Using an error-correction model to test whether endogenous long-run growth exists

Sau-Him Paul Lau*

School of Economics and Finance, University of Hong Kong, Pokfulam, Hong Kong

Received 7 July 2006; accepted 5 June 2007
Available online 28 June 2007

Abstract

A major empirical interest in growth studies is whether permanent changes in economic fundamentals affect the long-run growth rate or not. However, a direct time series analysis of this hypothesis may not always be feasible because the permanence of many such changes is rather questionable. This paper explains why examining the long-run effects of temporary changes in investment share on per capita output provides indirectly the answer regarding the effects of (possibly hypothetical) permanent changes in investment share, when log per capita output and log per capita investment are cointegrated. Applying the proposed method to the post-war data of major industrial countries, it is found that a disturbance to investment share does not produce a positive long-run effect in each of the three countries – France, Japan and the United Kingdom – in which log per capita output and log per capita investment are cointegrated. The evidence is unfavorable to the class of endogenous growth models.

JEL classification: O40; E22

Keywords: Error-correction model; Endogenous long-run growth
1. Introduction

Whether a permanent change in economic fundamentals affects the long-run growth rate of an economy is an empirical question that many researchers and policy makers are interested in. Moreover, it is a distinguishing characteristic between endogenous and exogenous growth models because the change leads to a growth effect in the former class of models but only a level effect in the latter; see, for example, Romer (1986) and Lucas (1988). Based on this implication, Jones (1995) performs empirical analysis and concludes that the evidence on major industrial countries is unfavorable to the class of endogenous growth models. Similarly, Stokey and Rebelo (1995) conclude that income taxes do not have a growth effect according to the evidence provided by the tax reform ‘experiment’ in the United States of America (USA): income tax revenues increased dramatically from 2% to 15% of output in the early 1940s, but there was no change in per capita output growth.

While the endogenous and exogenous growth models imply different long-run effects of permanent changes in economic fundamentals, a direct examination of this hypothesis may not always be feasible because the permanence of many such changes is rather questionable. As an example, one of the frequently cited evidence against endogenous growth models is that the growth rates of per capita output (in USA and other industrial countries after World War II) are essentially trendless, but many investment share (i.e., investment-output ratio) series, based on total investment or producer durables investment, contain either strong positive trends or unit roots (Jones, 1995, Table IV). While the evidence regarding stationary output growth is expected, the conclusion of non-stationary investment shares in many industrial countries is quite different from those in several well-known empirical studies such as King et al. (1991). Moreover, the stationarity of some ‘great ratios’ such as the consumption-output ratio and investment-output ratio is regarded by many researchers as a stylized fact; see King et al. (1991) and especially Cochrane (1994). One may expect that many economists and econometricians, trained to be critical, would demand more evidence before deciding whether the endogenous growth models are empirically relevant or not.

By assuming explicitly that a permanent change in investment share is absent (or at least cannot be established affirmatively) in the data, this paper takes a complementary approach to deal with the question regarding the presence or absence of a growth effect of a (possibly hypothetical) permanent change. It examines the long-run effect of a temporary change in investment share on per capita output, and explains why the proposed method provides indirectly the answer to the above question, when log per capita output and log per capita investment are cointegrated.

The connection between these two apparently distinct questions (long-run effects of permanent and temporary changes in investment share, respectively) is implied by the theoretical results on the time series properties of stochastic endogenous and exogenous growth models. Lau (1997) shows that permanent changes in economic fundamentals lead to growth effects, and temporary changes cause permanent level effects for endogenous growth models. On the other hand, permanent changes in
economic fundamentals lead only to level effects, and temporary changes may cause either permanent or temporary level effects for exogenous growth models. Moreover, Lau (1999, p. 18) points out a systematic difference between endogenous and exogenous growth models in terms of the long-run effect of a temporary change in economic fundamentals when the observed variables are cointegrated. Specifically, for a system of $n$ variables with $r$ ($1 \leq r \leq n - 1$) cointegrating vectors, the long-run multiplier matrix for the structural vector moving-average (VMA) representation, which summarizes the long-run effects of the structural disturbances on the level of observed variables, is of reduced rank of $n - r$. If this cointegrated system is generated by an exogenous growth model, then exactly $r$ columns of the structural long-run multiplier matrix are zero. On the other hand, if this cointegrated system is generated by an endogenous growth model, then there is no column of zero in the structural long-run multiplier matrix.

This paper exploits the above systematic difference between endogenous and exogenous growth models and examines the long-run effect of a disturbance to investment share on per capita output. An important paper, Levine and Renelt (1992), examines the results of cross-section growth studies (such as Barro, 1991; Mankiw et al., 1992) and finds that most ‘statistically significant’ regression results are fragile with respect to minor changes in specification. However, they do identify a positive and robust correlation between economic growth and the share of investment in output. (This point is also mentioned in Mankiw, 1995, p. 302.) It is interesting to investigate whether this relationship is found in time series studies.

Since the long-run effects of temporary changes in investment share on per capita output for endogenous and exogenous growth models have testable implications when log per capita output and log per capita investment are cointegrated, the empirical method proposed in this paper is designed for a bivariate cointegrated system. The procedure is then applied to post-war data of several major industrial

---

1Kocherlakota and Yi (1996, p. 132) mention that ‘In exogenous growth models temporary policy changes do not permanently affect GNP, while in endogenous growth models temporary policy changes can permanently affect GNP.’ As made clear in Lau (1997, Eq. (21)), a temporary change in economic fundamentals in exogenous growth models may produce either temporary effect (if the associated external impulse process is $I(0)$) or permanent effect (if the associated external impulse process is $I(1)$) on the level of output.

2In essence, a ‘reduced rank of $n - r$’ (i.e., $n - r$ independent columns) and ‘$r$ columns of zero’ in the structural long-run multiplier matrix are not necessarily the same. The first condition is related to the number of independent cointegrating relations, while the second condition is related to the zero long-run effect of some structural disturbances. The first condition only implies that at most $r$ structural disturbances produce zero long-run effect; it does not pin down the precise number of structural disturbances that produce zero long-run effect. However, this paper shows that testable implications appear when the cointegration feature is combined with the time series properties of stochastic endogenous and exogenous growth models.

3While different factors such as human capital, trade, government investment and tax rates have been emphasized in the growth literature (such as Lucas, 1988; Glomm and Ravikumar, 1994; Collard, 1999), the one variable which is present in most, if not all, growth models is investment in physical capital. Jones (1995, p. 505) suggests that a test based on investment data may, arguably, be regarded as providing evidence with respect to the endogenous versus exogenous growth debate for the whole class of growth models, rather than for a specific mechanism only.
countries. It is found that a temporary change in investment share does not produce a positive long-run effect on per capita output in France, Japan and the United Kingdom (UK). The conclusions based on the time series analysis of this paper, as well as those of Jones (1995), are quite different from the positive and significant relationship between investment share and output growth found in cross-section studies.

This paper is organized as follows. Section 2 summarizes the different properties of endogenous and exogenous growth models regarding the long-run effects of permanent and temporary changes in economic fundamentals, respectively. Starting with the systematic difference between endogenous and exogenous growth models about the long-run multiplier matrix of the structural VMA representation when log per capita output and log per capita investment are cointegrated, Section 3 suggests a method to investigate whether a disturbance to investment share produces zero or positive long-run effect. Section 4 presents the empirical results. Section 5 relates the results of this paper to the literature and provides the conclusions.

2. Long-run effects of permanent and temporary changes in economic fundamentals

The long-run effects of permanent and temporary changes in economic fundamentals for stochastic endogenous and exogenous growth models have been examined in Lau (1997, 1999). To illustrate the relevant idea of these two papers in a framework which is useful to the subsequent empirical analysis, this section studies, respectively, the stochastic Solow–Swan model (Solow, 1956; Swan, 1956) and its endogenous growth counterpart.

Consider first a stochastic version of the Solow–Swan model with exogenous technological progress. A closed economy is populated by a constant number (equal to labor input, $N$) of identical agents. The supply side of the economy is represented by the following Cobb–Douglas production function of a representative agent

$$Y_t = AK_t^\lambda [(1 + \tau^t)N]^{1-\lambda} e^p_t,$$

(1)

where $0 < \lambda < 1$, $\tau > 0$, $Y_t$ is output at time $t$, $K_t$ is capital input at $t$, and $e^p_t$ is an impulse process to productivity. Parameter $\tau$ is the average rate of exogenous labor-augmenting technological progress.

The demand side of the economy is represented by

$$\frac{I_t}{Y_t} = s e^I_t,$$

(2)

where $I_t$ is investment at time $t$, $s (0 < s < 1)$ is the average investment share in output, and $e^I_t$ is an external impulse to the investment share at time $t$.

The two mean-zero external impulse processes are assumed to have the following form:

$$
\begin{bmatrix}
(1 - L)^c q_p(L) & 0 \\
0 & (1 - L)^c q_I(L)
\end{bmatrix}
\begin{bmatrix}
\ln e^p_t \\
\ln e^I_t
\end{bmatrix} =
\begin{bmatrix}
e^p_t \\
e^I_t
\end{bmatrix},
$$

(3)
where $L$ is the lag operator ($L \ln e_t^j = \ln e_{t-j}^j, j = P$ or $I$), parameter $c_j$ is either 0 or 1, $q_j(L)$ contains only roots strictly outside the unit circle, and $e_t^P$ and $e_t^I$ are uncorrelated white noise structural disturbances.\footnote{The distinction of the two terms ‘external impulse’ ($\ln e_t^j$) and ‘structural disturbance’ ($e_t^j$), which are related by (3), is maintained throughout the analysis. Specifically, while a structural disturbance process is restricted to be white noise, an external impulse process may be $I(0)$ or $I(1)$. Nevertheless, a structural disturbance may produce temporary or permanent effects, depending on other aspects of the growth model.} The impulse process $\ln e_t^j$ is $I(0)$ when $c_j = 0$ or is $I(1)$ without drift when $c_j = 1$.

Capital stock changes over time according to

$$K_{t+1} = (1 - \delta)K_t + I_t,$$

where $\delta \ (0 < \delta < 1)$ is the depreciation rate per period.

Denote in lower case letter the corresponding variable per capita. In order for log per capita output and log per capita investment to be difference-stationary and cointegrated in this model, it can be shown that one impulse process should be $I(1)$ without drift and the other $I(0)$; see, for example, Lau (1997, Section 5.1). Assuming that the productivity and investment share impulses are $I(1)$ without drift and $I(0)$, respectively, then $c_P = 1$ and $c_I = 0$ in (3). In this case, the log-linearized equations of motion near the steady-state growth path have the following VMA form:

$$\begin{bmatrix} (1 - L) \ln y_t \\ (1 - L) \ln i_t \end{bmatrix} = \begin{bmatrix} \tau \\ \tau \end{bmatrix} + \begin{bmatrix} (1 - (1 + \tau) - (1 - \delta)(\tau + \delta) L)^{-1} \\ (1 - (1 + \tau) - (1 - \delta)(\tau + \delta) L)^{-1} \end{bmatrix} \begin{bmatrix} (1 - L) q_P^{-1}(L) q_I^{-1}(L) \\ (1 - L) q_P^{-1}(L) q_I^{-1}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^P \\ \varepsilon_t^I \end{bmatrix}.$$

(5)

It can be observed that the long-run growth rate in this model is affected by the rate of technological progress ($\tau$) only. The long-run effects on output and investment level of a disturbance to productivity and investment share, respectively, will be discussed later.

The simplest way to ‘endogenize’ the source of growth is the AK model (Rebelo, 1991). The (exogenous investment-share version of) stochastic AK model consists of (2)–(4) and

$$Y_t = AK_t e_t^P.$$

(6)

It can be shown that if the order of integration of log per capita output or log per capita investment is at most one, then both external impulses should be $I(0)$ in this model, i.e., $c_P = c_I = 0$ in (3). See, for example, Lau (1997, Section 5.2).
In this case, the log-linearized equations of motion near the steady-state growth path have the following VMA representation:

\[
\begin{bmatrix}
(1 - L) \ln y_t \\
(1 - L) \ln i_t
\end{bmatrix}
= \begin{bmatrix}
\ln(1 - \delta + sA) \\
\ln(1 - \delta + sA)
\end{bmatrix}
+ \begin{bmatrix}
(1 - \frac{1 - \delta}{1 - \delta + sA}L) q_p^{-1}(L) & \frac{sA}{1 - \delta + sA} q_I^{-1}(L) \\
(1 - \frac{1 - \delta}{1 - \delta + sA}L) q_p^{-1}(L) & (1 - \frac{1 - \delta}{1 - \delta + sA}L) q_I^{-1}(L)
\end{bmatrix}
\begin{bmatrix}
\varepsilon_t^p \\
\varepsilon_t^I
\end{bmatrix},
\] (7)

It is easy to observe from (7) that a permanent change in the investment share (represented by a change in parameter s) will produce growth effects on both output and investment.

The above analysis shows that both (5) of the Solow–Swan model and (7) of the AK model can be represented by the following first-difference VMA form:\n
\[
\begin{bmatrix}
\Delta x_{1t} \\
\Delta x_{2t}
\end{bmatrix} = \text{constant} + M(L) \varepsilon_t = \text{constant} + \begin{bmatrix}
M_{11}(L) & M_{12}(L) \\
M_{21}(L) & M_{22}(L)
\end{bmatrix} \varepsilon_t,
\] (8)

where \( \Delta \equiv 1 - L \) is the first-difference operator, \( x_1 \) and \( x_2 \) stand for log per capita output and log per capita investment, respectively, \( \varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})' \) is a \( 2 \times 1 \) vector of structural disturbances such that its components are serially and mutually uncorrelated, the \( 2 \times 2 \) matrix of lag polynomial \( M(L) \) is defined by \( M(L) = \sum_{j=0}^{\infty} M_j L^j \), and the ‘constant’ vector is non-zero (in order for the variables to demonstrate long-run growth). How this ‘constant’ vector is related to the behavioral parameters is important to the long-run effects of permanent changes in these parameters, as illustrated in the above analysis and in Lau (1997, Section 4). Other than this issue, its precise value is inconsequential to the analysis in this paper and therefore is left unspecified.

Following the terminology in the vector autoregression (VAR) literature, the time-invariant system (8) is called the structural model. Since these relationships are structural, the off-diagonal elements of the leading VMA parameter matrix \( M_0 \) may be non-zero; see, for example (5) and (7). For a meaningful structural model, the inverse of \( M_0 \) exists. The long-run multiplier matrix of the structural VMA representation (8) is given by

\[
M(1) = \sum_{j=0}^{\infty} M_j,
\] (8a)
and its various elements contain information about the long-run effects of structural disturbances on the level of observed variables.

It can be observed from (5) that the long-run multiplier matrix of the stochastic Solow–Swan model is given by

$$M(1) = \begin{bmatrix} \frac{\tau + \delta}{1 + \tau} q_p^{-1}(1) & 0 \\ \frac{\tau + \delta}{1 + \tau} q_p^{-1}(1) & 0 \end{bmatrix}. \quad (5a)$$

The first column of the long-run multiplier matrix in (5a) is non-zero and the second column is zero, reflecting that a productivity disturbance produces permanent effects and a disturbance to investment share produces temporary effects. On the other hand, the long-run multiplier matrix of the stochastic AK model is given by

$$M(1) = \begin{bmatrix} \frac{sA}{1 - \delta + sA} q_p^{-1}(1) & \frac{sA}{1 - \delta + sA} q_I^{-1}(1) \\ \frac{sA}{1 - \delta + sA} q_p^{-1}(1) & \frac{sA}{1 - \delta + sA} q_I^{-1}(1) \end{bmatrix}. \quad (7a)$$

Both columns of the long-run multiplier matrix in (7a) are non-zero, reflecting that the effect of either a disturbance to productivity or investment share is permanent. Moreover, it can be observed from (5a) and (7a) that the $2 \times 2$ long-run multiplier matrix $M(1)$ of either model is of reduced rank of one, meaning that log per capita output and log per capita investment are cointegrated.

The above analysis shows that if the cointegrated system (8) is generated by an endogenous growth model, then both external impulses are $I(0)$ but each of them produces non-zero long-run effects and therefore both columns of $M(1)$ are non-zero; see (7) and (7a). Moreover, a positive disturbance to productivity (respectively, investment share), which increases current per capita output (respectively, current per capita investment), is expected to produce positive (instead of negative) long-run effects on either variable for a meaningful endogenous growth model. Therefore, both columns of $M(1)$ are positive. On the other hand, if the cointegrated system (8) is generated by an exogenous growth model, then one external impulse process is $I(0)$ and the other is $I(1)$. As $I(0)$ impulses produce only temporary effects for an exogenous growth model, one column of the long-run multiplier matrix $M(1)$ will be zero; see (5) and (5a).

In summary, the long-run effects of permanent and temporary changes in economic fundamentals differ for endogenous and exogenous growth models. Moreover, the long-run effects of permanent and temporary changes are related as

---

6If both impulses are $I(1)$ for the exogenous growth model, then no column of $M(1)$ in (8a) is zero and the system is not cointegrated. The systematic difference between endogenous and exogenous growth models regarding the long-run effects of structural disturbances disappears for an $I(1)$ but non-cointegrated system.

7For convenience, in this paper I use the Solow–Swan exogenous growth model and the AK endogenous growth model to show the different long-run effects of permanent and temporary changes in investment share. These results can be generalized to other exogenous and endogenous growth models, since the
each is caused by the presence or absence of the factor \( (1 - L) \) in the determinant of the polynomial matrix of the behavioral systems; see, for example Lau (1997, Eqs. (37) and (32)). The difference of the long-run effects of permanent changes in economic fundamentals between these two classes of models is a very sharp distinction: a permanent change in some behavioral parameters (such as the average investment share \( s \)) will lead to a growth effect for an endogenous growth model, whereas a permanent change in the underlying parameters, other than the exogenous rate of technological progress, leads only to a level effect for an exogenous growth model. Unfortunately, this implication may not be testable as it is difficult to find interesting events of permanent changes in behavioral parameters.

On the other hand, temporary changes in economic fundamentals are easier to find. Regarding the long-run effects of these temporary changes, there is also a systematic difference between these two classes of models when the variables are cointegrated. From the next section onwards, the difference of the long-run effects of temporary changes between endogenous and exogenous growth models will be exploited to develop a testing procedure, and the focus will be on the long-run multiplier matrix for the structural VMA representation (8) of a cointegrated system. As the above analysis makes clear, testing the long-run effect of a temporary change in economic fundamentals provides an indirect answer regarding the effect of a permanent change.

3. Investigating the long-run effect of a disturbance to investment share

The analysis of Section 2 shows that for a bivariate cointegrated system of log per capita output and log per capita investment, both columns of the long-run multiplier matrix \( M(1) \) in (8) are positive if the data are generated by an endogenous growth model, but one column of \( M(1) \) is positive and the other is zero if the data are generated by an exogenous growth model. Provided that the structural model is identified, this is a testable hypothesis.

This section suggests a method to examine whether the long-run effect of an investment share disturbance on per capita output is zero or positive. Specifically, Section 3.1 discusses how identification is achieved in this paper. Under the particular identifying assumption, Section 3.2 suggests a simple way to test the hypothesis that an investment share disturbance produces a zero long-run effect. Combining all the results, Section 3.3 summarizes the empirical procedure used in this paper. Prior to these subsections, the next paragraph discusses the validity of the implicit assumption made in this paper that productivity disturbances always produce permanent effects.

When log per capita output and log per capita investment are cointegrated (and thus the rank of \( M(1) \) in (8) is one), at least one structural disturbance always

(footnote continued)

systematic differences in the long-run effects of these changes in economic fundamentals can be traced back to the different growth mechanisms of these two classes of models (Lau, 1997, 1999).
produces permanent effects. It is assumed in this paper that the productivity disturbance always produces a permanent effect but the investment share disturbance may produce a temporary or permanent effect. In the notation of this paper, the first column of $M(1)$ is non-zero but the second column may be zero or non-zero. This specification is justified on two grounds. First, such a specification is flexible to allow the data to decide whether a disturbance to investment share produces a temporary or permanent effect, which is the main objective of this paper. Changing the roles of the two structural disturbances just presupposes the answer. Second, if the investment share disturbance is interpreted according to (2) and (3), then the above assumption regarding the roles of the two structural disturbances is equivalent to assuming that the external impulse to the investment share is $I(0)$. In this case, $c_I = 0$ in (3) and the investment-output ratio is stationary, which is consistent with the empirical evidence on industrial countries; see, for example, King et al. (1991).

3.1. Identifying assumptions

Regarding the assumptions used to identify the structural model, this paper makes use of the idea in the theoretical growth literature that current output is determined in the supply side by factor inputs such as capital and labor. Specifically, it is assumed in many growth models that current output is determined in the supply side according to the production function, as illustrated in (1) of the Solow–Swan model or (6) of the AK model. On the other hand, the decision between current consumption and investment for future capital is made on the basis of some demand-side or preference factors, such as the simple investment rule used in the Solow–Swan model or a more complicated mechanism based on optimizing an intertemporal objective function. As a result, variables (such as investment) which are absent in the production function do not affect current output but may still affect future output through, for example, capital accumulation.

When the idea of supply-side determination of current output is interpreted in a stochastic setting, it implies a recursive ordering regarding the structural disturbances. Only disturbances appearing in the production function ($e_t^P$ in the models of Section 2) or the inputs will affect output in the current period, whereas other variables such as investment are contemporaneously affected by all disturbances. (The assumptions used in Kocherlakota and Yi (1996, footnote 4) also reflect the supply-side determination of current output.) For the bivariate system studied in this paper, the disturbance to investment share does not affect per capita output contemporaneously (i.e., $m_{0,12} = 0$ in the notation of this paper, where $m_{0,ij}$ is the row-$i$, column-$j$ element of the leading VMA parameter matrix $M_0$); see (5) or (7). As a result, the use of Sims’ (1980) recursive identification scheme (with per capita output ordered first) is valid when current output is determined in the supply side.8

8It should be emphasized that this recursive ordering depends only on the assumption that current output is determined in the supply side, and therefore may be present in an optimizing model or in an exogenous investment share model. To make the analysis relatively simple and intuitive, Section 2 uses an
Of course, no identifying assumption is appropriate in all circumstances or is consistent with all theoretical models. The above identification scheme is chosen as it reflects the maintained hypothesis that the data are generated by some growth models. Moreover, this choice is neutral with respect to the exogenous versus endogenous growth debate.9

3.2. A test of zero long-run effect

Under the assumption that a disturbance to productivity will always produce permanent effects, the analysis in Section 2 shows that the long-run effect of a disturbance to investment share is zero (i.e., $M_{12}(1) = M_{22}(1) = 0$) for an exogenous growth model, but is positive for an endogenous growth model. Thus, the rejection of zero long-run effect of an investment share disturbance is a necessary (but not sufficient) condition for the cointegrated system (8) to be generated by an endogenous growth model. This subsection shows that under the Sims’ recursive ordering, there is a simple way to test the null hypothesis of zero long-run effect. The test is based on the estimated coefficients of the following reduced-form bivariate vector error-correction system10:

$$
\begin{bmatrix}
\Delta x_{1t} \\
\Delta x_{2t}
\end{bmatrix}
= 
\begin{bmatrix}
\mu_1^R \\
\mu_2^R
\end{bmatrix} + 
\sum_{j=1}^{p} 
\begin{bmatrix}
\pi_{j,11}^R & \pi_{j,12}^R \\
\pi_{j,21}^R & \pi_{j,22}^R
\end{bmatrix}
\begin{bmatrix}
\Delta x_{1,t-j} \\
\Delta x_{2,t-j}
\end{bmatrix}
+ 
\begin{bmatrix}
\varphi_1^R \\
\varphi_2^R
\end{bmatrix}
(x_{2,t-1} - x_{1,t-1}) + 
\begin{bmatrix}
\varepsilon_{1t}^R \\
\varepsilon_{2t}^R
\end{bmatrix},
$$

(9)

(exogenous investment share assumption, and thus the structural disturbance $\varepsilon^e$ in (2) is naturally interpreted as a disturbance to investment share. Alternatively, if the bivariate system of per capita output and per capita investment is generated by a more elaborate model (such as an optimizing model with government), then the structural disturbance $\varepsilon^e$ may be interpreted as a mixture of disturbances to government policy and the agent’s utility function. For example, a negative disturbance to $\varepsilon^e$ in the bivariate framework of this paper may capture a positive disturbance to income tax revenue (the type of disturbances considered in Stokey and Rebelo, 1995).

9In subsequent analysis, I focus on developing testing procedures to distinguish between endogenous and exogenous growth models under the Sims’ recursive ordering. Despite the justifications for the use of recursive ordering in this paper, it should be pointed out that actual output data may also be affected by demand factors contemporaneously. A possible extension in the future is to develop testable implications under other identifying assumptions and to see whether the conclusions of this paper are robust to other identifying assumptions or not.

10In studying a bivariate system, it is implicitly assumed that the specification of only two structural disturbances is meaningful; see, for example, Cochrane (1994), Lastrapes and Selgin (1994), Quah and Vahey (1995) and, especially, Blanchard and Quah (1989). When the number of underlying disturbances is more than two, the proposed test in a bivariate cointegrated system to distinguish between the endogenous and exogenous growth models is still valid under the assumptions that (a) current output is determined in the supply side and (b) the ‘other disturbance’ ($\varepsilon^o$), which captures all disturbances orthogonal to the output disturbance and is identified as having no contemporaneous effect on output, produces a zero long-run effect on output for an exogenous growth model.
where $p$ is the lag length, and $z^R = (z_1^R, z_2^R)'$ is the reduced-form adjustment vector. Note that the components of $\varepsilon^R_t = (\varepsilon_1^R, \varepsilon_2^R)'$ in this reduced-form system are serially uncorrelated (which is a reasonable assumption provided that there are enough lag terms) but may be contemporaneously correlated with each other.

Under the identifying assumption of $m_{0,12} = 0$, it can be shown that testing whether $z_1^R$, the first element of the adjustment vector of the reduced-form vector error-correction model (9), is zero or non-zero can be used to examine whether the second column of $M(1)$ is zero (i.e., $M_{12}(1) = M_{22}(1) = 0$) or not for the structural model. More generally, the result can be stated as:

**Proposition 1.** If Sims’ $x_1$-first ordering is adopted for the bivariate cointegrated model (9) of variables $x_1$ and $x_2$, then testing whether $z_1^R$ is zero or not can be used to examine whether the long-run effect of a structural $x_2$-disturbance is zero or not. By symmetry, if Sims’ $x_2$-first ordering is adopted, then testing whether $z_2^R$ is zero or not can be used to examine whether the long-run effect of a $x_1$-disturbance is zero or not.

The proof is given in Appendix A.1. A brief discussion of the underlying idea of Proposition 1 is as follows. There is an assumption regarding the statistical property of the data (that the two variables are cointegrated), and there is an identifying assumption regarding the structural disturbances (that there is no contemporaneous effect of a $x_2$-disturbance on the other variable $x_1$).

Since the rank of the long-run multiplier matrix for the structural VMA representation (8) of a bivariate cointegrated system is one, at least one column of the matrix must be non-zero. Through the relationships among various representations of a cointegrated system, it can be shown that the null hypothesis of $M_{12}(1) = M_{22}(1) = 0$ implies that $z_1$ is zero, where $z_1$ is the coefficient of the error-correction term in the first equation of the following structural vector error-correction model corresponding to (9):

$$
\Pi_0 \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} = \begin{bmatrix} \pi_{0,11} & \pi_{0,12} \\ \pi_{0,21} & \pi_{0,22} \end{bmatrix} \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}
+ \sum_{j=1}^{p} \begin{bmatrix} \pi_{j,11} & \pi_{j,12} \\ \pi_{j,21} & \pi_{j,22} \end{bmatrix} \begin{bmatrix} \Delta x_{1,t-j} \\ \Delta x_{2,t-j} \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} (x_{2,t-1} - x_{1,t-1})
+ \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.
$$

(10)

Note that the two structural disturbances, $\varepsilon_{1t}$ and $\varepsilon_{2t}$, are uncorrelated. Moreover, the variance of each of these structural disturbances can be normalized to one without loss of generality.
Finally, the relationship between the structural model (10) and the reduced-form model (9) implies the following result:\(^{11}\):

\[
\begin{bmatrix}
\mathbf{x}_1^R \\
\mathbf{x}_2^R
\end{bmatrix}
\equiv \mathbf{x}_R = M_0 \mathbf{x} =
\begin{bmatrix}
m_{0,11} x_1 + m_{0,12} x_2 \\
m_{0,21} x_1 + m_{0,22} x_2
\end{bmatrix}.
\]

Combining the identifying assumption of \(m_{0,12} = 0\) with the hypothesis of \(\mathbf{z}_1 = 0\) about the unobserved structural model leads to a testable implication of \(\mathbf{z}_1^R = 0\) about the reduced-form model, according to (11).\(^{12}\)

### 3.3. The empirical procedure

The above results suggest the following procedure to examine the long-run effect of an investment share disturbance, when log per capita output and log per capita investment are cointegrated with the cointegrating vector \((1, -1)'\). The first step is to estimate the vector error-correction model (9). The next steps are to test \(\mathbf{z}_1^R = 0\) and to obtain the long-run response of an investment share disturbance on per capita output, \(M_{12}(1)\), under the identifying assumption \(m_{0,12} = 0\). The long-run effect \(M_{12}(1)\) is obtained according to the following Proposition.\(^{13}\) (The proof is given in Appendix A.2.)

**Proposition 2.** If Sims’ x₁-first ordering is adopted for the bivariate cointegrated model (9) of variables \(x_1\) and \(x_2\), then the long-run effect \(M_{12}(1)\), which is equal to \(M_{22}(1)\), is obtained according to the following Proposition.\(^{13}\) (The proof is given in Appendix A.2.)

\(^{11}\)Premultiplying the structural vector error-correction model (10) by \((\Pi_0)^{-1}\) and comparing it with its reduced-form counterpart (9) give (11) and \(\mathbf{z}_R^i = (\Pi_0)^{-1} \mathbf{z}_0 = M_0 \mathbf{z}_0\), where \(M_0 = (\Pi_0)^{-1}\) can be shown to hold. See, for example, Cochrane (1994). Moreover, the above relationship and the normalization of variances of the structural disturbances imply that the reduced-form variance–covariance matrix, \(E(\mathbf{z}_R^i \mathbf{z}_R^i')\), is related to \(M_0\) according to (A.16) in Appendix A.2. For a bivariate system, this relationship gives three independent restrictions. With one more restriction (such as \(m_{0,12} = 0\)), the four elements of \(M_0\) can be recovered and thus the structural model is identified.

\(^{12}\)Proposition 1 can be generalized to the case that the cointegrating vector is estimated instead of imposed. When the cointegrating vector is estimated, the reduced-form model (9) is replaced by

\[
\begin{bmatrix}
\Delta x_{1,t} \\
\Delta x_{2,t}
\end{bmatrix}
\equiv
\begin{bmatrix}
\mathbf{\mu}_1^R \\
\mathbf{\mu}_2^R
\end{bmatrix} + \sum_{j=1}^{p} \begin{bmatrix}
\pi_{1,11}^R & \pi_{1,12}^R \\
\pi_{2,21}^R & \pi_{2,22}^R
\end{bmatrix} \begin{bmatrix}
\Delta x_{1,t-j} \\
\Delta x_{2,t-j}
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{\beta}_1^R \\
\mathbf{\beta}_2^R
\end{bmatrix} \begin{bmatrix}
x_{2,t-1} \\
x_{1,t-1}
\end{bmatrix} + \begin{bmatrix}
\mathbf{\epsilon}_{1,t} \\
\mathbf{\epsilon}_{2,t}
\end{bmatrix}.
\]

\(^{13}\)The long-run effect can also be obtained by standard methods for impulse-response function. In fact, dynamic responses at different lags can easily be calculated by these numerical methods. Since the distinguishing feature between endogenous and exogenous growth models lie on the long-run response, the impulse responses at other lags are not presented.
is estimated by

\[
\frac{\left(\sqrt{\Omega_{22}^{R} - (\hat{\Omega}_{12}^{R})^2/\hat{\Omega}_{11}^{R}}\right)\tilde{\beta}_{1}^{R}}{(1 - \sum_{j=1}^{p} \hat{\beta}_{j,21}^{R} - \sum_{j=1}^{p} \hat{\beta}_{j,22}^{R})\tilde{\beta}_{1}^{R} - (1 - \sum_{j=1}^{p} \hat{\beta}_{j,11}^{R} - \sum_{j=1}^{p} \hat{\beta}_{j,12}^{R})\tilde{\beta}_{2}^{R}},
\]

where \(\tilde{\beta}_{1}^{R}, \tilde{\beta}_{2}^{R}, \hat{\beta}_{j,11}^{R}, \hat{\beta}_{j,12}^{R}, \hat{\beta}_{j,21}^{R} \) and \(\hat{\beta}_{j,22}^{R} \) are estimated coefficients of the vector error-correction model (9), and \(\hat{\Omega}_{11}^{R}, \hat{\Omega}_{22}^{R} \) and \(\hat{\Omega}_{12}^{R} \) are estimated variance and covariance terms of the least-squares residuals of the two equations in (9).

There are two different cases regarding the empirical results. If the null hypothesis \(\beta_{1}^{R} = 0 \) is rejected and the estimated long-run response of an investment share disturbance on per capita output, \(M_{12}(1)\), is positive, then the evidence is favorable to an endogenous growth model. On the other hand, if either (a) the null hypothesis \(\beta_{1}^{R} = 0 \) is not rejected, or (b) the null hypothesis \(\beta_{1}^{R} = 0 \) is rejected but the estimated long-run response \(M_{12}(1)\) is negative, then the evidence is not favorable to an endogenous growth model.

4. Empirical analysis

The empirical analysis focuses on four major industrial countries – France, Japan, UK and USA – in the post World War II period.\(^{14}\) The data are taken from the Penn World Table version 6.1 (PWT6.1); a description of an earlier version 5 is given in Summers and Heston (1991). The per capita real GDP figures are the ‘RGDPL’ series, and the per capita real investment figures are obtained from combining the ‘RGDPL’ and ‘KI’ series (real gross domestic investment as percentage of real GDP). The various series for these four countries are plotted in Figs. 1–4. All the (annual) series are from 1950 to 2000, and are measured in 1996 international prices. The post-war period is chosen primarily because of compatibility with previous studies on the effect of investment share on output, such as of Levine and Renelt (1992) and, especially, Jones (1995, Section 3). The former study uses cross-country data from 1960 to 1989, and the latter uses annual series of industrial countries from 1950 to 1988 and mentions that the choice of that sample avoids changes in the stochastic properties associated with World War II (p. 501). In this paper, estimation of the various models runs from 1955 to 2000, with the earlier observations used as starting values.

\(^{14}\)The real GDP per worker series for Germany – the remaining member of the Group of Five (G5) industrial countries – starts from 1990 and other variables start from 1970 in the PWT6.1 data set. As these series are too short for meaningful analysis, Germany is not examined in this paper. Moreover, while it is possible to apply the empirical analysis to other advanced countries (which are likely to be close to their balanced growth paths), I only focus on the most important G5 countries in this paper so as to conserve space.
4.1. Are the bivariate system cointegrated?

The empirical procedure of this paper is developed for a bivariate system in which log per capita output and log per capita investment are $I(1)$ and cointegrated with the cointegrating vector $(1, -1)$. A preliminary step is to check whether the above specification is consistent with the data or not.

A commonly used cointegration test is the Johansen (1991, 1995) procedure. As the log per capita output and log per capita investment series clearly exhibit upward trends (see Figs. 1–4), it is appropriate to use the version of Johansen’s test which allows for the possibility of a linear trend in the data and in the cointegrating vector (Johansen, 1995, Theorem 6.2). The analysis proceeds as follows. First, determine whether the cointegrating rank is one or not. Second, if the cointegrating rank is one, test the joint hypothesis that the sum of the coefficients of log per capita output and log per capita investment in the cointegrating relation is zero and the coefficient of the linear trend is zero.

The results of the cointegration tests for various countries, with the lag length determined by the Akaike information criterion (AIC) or Baynesian information criterion (BIC), are presented in Table 1. It can be observed that the null hypothesis of no cointegration is consistent with the data of USA. On the other hand, there is
evidence of one cointegrating relation and stationary log investment-output ratio in
the other three countries, even though the evidence is not very strong. Specifically,
according to the trace tests, it is concluded that there is one cointegration relation for
Japan at a 1% significance level. Conditional on one cointegrating relation, the
hypothesis of stationary investment-output ratio is not rejected at a 1% significance
level (but it is marginally rejected at a 5% significance level) for the model with two
lags. For France, the cointegrating rank is concluded to be one if a 1% significance
level is used. Moreover, the null hypothesis of stationary investment-output ratio is
not rejected at a 1% significance level. For the UK model with one lag, it is
concluded that the cointegrating rank is one at a 5% significance level, and the null
hypothesis of stationary investment-output ratio is not rejected at a 1% significance
level (but it is rejected at a 5% significance level).

An alternative procedure to test whether log per capita output and log per capita
investment are cointegrated with the prespecified cointegrating vector \((1, -1)\)' is the
Horvath–Watson test, which examines the null hypothesis of no cointegration
against the composite alternative of cointegration with a known cointegrating
vector. Horvath and Watson (1995) show that their test possesses better power
properties when compared to other tests (such as Johansen) that do not impose the
cointegrating vector.
The results of the Horvath–Watson test are presented in Table 2. It can be observed that the null hypothesis of no cointegration is rejected at a 1% significance level for France and Japan, is rejected at a 10% significance level for UK when there are two lags (and is rejected at a 5% significance level when there is one lag), but is not rejected even at a 10% significance level for USA.

To summarize, both the Johansen and Horvath–Watson tests indicate that there is no cointegration for USA. On the other hand, the evidence based on the Horvath–Watson tests (and, to a lesser extent, the Johansen tests) are favorable to the hypothesis that log per capita output and log per capita investment are cointegrated for France, Japan and UK, with weaker evidence for UK. The remaining empirical procedure of this paper is, therefore, applied only to these three countries but not to USA; see footnote 6 as well.

4.2. Vector error-correction models: estimation and testing

The regression results of the vector error-correction model (9) for France, Japan and UK, with the lag length determined by AIC, are presented in Table 3. (To conserve space, the estimation results for the models chosen by the BIC are not reported, but the robustness of the conclusion will be examined later.) For each of the equations given in Table 3, the correlograms and the Ljung-Box $Q$-statistics
Fig. 4. US.

Table 1
Johansen tests

<table>
<thead>
<tr>
<th>Country</th>
<th>Lag length</th>
<th>Trace statistic</th>
<th>Likelihood ratio statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$r = 0$</td>
<td>$r = 1$</td>
</tr>
<tr>
<td>France</td>
<td>1 (A, B)</td>
<td>32.09***</td>
<td>12.74**</td>
</tr>
<tr>
<td>Japan</td>
<td>2 (A)</td>
<td>31.58***</td>
<td>7.77</td>
</tr>
<tr>
<td>Japan</td>
<td>1 (B)</td>
<td>37.31***</td>
<td>9.75</td>
</tr>
<tr>
<td>UK</td>
<td>2 (A)</td>
<td>22.53</td>
<td>8.35</td>
</tr>
<tr>
<td>UK</td>
<td>1 (B)</td>
<td>30.05**</td>
<td>8.62</td>
</tr>
<tr>
<td>USA</td>
<td>2 (A)</td>
<td>15.38</td>
<td>6.00</td>
</tr>
<tr>
<td>USA</td>
<td>1 (B)</td>
<td>19.25</td>
<td>4.75</td>
</tr>
</tbody>
</table>

Note: The lag length is determined by the AIC (denoted by A) or BIC (denoted by B). The critical values of the trace test statistic for the null hypothesis of no cointegration ($r = 0$) are 22.95 at a 10% significance level, 25.47 at a 5% significance level, and 30.65 at a 1% significance level, respectively. The critical values of the trace test statistic for the null hypothesis of one cointegrating relation ($r = 1$) are 10.56 at a 10% significance level, 12.39 at a 5% significance level, and 16.39 at a 1% significance level, respectively (see, for example, Johansen, 1995, Table 15.4). The likelihood ratio test is used to test the joint hypothesis that, conditional on $r = 1$, the coefficient of the linear trend is zero and the sum of the coefficients of log per capita output and log per capita investment is zero. The likelihood ratio test statistic is Chi-square distributed with two degrees of freedom under the null hypothesis. The symbols *, ** and *** represent statistical significance at 10%, 5%, and 1%, respectively.
indicate that the residuals are not autocorrelated and there is no evidence of misspecification. Based on the estimated coefficients and the identifying assumption that a disturbance to investment share does not affect current per capita output, the long-run responses of the level of per capita output (or investment) to both structural disturbances can be obtained. They are also given in Table 3.

It is observed from Table 3 that the \( t \)-statistic of the estimated coefficient of the error-correction term of the per capita output equation is \(-0.95\) for UK. Thus, the coefficient \( x_1^R \) is not statistically different from zero at a 5\% significance level.\(^{15}\) On the other hand, the \( t \)-statistic of the estimated coefficient of the error-correction term of the per capita output equation is \(-2.79\) for France and \(-3.42\) for Japan, but the estimated long-run response, \( M_{12}(1) \), is negative in each case (\(-0.014\) for France and \(-0.038\) for Japan). Thus, the null hypothesis that the data are generated by an exogenous growth model is not rejected by the post-war data of each of the three G5 countries in which the cointegrated system (9) is appropriate.

### 4.3. Robustness analysis

A practical virtue of the simple (least-squares) estimation procedure used in Section 4.2 is that the results are quite robust with respect to the number of lag terms

---

\(^{15}\)In the presence of non-stationary time series, a natural question is whether the distributions of the various test statistics (especially those of the error-correction coefficients) are standard or not under the null hypothesis. The results in Sims et al. (1990) make clear that under the maintained assumption that the variables are cointegrated, the \( t \)-statistic for the null hypothesis follows standard distribution asymptotically because all the right-hand side variables of the regressions are \( I(0) \). Note that the test \( x_1^R = 0 \) is a test of zero long-run effect under the maintained assumption of cointegration, and is neither a unit root nor cointegration test. On the other hand, the asymptotic distribution of the null hypothesis of no cointegration (i.e., \( x_1^R = x_2^R = 0 \)) is non-standard; see Table 2.
The estimated coefficients and the $t$-statistics for the error-correction terms, and the long-run responses of the structural disturbances (under the recursive ordering with per capita output first) for France, Japan or UK with various lags are presented in the upper panel of Table 4. It can be observed that the estimated coefficients of the error-correction terms and the associated $t$-statistics for the models with different lags remain relatively unchanged in each country. For the various models with 1–4 lags in each country, it is still concluded that a disturbance to investment share does not have a positive long-run effect on per capita output.

In the theoretical models in Section 2, the simplifying assumption of inelastic labor supply is used, as in most growth models. In empirical papers examining the relevance of theoretical growth models, researchers have used either the data based on total population or total number of workers. To see whether the above conclusion is robust with respect to the choice of variables, this paper also uses the real GDP per worker series (‘RGDPWOK’ series in PWT61) and the real investment per worker series (by combining the ‘RGDPWOK’ and ‘KI’ series).

In the lower panel of Table 4, it is observed that the $t$-statistic of the estimated coefficient of the error-correction term of the output per worker equation is −1.21

---

Table 3
Estimated coefficients and $t$-statistics for France, Japan and UK

<table>
<thead>
<tr>
<th>Country</th>
<th>France</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left-hand variable</td>
<td>$\Delta l_y$</td>
<td>$\Delta l_i$</td>
<td>$\Delta l_y$</td>
</tr>
<tr>
<td>Notes: The variables $l_y$ and $l_i$ represent, respectively, log per capita output and log per capita investment. For each regressor, the number not in parentheses is the estimated coefficient and the number in parentheses is $t$-statistic. The long-run responses $M_{11}(1)$ and $M_{12}(1)$ are obtained under the identifying assumption of $m_{0,12} = 0$.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

16 According to unreported results, $M_{11}(1)$ is positive in all cases. Thus, the implicit assumption that the productivity disturbance produces positive long-run effects is satisfied.
for UK. Thus, the coefficient $a_{R1}$ is not statistically different from zero at a 5% significance level. On the other hand, the $t$-statistic of the estimated coefficient of the error-correction term of the output per worker equation is $\frac{17}{10}$ for France and $\frac{32}{10}$ for Japan, but the estimated long-run response, $M_{12}(1)$, is negative in each case ($-0.023$ for France and $-0.038$ for Japan). Again, the null hypothesis that the data, based on total number of workers, are generated by an exogenous growth model is not rejected by the post-war data of France, Japan and UK.

### 4.4. Summary

To summarize, when log per capita output and log per capita investment are cointegrated and Sims’ output-first ordering is valid, the test of $a_{R1} = 0$ in (9) and the sign of $M_{12}(1)$ can be used to examine whether a disturbance to investment share produces a zero or positive effect on per capita output. For France, Japan and UK, it is concluded that a temporary change in investment share does not produce a positive long-run effect on per capita output. The above empirical results are more consistent with the class of exogenous growth models than with the class of endogenous growth models. The conclusion holds whether per capita series or per worker series are used, and is also robust to the number of lag terms included.
5. Conclusion

A major empirical interest, and a distinguishing characteristic between endogenous and exogenous growth models, is whether a permanent change in investment share produces a growth effect or only a level effect. Unfortunately, a direct time series analysis of this hypothesis may not always be possible in many countries due to a lack of permanent changes in investment share. In fact, the investment-output ratios fluctuate around a constant mean and there is no evidence of a permanent change in this variable for France, Japan and UK (Figs. 1–3); see also the discussion in King et al. (1991) and Cochrane (1994). To prevent determining the empirical relevance of the endogenous growth models by examining the long-run effects of a (possibly non-existent) permanent change in investment share, this paper takes a complementary approach. It assumes that such a permanent change is absent (or at least cannot be established affirmatively) in the data, and considers the long-run effect of a temporary change in investment share.

Using the systematic difference in time series properties of endogenous and exogenous growth models (in Section 2) and the relationships of different representations of a cointegrated system (in Section 3), Proposition 1 suggests a simple test of zero long-run effect of a disturbance to investment share, under the assumption that an investment share disturbance does not affect per capita output contemporaneously. Moreover, Proposition 2 expresses the long-run effect of an investment share disturbance on per capita output in terms of the estimated coefficients and variance–covariance matrix of model (9).

The empirical framework of this paper shares some similarities with Kocherlakota and Yi (1996), as the long-run effects of temporary changes in economic fundamentals are examined in both papers. However, there are also a number of differences between the two papers. Kocherlakota and Yi (1996) focus on a set of policy variables for USA, while investment shares of several industrial countries are examined in this paper. Kocherlakota and Yi (1996, p. 127) provide statistical justifications for the three cases in which their methods are appropriate, whereas this paper combines economic analysis and time series properties of the data to suggest a method to distinguish between endogenous and exogenous growth models when per capita output and per capita investment are cointegrated. Finally, Kocherlakota and Yi (1996) use a single-equation approach, but this paper uses a system approach.17

Applying the proposed empirical procedure to France, Japan and UK, it is found that a disturbance to investment share does not have a positive long-run effect on per capita output for each of these countries. There is no evidence favorable to the endogenous growth models. These results are quite robust with respect to lag length and to the choice of variables. Even though this paper assumes that the

---

17While Proposition 1 suggests that only $x_t^p$ of the output equation in (9) is required to test the null hypothesis of zero long-run effect of an investment share disturbance under the assumption that an investment share disturbance does not affect per capita output contemporaneously, Proposition 2 shows that both equations of (9) are required to estimate the long-run response $M_{12}(1)$. Moreover, the evidence based on both equations is useful in determining the lag length according to the AIC or BIC.
investment-output ratio is stationary and therefore does not share with a major premise in Jones (1995) regarding the presence of deterministic or stochastic trend in this ratio, the conclusions obtained in these two papers are similarly unfavorable to endogenous growth models. Moreover, they are complementary in the sense that no positive growth effect associated with a permanent change in investment share is found in one study and no positive permanent level effect associated with a temporary change in investment share is found in the other. The time series evidence appears to be quite different from the positive and significant relationship between investment share and output growth found in cross-section studies.

With the focus on examining whether the long-run effect of a temporary change in investment share on per capita output is zero or positive, the objectives of this paper are to derive testable implications of some theoretical growth models and then to perform empirical analysis for post-war data of major industrial countries. Other interesting questions are not investigated in this paper and left to future research. One possible extension is to examine the above hypothesis by combining the information of France, Japan and UK (and possibly other industrial countries as well, provided that the stationarity assumption for investment share is reasonable). While this could simply be done by testing the joint hypothesis that the coefficients of the error-correction terms for the output equations in all three countries are zero, it may be better to first derive and then test the implications of theoretical growth models allowing for capital and/or labor mobility. It is interesting to see whether the empirical evidence based on testing the implications of growth models with factor mobility is consistent with those examining growth models in which each economy is implicitly assumed to evolve independently of others. Another, and a more ambitious, task would be to reconcile the different methodologies and conclusions used in cross-section studies (such as Barro, 1991; Mankiw et al., 1992; Levine and Renelt, 1992) and time series studies (such as Jones (1995) and this paper) regarding the relationship between investment share and per capita output growth. This investigation would potentially parallel the important work of Bernard and Durlauf (1996) which provides a unified framework for interpreting the diverse definitions, methods and results of cross-section and time series tests of the convergence hypothesis.

Acknowledgment

I am grateful to Trevor Breusch, Yin-Wong Cheung, David Harris, John Hutton, Chung-Ming Kuan, W. K. Li, Jin-Lung Lin, Vai-Lam Mui, Nilss Olekalns, Peter

---

18 The conclusions of these two papers using time series analysis are not necessarily inconsistent with the evidence of Kocherlakota and Yi (1996) – another time series study which, however, does not examine the relationship between investment share and output growth – even though they have found some evidence in favor of endogenous growth models. They examine US data starting from 1910s for taxes, tariffs, money growth rate, government equipment and structural capital (both military and non-military) and find robust evidence supporting endogenous growth models based on government non-military structural capital but not the other six variables.
Appendix A

A.1. Proof of Proposition 1

Rewrite the reduced-form vector error-correction model (9) in matrix form as

\[
P_R(L)\Delta x_t - x^R z_{t-1} \equiv \begin{bmatrix} \Pi_{11}^R(L) & \Pi_{12}^R(L) \\ \Pi_{21}^R(L) & \Pi_{22}^R(L) \end{bmatrix} \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} - \begin{bmatrix} z_{1t}^R \\ z_{2t}^R \end{bmatrix} z_{t-1} \\
= \text{constant} + \epsilon_t^R,
\]

(A.1)

where

\[
z_t = x_{2t} - x_{1t},
\]

(A.2)

\[P_R(L) = \sum_{j=0}^{p} \Pi_j^R L^j\]

such that \(\Pi_j^R (0 \leq j \leq p)\) is a \(2 \times 2\) matrix of coefficients, and the coefficients of \(P_R(L)\) are related to \(\pi_{j,11}^R, \pi_{j,12}^R, \pi_{j,21}^R,\) and \(\pi_{j,22}^R\) of (9) in a straightforward way. Note that the leading parameter matrix \(P_0^R\) is an identity matrix.

Similarly, rewrite the structural vector error-correction model (10) as

\[
P(L)\Delta x_t - x z_{t-1} \equiv \begin{bmatrix} \Pi_{11}(L) & \Pi_{12}(L) \\ \Pi_{21}(L) & \Pi_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} - \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} z_{t-1} \\
= \text{constant} + \epsilon_t,
\]

(A.3)

where \(P(L) = \sum_{j=0}^{p} \Pi_j L^j\), and the coefficients of \(P(L)\) are related to \(\pi_{j,11}, \pi_{j,12}, \pi_{j,21},\) and \(\pi_{j,22}\) of (10) in a straightforward way.

Three other structural representations are particularly useful for subsequent analysis. They are the triangular VAR representation:

\[
D(L)\begin{bmatrix} \Delta x_{1t} \\ z_t \end{bmatrix} \equiv \begin{bmatrix} D_{11}(L) & D_{12}(L) \\ D_{21}(L) & D_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta x_{1t} \\ z_t \end{bmatrix} = \text{constant} + \epsilon_t,
\]

(A.4)

Some of the derivations of the structural relationships make use of the idea in Engle and Granger (1987), Johansen (1991) and Phillips (1991), all of which focus on a reduced-form cointegrated system.
the triangular VMA representation:
\[
\begin{bmatrix}
\Delta x_{1t} \\
z_t
\end{bmatrix} = \text{constant} + C(L)e_t \equiv \text{constant} + \begin{bmatrix}
C_{11}(L) & C_{12}(L) \\
C_{21}(L) & C_{22}(L)
\end{bmatrix} e_t,
\]
(A.5)
and the first-difference VMA representation (8) in the main text.\(^{20}\)

From the definition in (A.2), it is easy to obtain
\[
\Delta x_{2t} = \Delta z_t + \Delta x_{1t}.
\]
Using this relationship, it can be shown that (A.3) is equivalent to the triangular VAR representation (A.4) such that the various coefficients in (A.3) and (A.4) are related by
\[
C(L) = D^{-1}(L).
\]
(A.8)
Premultiplying (A.5) by \(F(L)\) gives rise to the first-difference VMA representation (8) such that the coefficients in (A.5) and (8) are related by\(^{22}\)
\[
M(L) = F(L)C(L)
\]
\[
\begin{bmatrix}
C_{11}(L) & C_{12}(L) \\
C_{11}(L) + (1 - L)C_{21}(L) & C_{12}(L) + (1 - L)C_{22}(L)
\end{bmatrix}.
\]
(A.9)
\(^{20}\)If the variables are \(I(1)\) but not cointegrated (as in Lastrapes and Selgin, 1994; Quah and Vahey, 1995), the first-difference VMA representation can easily be obtained by inverting the first-difference VAR representation. However, a first-difference VAR representation does not exist when the variables are cointegrated, since at least one element of \(a\) in (A.3) is non-zero. The crucial step in the subsequent derivations is to transform the vector error-correction representation (A.3), which corresponds to the presence of cointegration, into a triangular VAR representation (see Phillips, 1991). Since an inverse exists for the long-run multiplier matrix of the triangular VAR representation, the triangular VMA representation can be obtained by inversion. The first-difference VMA representation is then recovered from the triangular VMA representation.

\(^{21}\)A necessary condition for the non-singularity of \(D(1)\) is that at least one of \(a_1\) or \(a_2\) in (A.3) is non-zero, or equivalently, the two variables are cointegrated.

\(^{22}\)Equivalently, taking the first difference of the second equation of (A.5) and combining it with the first equation gives rise to the second equation of (8).
The above relationships among various structural representations are useful in developing expressions for subsequent analysis. Specifically, (A.6) and (A.7) lead to

\[
\begin{bmatrix}
D_{11}(1) & D_{12}(1) \\
D_{21}(1) & D_{22}(1)
\end{bmatrix} = D(1) = \Pi(1)F(1) - \begin{bmatrix} 0 & \alpha_1 \\ 0 & \alpha_2 \end{bmatrix} = \begin{bmatrix} \Pi_{11}(1) + \Pi_{12}(1) & -\alpha_1 \\ \Pi_{21}(1) + \Pi_{22}(1) & -\alpha_2 \end{bmatrix},
\]

(A.6a)

and (A.8) leads to

\[
\begin{bmatrix}
C_{11}(1) & C_{12}(1) \\
C_{21}(1) & C_{22}(1)
\end{bmatrix} = C(1) = D^{-1}(1) = \frac{1}{|D(1)|} \begin{bmatrix} D_{22}(1) & -D_{12}(1) \\ -D_{21}(1) & D_{11}(1) \end{bmatrix},
\]

(A.8a)

where \(|D(1)|\) is the determinant of \(D(1)\). Eq. (A.9) leads to

\[
\begin{bmatrix}
M_{11}(1) & M_{12}(1) \\
M_{21}(1) & M_{22}(1)
\end{bmatrix} = M(1) = \begin{bmatrix} C_{11}(1) & C_{12}(1) \\ C_{11}(1) & C_{12}(1) \end{bmatrix}.
\]

(A.9a)

Moreover, the leading parameter matrices of various representations are related by (A.6), (A.8) and (A.9) in the following way: \(D_0 = \Pi_0F(0), \ (D_0)^{-1} = C_0, \) and \(M_0 = F(0)C_0, \) where

\[
F(0) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.
\]

Simple matrix manipulations then give

\[
(M_0)^{-1} = \Pi_0.
\]

(A.10)

From (A.6a), (A.8a) and (A.9a), it can be shown that

\[
M_{11}(1) = M_{21}(1) = C_{11}(1) = \frac{D_{22}(1)}{|D(1)|} = \frac{-\alpha_2}{|D(1)|},
\]

(A.11)

and

\[
M_{12}(1) = M_{22}(1) = C_{12}(1) = \frac{-D_{12}(1)}{|D(1)|} = \frac{\alpha_1}{|D(1)|}.
\]

(A.12)

Note that \(|D(1)|\) is non-zero when \(D(1)\) is non-singular.

The structural and reduced-form models are closely related; see, for example, Cochrane (1994). In particular, premultiplying the structural vector error-correction model (A.3) by \((\Pi_0)^{-1}\) and comparing it with its reduced-form counterpart (A.1) gives

\[
\varepsilon_t^R = (\Pi_0)^{-1}\varepsilon_t = M_0\varepsilon_t,
\]

(A.13)

\[
\Pi(L) = \Pi_0\Pi^R(L) = (M_0)^{-1}\Pi_0\Pi^R(L)
\]

(A.14)
and
\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix} = \alpha = (M_0)^{-1}\alpha^R = \frac{1}{|M_0|} \begin{bmatrix}
m_{0,22} & -m_{0,12} \\
-m_{0,21} & m_{0,11}
\end{bmatrix} \begin{bmatrix}
\alpha_1^R \\
\alpha_2^R
\end{bmatrix}
\]
\[
= \frac{1}{|M_0|} \begin{bmatrix}
m_{0,22}\alpha_1^R - m_{0,12}\alpha_2^R \\
-m_{0,21}\alpha_1^R + m_{0,11}\alpha_2^R
\end{bmatrix}.
\]  

(A.15)

Note that (A.10) has been used to obtain (A.13)–(A.15), and that (A.15) is equivalent to (11) in the main text.

Under the identifying assumption \(m_{0,12} = 0\), (A.12) becomes
\[
M_{12}(1) = \frac{m_{0,22}\alpha_1^R}{|D(1)||M_0|}.
\]

This proves Proposition 1.

A.2. Proof of Proposition 2

Eqs. (A.10), (A.13) and the normalization of the variances of the structural disturbances imply that the reduced-form variance–covariance matrix, \(E(\epsilon_i^R, \epsilon_i'^R)\), is related to \(M_0\) according to
\[
E(\epsilon_i^R, \epsilon_i'^R) = M_0 E(\epsilon_i, \epsilon_i') M'_0 = M_0 M'_0.
\]  

(A.16)

To express \(|D(1)|\) in (A.11) and (A.12) in terms of estimated parameters, note from (A.14) that
\[
\Pi(1) = (M_0)^{-1}\Pi^R(1) = \frac{1}{|M_0|} \begin{bmatrix}
m_{0,22} & -m_{0,12} \\
-m_{0,21} & m_{0,11}
\end{bmatrix} \begin{bmatrix}
\Pi_{11}^R(1) & \Pi_{12}^R(1) \\
\Pi_{21}^R(1) & \Pi_{22}^R(1)
\end{bmatrix}
\]
\[
= \frac{1}{|M_0|} \begin{bmatrix}
m_{0,22}\Pi_{11}^R(1) - m_{0,12}\Pi_{12}^R(1) & m_{0,22}\Pi_{12}^R(1) - m_{0,12}\Pi_{22}^R(1) \\
-m_{0,21}\Pi_{11}^R(1) + m_{0,11}\Pi_{21}^R(1) & -m_{0,21}\Pi_{12}^R(1) + m_{0,11}\Pi_{22}^R(1)
\end{bmatrix}.
\]  

(A.17)

Substituting (A.17) into (A.6a) gives
\[
D(1) = \begin{bmatrix}
\frac{m_{0,22}[\Pi_{11}^R(1) + \Pi_{12}^R(1)] - m_{0,12}[\Pi_{21}^R(1) + \Pi_{22}^R(1)]}{|M_0|} & -\alpha_1 \\
-\frac{m_{0,21}[\Pi_{11}^R(1) + \Pi_{12}^R(1)] + m_{0,11}[\Pi_{21}^R(1) + \Pi_{22}^R(1)]}{|M_0|} & -\alpha_2
\end{bmatrix}.
\]
Thus,

\[
|D(1)| = \frac{[\Pi^R_{21}(1) + \Pi^R_{22}(1)]m_{0,12}x_1 + m_{0,12}x_2] - [\Pi^R_{11}(1) + \Pi^R_{12}(1)]m_{0,21}x_1 + m_{0,22}x_2]}{|M_0|}
\]

\[
= \frac{[\Pi^R_{21}(1) + \Pi^R_{22}(1)]x_1^R - [\Pi^R_{11}(1) + \Pi^R_{12}(1)]x_2^R}{|M_0|},
\]

(A.18)

where (11) has been used in the last step.

Substituting (A.15) and (A.18) into (A.11) and (A.12), respectively, gives

\[
M_{11}(1) = \frac{m_{0,21}x_1^R - m_{0,11}x_2^R}{[\Pi^R_{21}(1) + \Pi^R_{22}(1)]x_1^R - [\Pi^R_{11}(1) + \Pi^R_{12}(1)]x_2^R}
\]

(A.11a)

and

\[
M_{12}(1) = \frac{m_{0,22}x_1^R - m_{0,12}x_2^R}{[\Pi^R_{21}(1) + \Pi^R_{22}(1)]x_1^R - [\Pi^R_{11}(1) + \Pi^R_{12}(1)]x_2^R}.
\]

(A.12a)

Under the identifying assumption \(m_{0,12} = 0\), (A.16) gives

\[
\Omega^R = \begin{bmatrix}
\Omega^R_{11} & \Omega^R_{12} \\
\Omega^R_{21} & \Omega^R_{22}
\end{bmatrix} = E(c_i^R; c_i^R') = M_0 M_0'
\]

\[
= \begin{bmatrix}
m_{0,11} & 0 \\
m_{0,21} & m_{0,22}
\end{bmatrix}
\begin{bmatrix}
m_{0,11} & m_{0,21} \\
0 & m_{0,22}
\end{bmatrix} = \begin{bmatrix}
(m_{0,11})^2 & m_{0,11}m_{0,21} \\
m_{0,11}m_{0,21} & (m_{0,21})^2 + (m_{0,22})^2
\end{bmatrix}.
\]

(A.19)

Thus, Cholesky factorization of \(\Omega^R\) gives \(m_{0,11}, m_{0,21}\) and \(m_{0,22}\). Specifically,

\[
m_{0,11} = \sqrt{\Omega^R_{11}},
\]

(A.19a)

\[
m_{0,21} = \frac{\Omega^R_{12}}{\sqrt{\Omega^R_{11}}}
\]

(A.19b)

and

\[
m_{0,22} = \sqrt{\Omega^R_{22} - \left(\frac{\Omega^R_{12}}{\Omega^R_{11}}\right)^2}.
\]

(A.19c)

Finally, it can be seen from (A.1) that

\[
\Pi^R_0 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}.
\]

Thus,

\[
\Pi^R_{11}(1) = 1 - \sum_{j=1}^{p} \pi^R_{j,11},
\]

(A.1a)
\[ \Pi_{12}^R(1) = - \sum_{j=1}^{p} \pi_{j,12}^R, \]  
(A.1b)

\[ \Pi_{21}^R(1) = - \sum_{j=1}^{p} \pi_{j,21}^R, \]  
(A.1c)

and

\[ \Pi_{22}^R(1) = 1 - \sum_{j=1}^{p} \pi_{j,22}^R. \]  
(A.1d)

Substituting (A.19c), (A.1a)–(A.1d), and the identifying assumption \( m_{0,12} = 0 \) into (A.12a) gives

\[ M_{12}(1) = \frac{\left( \sqrt{\Omega_{22}^R - (\Omega_{12}^R)^2 / \Omega_{11}^R} \right) z_{1}^R}{(1 - \sum_{j=1}^{p} \pi_{j,21}^R - \sum_{j=1}^{p} \pi_{j,22}^R) z_{1} - (1 - \sum_{j=1}^{p} \pi_{j,11}^R - \sum_{j=1}^{p} \pi_{j,12}^R) z_{2}^R}. \]

Replacing the various parameters in the right-hand side of this equation by their estimates, one obtains Eq. (12).

References


