paper, Bennett and Iossa (2006) show that positive externality between the investments in the two stages favors contract bundling. Under different contract enforceability assumptions, Chen and Chiu (2010) show that complementarity (a specific kind of positive externality) between the two investments actually favors contract unbundling. In this paper, adopting the same contract enforceability assumption as in Bennett and Iossa, we establish results similar to those in Chen and Chiu with nonetheless a different underlying mechanism.

Keywords: Complementarity; Incomplete contracts; Public-private partnership; Substitutability

JEL classification: D23; H11; L33

1 Introduction

Nowadays, it is common across countries that governmental agencies collaborate with the private sector to deliver public services; in some cases, even the whole project is contracted out to a single firm that takes responsibilities for all involved tasks, say building and maintaining the facility. In the literature of public-private partnerships (PPPs), as this

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practice is usually referred to, an important issue is whether these tasks should be handled by a single consortium (in case of bundling) or by separate firms (in case of unbundling).

In a widely cited paper, Bennett and Iossa (2006) (BI, hereafter) study a model in which the two innovation activities (or investments in short), one in the building stage and the other in the operating stage, are both non-contractible. Assuming a sort of task externality such that the investment in the building stage may increase or decrease the cost in the operating stage, the paper shows that positive externality favors contract bundling and negative externality favors contract unbundling.\(^1\)

The relationship between the two tasks may be even richer, however. They may be substitutable so that making more of one investment will decrease the returns of making more of another investment. For example, a hospital may be built in a more specified manner so that, while the subsequent operational cost is generally lower (i.e., positive externality), further enhancement of quality or alternation of usage would be more difficult to achieve. The two tasks may be complementary so that making more of one investment will increase the returns of making more of another investment. For example, a school may be built with better-quality and more-expensive-glass windows so that, whereas the subsequent operational cost is generally lower (i.e., positive externality), an increase in guard services during the operating stage may be more valuable as it prevents a greater loss from pupils' vandalism. Chen and Chiu (2010) (CC, hereafter) show that task complementarity—a kind of positive externality—favors unbundling, in contrast with the insight in BI that positive externality favors coordination and hence bundling of the two tasks in a single contract.

That said, the contrast between their results is blurred thanks to other differences in the models of the two papers. Most importantly, while BI assume that the two investments are non-contractible, CC assume that, while non-contractible at the outset, the second investment becomes contractible once the first investment has been made. Because of this, their results are not directly comparable. It is just presumptuous to believe that the same insight in CC will carry over to the contract enforcement environment in BI.

In this paper, we revisit the role of task interdependence on optimal contractual

\(^1\)In another paper where both operational costs and service quality are contractible, Martimort and Pouyet (2008) also show similar results.
arrangements. Using the contract enforceability assumption in BI, we show that the
counterintuitive insight in CC is indeed robust and general. Although bundling versus
unbundling is a key issue in the PPP literature, also important is whether the facility
to be built should be owned privately or publicly. For this reason, we also consider this
problem and obtain results in our analysis.

The robustness result is useful for two reasons. First, it fills a gap in the theoretical
literature. The second reason is related to empirical and policy relevance. It is often
hard to distinguish in a real world problem what is the right model to use, for instance,
whether the second investment is non-contractible throughout the process (as in BI) or
will become contractible subsequently (as in CC). The result enables us to discuss the
empirical relevance and to give policy suggestions more comfortably, despite minimum
knowledge of the public project at hand.

There is now a large literature on PPPs. According to modern firm theory, ownership
structure and control right matter when the investment incentives are non-contractible.
Inspired by the seminal paper of Hart, Shleifer and Vishny (1997) on prison, much re-
cent work (e.g., Besley and Ghatak 2001) applies the property right approach to study
the organization of public good provision. As noted by Hart (2003), in the context of
public-private partnerships (PPPs), the government usually bundles tasks such as build-
ing and operation of a facility in a single contract. Thus, given the richness of contractual
arrangement, other factors may play a role on the determination of the optimal regime.
BI study the role of externality in particular.² It should be noted that incomplete contract
approach is not the only approach to studying PPPs. In a complete contracting approach
and with risk averse agents, Martimort and Pouyet (2008) show similar results as BI do
that positive externality between the two tasks favors contract bundling.

The rest of this paper is organized as follows. Section 2 presents the model. Sections 3
and 4 analyze and compare the chosen investments under the private and public ownership,
respectively. Section 5 derives the conditions which underlie the optimum of traditional
procurement or PPP. It also discusses the policy implications and empirical relevance of

²More recent work along this line include Hoppe and Schmitz (2010) and Francesconi and Muthoo
(2011). The former paper studies the provision of a good of variable quantity, finding that the main
determinants of the optimal governance structure are the relative importance of cost reduction versus
quality innovation, as well as the involved parties’ bargaining power. Francesconi and Muthoo (2011)
study a “complex” partnership which produces “impure” public goods, and find that the optimal allocation
of control right depends on the degree of impurity of public good.
the findings. Section 6 concludes.

2 Basic setup

We follow closely the model in BI, which is a variant of Hart, Shleifer, and Vishny. A governmental agency (hereafter, the government) is contemplating a project consisting of two sequential tasks, namely, "building" and "operating" of a facility. We use $a$ and $e$ to denote the levels (also the costs) of the innovative activities in the building stage and in the operating stage, respectively, and these two activities can be undertaken by two separate firms or by a consortium. The operational cost, borne by the manager in the operation stage, is

$$C(a, e) = C_0 - d(a, e),$$

where $C_0$ is the positive default cost, and $d(a, e)$ is the reduction of operational cost caused by the investments, $a$ and $e$, satisfying the following properties:

(i) $d(0, 0) = 0.$

(ii) $d_2(a, e) > 0$, $d_2(a, 0) = \infty$, $d_2(a, \infty) = 0$, $d_{22}(a, e) < 0$.

(iii) If $d_1(a, e) > 0$, then $d_1(0, e) = \infty$, $d_1(\infty, e) = 0$, $d_{11}(a, e) < 0$; if $d_1(a, e) \leq 0$, then $d_1(0, e) = 0$, $d_1(\infty, e) = -\infty$, $d_{11}(a, e) < 0$.

If $d_1(a, e) > 0$ ($< 0$), $a$ decreases (increases) the operational cost; these positive and negative externalities are studied in BI. If $d_{12}(a, e) > 0$ ($< 0$), $a$ and $e$ are complementary (substitutable) investments and an increase in $a$ increases (decreases) the marginal benefit of $e$. In BI, $d_{12}(a, e)$ is assumed to be zero.

Remark 1 To facilitate exposition, we will use the following nomenclature unless otherwise stated. Task or investment externality refers to the case that $d_1(a, e)$ is nonzero; in particular, positive (negative) externality refers where $d_1(a, e) > 0$ ($d_1(a, e) < 0$). Task or investment interdependence refers to the case that $d_{12}(a, e)$ is nonzero; in particular, complementarity (substitutability) refers to where $d_{12}(a, e) > 0$ ($d_{12}(a, e) < 0$).

Like in BI, the project yields the following social benefits:

$$B(a, e) = B_0 + u(a) + v(e),$$
where $B_0$ is the positive default benefit; the terms $u(a) \equiv \alpha U(a)$ and $v(e) \equiv \beta V(e)$ measure the part of social benefits caused by the investments of $a$ and $e$, respectively. $\alpha > 0$ and $\beta > 0$ are scale parameters and $U$ and $V$ are normalized functions satisfying:

$U(0) = V(0) = 0; U(1) = V(1) = 1; U''(a), V'(e) > 0; U''(a), V''(e) < 0; U'(0) = V'(0) = \infty; U'(\infty) = V'(\infty) = 0.$

Upon the project’s expiry, the owner of the facility claims its residual value, which equals

$$R(a) = R_0 + t(a),$$

where $R_0$ is the positive default residual value, and $t(a) \equiv \gamma T(a)$ measures the additional residual value generated by investment, $a$, where $\gamma$ is a scale parameter and $T$ is a normalized function satisfying $T(0) = 0; T(1) = 1; T'(a) > 0, T''(a) < 0; T'(0) = \infty; T'(\infty) = 0.$

To simplify matters, we make the following assumption.

**A1** $t'(a) + d_1(a, e) > 0$ for all $a$ and $e$.

This assumption implies that, although we allow $a$ to increase the operational cost, this negative externality is constrained to a moderate range. BI implicitly use a similar assumption when they analyze the negative externality case.

As in BI, both $a$ and $e$ are observable, non-verifiable, and hence non-contractible. The sequence of events is as follows (see Figure 1). At time 0, given the contractual regime, the government specifies the basic standards and contract payments when contracting with the firm(s). The various possible contractual regimes include: separation with builder ownership (SB), separation with manager ownership (SM), separation with public ownership (SP), integration with consortium ownership (IC), and integration with public ownership (IP) (where separation means unbundling and integration means bundling). At time 1, $a$ is undertaken. At time 1.5, negotiation needs occur over the adoption of $a$. At time 2, $e$ is undertaken. At time 2.5, negotiation may occur over on the adoption of $e$. At time 3, all the payoffs are realized. Note that investment costs of $a$ or $e$ are irreversible whether or not they will be subsequently adopted.
Figure 1: Time line of the game
We use $g$ to denote the government’s payoff, $f$ the consortium’s payoff, and $f_b$ and $f_m$ the builder’s and manager’s payoffs, respectively, when they are two separate agents. To simplify our presentation, we omit such default values as $C_0, B_0, R_0$ in the discussions. As in BI, negotiation is conducted through Nash bargaining with symmetric bargaining power.

Notice that the first-best investments $(a^*, e^*)$ satisfy:

\[ u'(a^*) + t'(a^*) + d_1(a^*, e^*) = 1 \]  
\[ v'(e^*) + d_2(a^*, e^*) = 1. \]

We assume that unique, interior solutions to these two equations exist. Likewise, we also assume that the equilibrium in every regime is also unique; such an assumption is standard in the literature even though it is not usually explicitly stated.

### 3 Private ownership

#### 3.1 Builder ownership

Suppose that the builder owns the facility and the manager is a separate agent. In this case, as explained by BI, $a$ will always be implemented without going through any bargaining, because the builder has control rights and receives the residual value generated by adopting $a$. However, in the operating stage, the builder negotiates with the manager on the approval of $e$, since he does not directly gain anything if he does not threaten to disallow the adoption of $e$. Notice that the net surplus generated by allowing the adoption of $e$ is $d(a, e) - d(a, 0)$. Given equal division of this net surplus (because of Nash bargaining), the ex post payoffs of the builder and the manager, respectively, are

\[ f_b = t(a) + \frac{1}{2} [d(a, e) - d(a, 0)] - a \] and \[ f_m = \frac{1}{2} [d(a, e) + d(a, 0)] - e. \]

\(^3\)To follow BI, we assume that the owner cannot adopt the investment without the investor’s consent. This is in contrast to Aghion and Tirole (1994), where it is assumed that once an innovation has been made, it can be used by the owner.
Given $a$, the manager maximizes $f_{m}$ by choosing $e = e_{SB}(a)$, and the FOC is

$$\frac{1}{2}d_2(a, e_{SB}(a)) = 1. \quad (3)$$

Totally differentiating it, we obtain $\frac{\partial e_{SB}}{\partial a} = -\frac{d_2(\ldots)}{d_2(\ldots)}$, which is positive (negative) if there is task complementarity (substitutability).

Foreseeing the manager’s behavior, the builder maximizes $f_{b}$ subject to (3) by choosing $a = a_{SB}$, and the first-order condition (FOC) is

$$t'(a_{SB}) + \frac{1}{2} [d_1(a_{SB}, e_{SB}) - d_1(a_{SB}, 0)] + \frac{\partial e_{SB}}{\partial a} = 1. \quad (4)$$

### 3.2 Manager ownership

In this case, the manager could unilaterally adopt his own $e$, without going through any bargaining. Thus, given $a$, the manager chooses $e = e_{SM}(a)$, and the FOC is

$$d_2(a, e_{SM}(a)) = 1. \quad (5)$$

After differentiation, we obtain $\frac{\partial e_{IC}}{\partial a} = -\frac{d_2(\ldots)}{d_2(\ldots)}$, which is positive (negative) if there is task complementarity (substitutability).

The builder and the manager will bargain over the adoption of $a$, however. If $a$ is not adopted, the builder will gain nothing except the default contract payment and the manager will have continuing payoff $[d(0, e_{SM}(0)) - e_{SM}(0)]$. If $a$ is adopted, the total surplus for the builder and the manager will be $[t(a) + d(a, e_{SM}(a)) - e_{SM}(a)]$. Therefore, under Nash bargaining solution, the builder’s ex post payoff is

$$f_{b} = \frac{1}{2} [t(a) + d(a, e_{SM}(a)) - e_{SM}(a) - d(0, e_{SM}(0)) + e_{SM}(0)] - a,$$

Foreseeing this, the builder maximizes $f_{b}$ subject to (5) by choosing $a = a_{SM}$, and the FOC is

$$\frac{1}{2} [t'(a_{SM}) + d_1(a_{SM}, e_{SM})] = 1. \quad (6)$$
3.3 Consortium ownership

Now that a consortium is in charge of investing both $a$ and $e$ and also owns the facility, he could adopt the investments of $a$ and $e$ without the government’s approval.\footnote{Following BI, we assume that if the builder and manager form a consortium, they act as if a single person. Needless to say, this view of integration is different from the view expressed in Grossman and Hart (1986).} According to A1, it is always profitable for him to adopt both two investments, and negotiation never happens. His payoff is $f = t(a) + d(a, e) - a - e$. Thus, the FOC for choosing $e = e_{IC}(a)$ is (5), the same as under $SM$, while the FOC for choosing $a = a_{IC}$ is

$$t'(a_{IC}) + d_1(a_{IC}, e_{IC}) = 1.$$  

(7)

3.4 Comparison

Under private ownership, there are three regimes to consider. The comparison between $IC$ and $SM$ is relatively easy because the FOCs for the choice of $e$ are the same (both represented by (5)). The first implication is that, in case $a$ and $e$ exhibit complementarity, a regime that corresponds to a greater $a$ also corresponds to a greater $e$, and vice versa. We next examine which regime leads to a greater $a$, without restricting to the case of complementarity. Comparing their FOCs for $a$ ((7) for $IC$ and (6) for $SM$), we notice that the builder under $SM$ has a smaller incentive than the consortium under $IC$ in investing $a$. In particular, whereas the consortium under $IC$ receives all of the marginal gain in residual value of the facility due to increase in $a$, the builder under $SM$ receives only half of it. This suggests that $IC$ always leads to a greater level of $a$. We thus obtain the following proposition (all proofs are relegated to the Appendix unless otherwise stated).

**Proposition 1** (i) Consortium ownership ($IC$) yields a greater $a$ than manager ownership ($SM$). (ii) If $a$ and $e$ are complementary investments, $IC$ also yields a greater $e$ than $SM$.

The fact that $SM$ is dominated by $IC$ as far as $a$ is concerned is supported by the prevalence of $IC$ over $SM$ in the real world. Because of this, we just need to compare regime $IC$ and regime $SB$, while omitting the comparison between $SM$ and $SB$. 


Proposition 2  Consortium ownership (IC) yields a greater $a$ than does builder ownership (SB), if and only if, for all $a \in [\min\{a_{IC},a_{SB}\},\max\{a_{IC},a_{SB}\}]$,

$$d_1(a,e_{IC}) > \frac{1}{2} [d_1(a,e_{SB}) - d_1(a,0)] - \frac{d_{21}(a,e_{SB})}{d_{22}(a,e_{SB})}. \quad (8)$$

According to (8), when $d_{21}(.) = 0$, both $(d_1(a,e_{SB}) - d_1(a,0))$ and $-\frac{d_{21}(a,e_{SB})}{d_{22}(a,e_{SB})}$ are zero and IC leads to a greater $a$ than SB if and only if $d_1(.) > 0$. This result is indeed Proposition 1 and Lemma 1 in BI, in which the choice of integration (separation) is preferred if and only if the building investment directly reduces (increases) the operational cost. Our proposition, however, is more general as it captures the role played by task interdependence measured by cross derivatives $d_{12}(a,e)$ or $d_{21}(a,e)$. According to (8), if $d_{12}(a,e) > 0$ (task complementarity), both $-\frac{d_{21}(a,e_{SB})}{d_{22}(a,e_{SB})}$ and $\frac{1}{2} (d_1(a,e_{SB}) - d_1(a,0))$ are positive; that means that, for integration to be the optimal regime, $d_1(a,e_{IC})$ has to be not only positive but also sufficiently large. On the contrary, if $d_{12}(a,e) < 0$ (task substitutability), both $-\frac{d_{21}(a,e_{SB})}{d_{22}(a,e_{SB})}$ and $\frac{1}{2} (d_1(a,e_{SB}) - d_1(a,0))$ are negative; i.e., for integration to lead to a greater $a$, $d_1(a,e_{IC})$ need not be positive, it suffices that it is not too negative.

The above analysis suggests that, as far as $a$ is concerned, task complementarity favors some kind of unbundling (in the form of SB) against bundling. The intuition is as follows. In case of SB, the builder could bargain with the manager. After the bargaining, the former party could share the benefits generated by the manager’s investment, while not bearing any cost incurred by such investment. Because of complementarity, a higher building investment leads to a higher operating investment, yielding a greater net surplus to be split. Anticipating more rents to be extracted from the manager’s investment, the builder has a greater incentive to invest. As a result, investment complementarity helps mitigate the builder’s under-investment problem. In the case of IC, on the contrary, when investing in the building stage, the consortium will internalize not only the benefits but also the costs of subsequent investment, resulting in a dampened investment incentive on his part. Because task complementarity can be viewed as a special kind of positive externality, this result sheds new, somewhat counter-intuitive, light to the issue.

One can make a strong observation regarding the comparison between IC and SB. Consider the case of complementarity. Suppose $a_{IC} > a_{SB}$ (because of sufficiently large
positive externality). If follows that \( e_{IC} > e_{SB} \) for two reasons. First, complementarity of \( a \) and \( e \) suggests that the optimal choice of \( e \) is increasing in \( a \); second, the incentive to invest \( e \) is stronger under IC than under SB. These two features reinforce each other, making \( e_{IC} > e_{SB} \). Notice that, however, according to Proposition 2, the positive externality must be strong enough. Absent such strong enough positive externality, we do not obtain \( a_{IC} > a_{SB} \) and in that case we cannot say too much about \( e_{IC} \) and \( e_{SB} \).

**Corollary 1** Suppose \( d_{12}(a,e) > 0 \). Relative to builder ownership (SB), consortium ownership (IC) yields greater \( a \) and \( e \) only if there exists strong enough positive externality.

**Proof.** Omitted. ■

The case of substitutability is more difficult. The reason is that whenever \( a \) is higher in one regime that in the other, substitutability between \( a \) and \( e \) will lead to a lower \( e \) in the former regime than in the latter, other things being equal. It is thus difficult to determine when one regime dominates the other in both \( a \) and \( e \).

4 Public ownership

4.1 Separation and public ownership

In this case, two different agents undertake building and operation, and the government owns the facility. Then the government will negotiate with the two agents separately, with respect to the adoption of \( a \) and \( e \).\(^5\) We first consider the bargaining with the manager. In case \( e \) is not adopted, the government gains nothing except the default values at this stage, and the manager ends up with \( d(a,0) \) due to the reduction in operational costs. Hence, the net surplus arising from the successful adoption of \( e \) is \([v(e) + d(a,e) - d(a,0)]\).

Using the Nash bargaining solution, we reckon the manager’s ex post payoff to be \( f_m = \frac{1}{2} [v(e) + d(a,e) + d(a,0)] - e \).

Thus, given \( a \), the manager maximizes \( f_m \) by choosing \( e = e_{SP}(a) \), and the FOC is

\[
\frac{1}{2} [v'(e_{SP}(a)) + d_2(a,e_{SP}(a))] = 1.
\]

\(^5\)We assume that the government’s payoff is equal to the social benefit minus payments made to the firm(s), plus the residual value if she is also the owner. In this regard, her objective is different from a benevolent social planner in typical welfare analysis.
After total differentiation, we obtain

\[
\frac{\partial e_{SP}}{\partial a} = -\frac{d_{21}(a, e_{SP})}{v'(e_{SP}) + d_{22}(a, e_{SP})},
\]

(10)

which is positive (negative) in case of complementarity (substitutability). Next, we notice that, given adopted \( a \), the government’s gain in the operating stage (denoted as \( g_2(a) \)) is

\[
g_2(a) = \frac{1}{2} [v(e_{SP}(a)) + d(a, e_{SP}(a)) - d(a, 0)].
\]

We then come back to consider the government’s bargaining with the builder. In case \( a \) is not adopted, she would end up with a continuing payoff of \( g_2(0) = \frac{1}{2} [v(e_{SP}(0)) + d(0, e_{SP}(0))] \), and the builder ends up with nothing except the default contract payment. The net surplus generated by the adoption of \( a \) is thus \([u(a) + t(a) + g_2(a) - g_2(0)]\). Through Nash bargaining, the builder’s expected payoff is

\[
f_b = \frac{1}{2} [u(a) + t(a) + g_2(a) - g_2(0)] - a
\]

\[
= \frac{1}{2} [u(a) + t(a)] + \frac{1}{4} [v(e_{SP}(a)) + d(a, e_{SP}(a)) - d(a, 0) - v(e_{SP}(0)) - d(0, e_{SP}(0))] - a.
\]

Foreseeing the bargaining outcome, the builder maximizes \( f_b \) subject to (9) by choosing \( a = a_{SP} \), and the FOC is

\[
\frac{1}{2} [u'(a_{SP}) + t'(a_{SP})] + \frac{1}{4} [d_1(a_{SP}, e_{SP}) - d_1(a_{SP}, 0)] + \frac{1}{2} \frac{\partial e_{SP}}{\partial a} = 1.
\]

(11)

**4.2 Integration and public ownership**

Suppose that a consortium takes charge of building and operation under the government’s ownership. There will be negotiation in both stages. We first consider the bargaining on the adoption of \( e \). In case \( e \) is not adopted, the government gains nothing except the default values in the operating stage, and the consortium gets \( d(a, 0) \). The net surplus arising from the adoption of \( e \) is thus \( v(e) + d(a, e) - d(a, 0) \). Through Nash bargaining solution, the consortium gets \( \frac{1}{2} [v(e) + d(a, e) + d(a, 0)] - e \), and the government gets \( \frac{1}{2} [v(e) + d(a, e) - d(a, 0)] \). Thus, given \( a \), the consortium’s optimal choice of \( e = e_{IP}(a) \)
satisfies (9), as under \( SP \).

We next examine the bargaining on the adoption of \( a \). In case \( a \) is not adopted, the consortium ends up with a continuing payoff of \( \frac{1}{2} [v(e_{IP}(0)) + d(0, e_{IP}(0))] - e_{IP}(0) \), and the government ends up with a continuing payoff of \( \frac{1}{2} [v(e_{IP}(0)) + d(0, e_{IP}(0))] \). If \( a \) is successfully adopted, the total surplus generated is

\[
u(a) + t(a) + d(a, e_{IP}(a)) + v(e_{IP}(a)) - e_{IP}(a)\].

Hence, taken into account Nash bargaining, the consortium’s ex post payoff is

\[
f = \frac{1}{2} [u(a) + t(a) + d(a, e_{IP}(a)) + v(e_{IP}(a)) - e_{IP}(a) - e_{IP}(0)] - a.
\]

Maximizing \( f \) by choosing \( a = a_{IP} \) subject to (9) gives the following FOC:

\[
\frac{1}{2} [u'(a_{IP}) + t'(a_{IP}) + d_1(a_{IP}, e_{IP})] + \frac{1}{2} \frac{\partial e_{IP}}{\partial a} = 1. \tag{12}
\]

\subsection*{4.3 Comparison}

There are only two regimes to study under public ownership, \( IP \) and \( SP \). We first notice that the FOCs for \( e \) under the two regimes are indeed identical, both represented by (9). So to compare the two regimes, we just need to focus on the choice of \( a \). The following proposition summarizes the result.

\begin{proposition}
Under public ownership, integration (\( IP \)) leads to a greater \( a \) than does separation (\( SP \)), if and only if, for all \( a \in [\min \{a_{IP}, a_{SP}\}, \max \{a_{IP}, a_{SP}\}] \),

\[
d_1(a, e_{IP}) > \frac{1}{2} [d_1(a, e_{SP}) - d_1(a, 0)]. \tag{13}
\]
\end{proposition}

According to (13), if \( d_{12}(a, e) > 0 \) (task complementarity), \( \frac{1}{2} [d_1(a, e_{SP}) - d_1(a, 0)] \) are positive; it means that, for integration to be the optimal regime, \( d_1(a, e_{IP}) \) should be not only positive but also sufficiently large. On the contrary, if \( d_{12}(a, e) < 0 \) (task substitutability), the term \( \frac{1}{2} [d_1(a, e_{SP}) - d_1(a, 0)] \) is negative; for integration to be the optimal regime, \( d_1(a, e_{IP}) \) need not be positive; it suffices that it is not too negative. Therefore,
as for the impacts of task interdependence on the bundling decision, the intuition in case of public ownership is similar to that in case of private ownership.

It is useful to notice the difference with CC though. In their model, since investment e is assumed to be contractible by following the investment a, SP and IP happen to make the same investment choices. In other words, task interdependence plays no role at all in the comparison between the two public ownership regimes. The difference between their result and ours is driven by two forces. First, the interim contractibility of operating investment deprives the bargaining power of the manager in case of unbundling. Whenever bargaining takes place, the total surplus generated by the building investment is always shared between the two parties: the government and the builder under SP or the government and the consortium under IP. However, in this model, bargainings happen sequentially among the three parties, including the manager. Second, in CC, the bargaining in the operating stage is triggered by the interim contractibility of e, so it happens before e is invested, while in our non-contractability setting, the bargaining on e happens ex post regarding its implementation.

Restricting to task complementarity, we obtain the following result.

Corollary 2 Suppose \( d_{12}(a, e) > 0 \). Under public ownership, separation (SP) yields greater a and e than integration (IP) if there exists negative externality or there is positive but small enough externality.

Proof. Omitted. ■

5 The optimal regime

5.1 Underinvestment of \( a \) and \( e \)

The three regimes under private ownership are SB, SM, and IC, and their decisions on \( a \) and \( e \) are given by (4) and (3), (6) and (5), and (7) and (5), respectively. Notice that, whereas neither \( u(.) \) nor \( v(.) \) appears in these conditions, both of them appear in the first best problem (see (1) and (2)). As a result, when the scale measure of social benefit, \( \alpha \) and \( \beta \), are large enough, both \( a \) and \( e \) are under-invested under any private ownership regime.
For public ownership, the choices of $a$ and $e$ are responsive, but only partially, to $u(.)$ and $v(.)$ (comparing (11) and (9) for $SP$, (12) and (9) for $IP$ to (1) and (2) for the first-best problem). Thus, there still exist underinvestments of $a$ and $e$ when $\alpha$ and $\beta$ are large enough. The following lemma is thus obtained.

**Lemma 1** When $\alpha$ and $\beta$ are sufficiently large, there are under-investments of $a$ and $e$ in all regimes.

**Proof.** Omitted. ■

This lemma justifies our exercise of comparing different regimes in terms of $a$; i.e., when one regime is found to correspond to a greater level of $a$, this regime is indeed a more efficient regime as far as the level of $a$ is concerned.

### 5.2 PPP versus traditional procurement

In previous real world practice, multiple stages involved in a public project were often organized through contracting out to separate firms, so this arrangement is often referred to as traditional procurement. However, recently, the Private Finance Initiative (PFI), a pioneer form of PPP launched in the UK, becomes a new trend in public service provision. According to Iossa and Martimort (2008), PPPs are being used across Europe, North American and many developing countries, covering the sectors of transport, energy, water, prisons, military training, waste management, schools and hospitals etc. So we want to understand whether the traditional procurement is still useful or should be replaced by PPPs with little qualifications. The following proposition compares PPP (i.e., the $IC$ regime) versus traditional procurement (i.e., $SP$ regime).

**Proposition 4** PPP ($IC$) leads to a greater $a$ than does traditional procurement ($SP$), if and only if, for all $a \in \{a_{IC}, a_{SP}\}$,  

\[
\frac{1}{2} [u'(a) - t'(a)] - d_1(a,e_{IC}(a)) < \frac{1}{4} [d_1(a,0) - d_1(a,e_{SP}(a))] + \frac{d_{21}(a,e_{SP}(a))}{2u''(e_{SP}(a)) + d_{22}(a,e_{SP}(a))}.
\]

\[ (14) \]

---

6One may ask stronger questions as to when $IC$ dominates all other regimes (rather than just $SP$) and when $SP$ dominates all other regimes (rather than just $IC$). We believe that the result is probably not enlightened enough given the space limitation.
Proof. The proof is similar to those of Propositions 2 and 3 and is omitted. ■

According to the left hand side, IC is more likely to dominate SP in terms of $a$ if the social benefit associated with the building task is smaller, the residual value of the project is greater, and the externality between $a$ and $e$ is more positive. This is the insight obtains under BI. However, what is new here is the right hand side (RHS) which depends on task interdependence. More specifically, both terms in the RHS are negative, positive, and zero in the presence of complementarity, substitutability, no interdependence, respectively. Thus complementarity (substitutability) makes it more difficult for the condition to hold, i.e., more difficult for PPP to dominate traditional procurement. This insight which appears recurrently in this paper continues to hold here, thus suggesting stronger and more general justification for the use of traditional procurement.

The next corollary give conditions under which either regime dominates the other in both $a$ and $e$, focusing on the case of task complementarity.

Corollary 3 Suppose $d_{12}(a,e) > 0$.

1. PPP (IC) leads to greater $a$ and $e$ than does traditional procurement (SP) if (i) (14) holds for all $a \in [\min \{a_{IC}, a_{SP}\}, \max \{a_{IC}, a_{SP}\}]$ and (ii) $\beta$ is sufficiently small.

2. Traditional procurement (SP) leads a greater $a$ and $e$ than PPP (IC) does if (i) (14) holds with "$<"$ for all $a \in [\min \{a_{IC}, a_{SP}\}, \max \{a_{IC}, a_{SP}\}]$ and (ii) $\beta$ is sufficiently large.

The corollary is easy to understand. Consider the first result. Condition (i) is to ensure that $a_{IC} > a_{SP}$, which is just a restatement of Proposition 4. In terms of $e$, there is a trade-off under the two regimes. The consortium under IC obtains the full marginal private benefit of $e$. While the manager under SP obtains half of such marginal private benefit, it also obtains the half of the social benefit associated with the investment. As a result, when this social benefit is small (i.e., $\beta$ is small), IC is the regime that provides stronger incentive for the investment. Condition (ii) is to ensure that $e_{IC} > e_{SP}$, which is easily shown under complementarity and under the condition that $a_{IC} > a_{SP}$. A similar result under task substitutability is more difficult to establish. The second result of the corollary is understood in the same manner.
Thus, according to Corollary 3, the usefulness of PPP is more limited as predicted by our model than by others. Ignoring task interdependence in their analysis, few existent studies are aware that task complementarity, another kind of positive externality, performs as a detriment of the desirability of PPP. Given abundant real life projects demonstrating task complementarity, practice might be too optimistic by following the fashion of adopting PPP regime and its suitability might be overstated in public good provision.

5.3 Discussions

Table 1 compares the prediction of optimal contractual arrangement of our model with BI. If we assume no task interdependence (the bottom row case), our results collapse into BI's. However, if we consider interplay between task interdependence and externality, clear-cut predictions are obtained when there exist positive externality and task substitutability, and when there exist negative externality and task complementarity; in other circumstances, the optimal regime is uncertain, in the sense that it depends not only on the nature of the interactions, but also on the relative strength of the forces.

It is worth noting that the findings in current paper also contrast with CC. They show that complementarity favors unbundling over bundling under private ownership while task interdependence plays no role under public ownership. In this paper, however, the impacts of task interdependence on the optimal contractual arrangement is pervasive whether or not the ownership is private or public. The disparity between their result and ours arises from the fact that they consider a different framework where the operating investment becomes contractible once the building investment is complete and, therefore, will be chosen efficiently under standard bargaining procedures. On the other hand, in this paper, the non-contractibility of operating investment will trigger bargaining after it is complete, hence other parties will share the benefit of the operator’s investment, rendering an underinvestment problem. More building investments, given task complementarity, will exacerbate such a holdup problem in the operating stage that in return shapes variant incentives of building investment under different contractual arrangements.
<table>
<thead>
<tr>
<th></th>
<th>positive externality</th>
<th>negative externality</th>
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<tr>
<td>complementarity</td>
<td>BI: bundling;</td>
<td>BI: unbundling;</td>
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<td>This paper: bundling or unbundling</td>
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<td>substitutability</td>
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<td>This paper: bundling</td>
<td>This paper: bundling or unbundling</td>
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<td>no interdependence</td>
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<td>This paper: bundling</td>
<td>This paper: unbundling</td>
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Table 1: Comparison with results in BI
Literature suggests that traditional procurement is desirable only if there exists negative externality across the project phases (see, e.g., BI, and Martimort and Pouyet). However, as pointed out by Iossa and Martimort, evidence of negative externalities is difficult to find. Alternatively, the prediction of our model relates the desirable role of traditional procurement to task complementarity, which seems more plausible. A well-designed and well-built facility would not by itself increase the operational costs; it may instead demand more efforts from the manager to engage in careful maintenance, since breakdown of these novel facilities may cost more for repairs.

Our results provide alternative explanation to various phenomena in public goods provision. According to a report by the Audit Commission, the quality of PFI schools is undesirable; particularly, they have few windows, poor acoustic and air quality, compared to traditionally procured schools. Using our framework, we do not necessarily attribute this phenomenon to negative externality, as like BI, because the improvement of school quality does not directly increase the maintenance cost; instead, it heightens the importance of the school manager’s work, and any failure to control vandalism causes more valuable school assets to be damaged. Hence, investment on cost reduction will be crowded in by investment on quality enhancing. Traditional procurement, relieving the solo builder of such concern, may take the advantage of restoring the builder’s incentive on quality innovation.

On the other hand, the desirability of PPP could be rationalized by task substitutability. Arthur Andersen and LSE (2000) estimate that significant cost savings were realized in the prison sector. As we illustrated construction of a prison before, positive externality and task substitutability coexist here. According to the National Audit Office (2003), “innnovative design solutions helped to reduce the level of ‘staffing’ needed to ensure security and this resulted in an overall cost reduction by approximately 30%.” Preferably, we translate "less staffing" into less necessity of cost-reduction effort in the operating stage, so it might be crowded out by effort on quality innovation, due to task substitutability.
6 Concluding remarks

In this paper, we have reconsidered the impacts of the role of task interdependence on optimal contractual arrangements in the literature of PPPs. Using the contract enforceability assumption in BI, we have shown the robustness of the counterintuitive insight in CC that task complementarity favors contract unbundling of the two tasks. In addition to filling in a gap in the theoretical literature, this robustness result has important empirical and policy relevance. As a matter of fact, it is hard to distinguish in a real world problem what is the right model to use, for instance, whether the second investment is non-contractible throughout the process or will become contractible subsequently. The robustness result thus enables us to discuss the empirical relevance and to give policy suggestions more comfortably, despite minimum knowledge of the public project or disagreement in the model to be used. Our paper, together with CC, argue that complementarity will favor unbundling of contacts no matter whether the sequential operating investment is contractible or not, hence providing additional justification for efficiency of traditional procurement.
References


Appendix

Proof of Proposition 1

Proof. To show the first result, we notice that when $a = a_{SM}$ is chosen under IC, the subsequent $e$ to be chosen will be $e_{SM}$. Substituting $(a_{SM}, e_{SM})$ into the first order condition for the IC problem (7), we realize that the marginal benefit of investing $a$ is two times the marginal cost of investing. As a result, the consortium finds it beneficial to expand $a$, and it follows that $a_{IC}$ is indeed greater than $a_{SM}$. The second result comes from the facts that both IC and SM have the same FOC for $e$ and that, in this FOC, because of complementarity, the choice of $e$ is positively related to $a$. ■

Proof of Proposition 2

Proof. Suppose that $a_{IC} > a_{SB}$. Notice that all the maximization problems are concave (second-order conditions are satisfied). Then, making use of (7), we know that, for $a \in [a_{SB}, a_{IC}]$,

$$t'(a) + d_1(a, e_{IC}(a)) \geq 1,$$

where the equality holds only when $a = a_{IC}$; making use of (4), we know that, for all $a \in [a_{SB}, a_{IC}]$,

$$t'(a) + \frac{1}{2} [d_1(a, e_{SB}(a)) - d_1(a, 0)] + \frac{\partial e_{SB}(a)}{\partial a} \leq 1,$$

where the equality holds only when $a = a_{SB}$. Therefore, subtracting the second inequality from the first and rearranging, we obtain result:

$$a_{IC} > a_{SB} \Rightarrow d_1(a, e_{IC}(a)) - \frac{1}{2} [d_1(a, e_{SB}(a)) - d_1(a, 0)] - \frac{\partial e_{SB}(a)}{\partial a} > 0 \text{ for all } a \in [a_{SB}, a_{IC}].$$

(15)

Suppose that $a_{IC} < a_{SB}$. The analysis is similar. Using (7) and (4), we know that, for $a \in [a_{IC}, a_{SB}]$,

$$t'(a) + d_1(a, e_{IC}(a)) \leq 1,$$
where the equality holds only when \( a = a_{IC} \); and, for all \( a \in [a_{IC}, a_{SB}] \),

\[
t'(a) + \frac{1}{2} [d_1(a, e_{SB}(a)) - d_1(a, 0)] + \frac{\partial e_{SB}(a)}{\partial a} \geq 1,
\]

where the equality holds only when \( a = a_{SB} \). Using these two inequalities, we obtain:

\[
a_{IC} < a_{SB} \Rightarrow d_1(a, e_{IC}(a)) - \frac{1}{2} [d_1(a, e_{SB}(a)) - d_1(a, 0)] - \frac{\partial e_{SB}(a)}{\partial a} < 0 \text{ for all } a \in [a_{IC}, a_{SB}].
\] (16)

Combining (15) and (16) and making use of (3), we show the claimed result. ■

**Proof of Proposition 3**

**Proof.** Suppose that \( a_{IP} > a_{SP} \). Notice that all the second-order conditions are satisfied. Then making use of (12), we know that, for \( a \in [a_{SP}, a_{IP}] \),

\[
\frac{1}{2} [u'(a) + t'(a) + d_1(a, e_{IP}(a))] + \frac{1}{2} \frac{\partial e_{IP}(a)}{\partial a} \geq 1,
\]

where the equality holds only when \( a = a_{IP} \); from (11), we know that, for \( a \in [a_{SP}, a_{IP}] \),

\[
\frac{1}{2} [u'(a) + t'(a)] + \frac{1}{4} [d_1(a, e_{SP}(a)) - d_1(a, 0)] + \frac{1}{2} \frac{\partial e_{SP}(a)}{\partial a} \leq 1,
\]

where the equality holds only when \( a = a_{SP} \). Subtracting the second inequality from the first, making use of the fact that \( \frac{\partial e_{IP}(a)}{\partial a} = \frac{\partial e_{SP}(a)}{\partial a} \) (\( e_{IP} \) and \( e_{SP} \) share the same FOC (i.e., (9)), and rearranging, we obtain:

\[
a_{IP} > a_{SP} \Rightarrow d_1(a, e_{IP}(a)) > \frac{1}{2} [d_1(a, e_{SP}(a)) - d_1(a, 0)] \text{ for all } a \in [a_{SP}, a_{IP}].
\] (17)

Suppose \( a_{IP} < a_{SP} \). Using the same procedure, we know that, for \( a \in [a_{IP}, a_{SP}] \),

\[
\frac{1}{2} [u'(a) + t'(a) + d_1(a, e_{IP}(a))] + \frac{1}{2} \frac{\partial e_{IP}(a)}{\partial a} \leq 1,
\]

where the equality holds only when \( a = a_{IP} \); and, for \( a \in [a_{IP}, a_{SP}] \),

\[
a_{IP} < a_{SP} \Rightarrow \frac{1}{2} [u'(a) + t'(a)] + \frac{1}{4} [d_1(a, e_{SP}(a)) - d_1(a, 0)] + \frac{1}{2} \frac{\partial e_{SP}(a)}{\partial a} \geq 1,
\]
where the equality holds only when \( a = a_{SP} \). Using these two inequalities, we obtain:

\[
d_1(a, e_{IP}(a)) < \frac{1}{2} [d_1(a, e_{SP}(a)) - d_1(a, 0)] \quad \text{for all } a \in [a_{IP}, a_{SP}].
\] (18)

Combining (17) and (18), we show the claimed result. ■

Proof of Corollary 3

Proof. We show (1) first. Condition (i) is to ensure that \( a_{IC} > a_{SP} \) and is evidently true from Proposition 5. Condition (ii) is to ensure that \( e_{IC} > e_{SP} \). Define \( \beta^* \) which satisfies \( \beta^* V'(e_{IC}) = d_2 (a_{IC}, e_{IC}) \). Since both \( a_{IC} \) and \( e_{IC} \), determined in (5) and (7), are independent of \( \beta \), \( \beta^* \) is well-defined. Given any \( \beta < \beta^* \), consider the investment of \( e \) under \( SP \) (see equation (9)). The marginal benefit of investing \( e = e_{IC} \) equals

\[
\frac{1}{2} (\beta V'(e_{IC}) + d_2 (a_{SP}, e_{IC})) < \frac{1}{2} (\beta^* V'(e_{IC}) + d_2 (a_{IC}, e_{IC}))
\]

\[= d_2 (a_{IC}, e_{IC})
\]

\[= 1,
\]

where the first line is due to \( \beta < \beta^* \), \( a_{SP} < a_{IC} \) ensured by (i), complementarity between \( a \) and \( e \); the second line due to the definition of \( \beta^* \); and the third line due to (5). As the marginal benefit is lower than the marginal cost, it follows that the \( e \) chosen under \( SP \) will be smaller than \( e_{IC} \), i.e., \( e_{SP} < e_{IC} \).

We show (2) here. Condition (i) is to ensure that \( a_{SP} > a_{IC} \) and is evidently true from Proposition 5. Condition (ii) is to ensure that \( e_{SP} > e_{IC} \). Use the same cutoff \( \beta^* \) as defined earlier. Given any \( \beta > \beta^* \), consider the investment of \( e \) under \( SP \) (see equation (9)). The marginal benefit of investing \( e \) equals

\[
\frac{1}{2} (\beta V'(e_{IC}) + d_2 (a_{SP}, e_{IC})) > \frac{1}{2} (\beta^* V'(e_{IC}) + d_2 (a_{IC}, e_{IC}))
\]

\[= d_2 (a_{IC}, e_{IC})
\]

\[= 1,
\]
where the first line is due to $\beta > \beta^*$, $a_{SP} > a_{IC}$ ensured by (i), complementarity between $a$ and $e$; the second line due to the definition of $\beta^*$; and the third line due to (5). As the marginal benefit exceeds the marginal cost, it follows that the $e$ chosen under $SP$ will exceed $e_{IC}$, i.e., $e_{SP} > e_{IC}$. This completes the proof. ■