Trade, Capital Redistribution and Firm Structure

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Abstract

A model of heterogeneous firms with multiple products is used to study how trade liberalization affects firm choices through both product and factor markets. We show that intensified competition in the product market may reduce factor prices, and that substitution between factors may cause labor productivity to drop. In response to trade liberalization, low efficiency firms always reduce their product scope, but high efficiency firms may expand their scope.

Keywords: firm heterogeneity, trade liberalization, multiproduct, multifactor, firm structure, scale, scope, mergers and acquisitions

JEL Code: F12, F13, F15, L11, L25

1 Introduction

This study adapts the workhorse firm heterogeneity model of Melitz and Ottaviano (2008) in two major ways: Each firm can produce multiple products, and factor prices are endogenized so that trade liberalization affects firm choices through both the product market and the factor market. It is found that, after trade liberalization, high efficiency firms may increase product scope, and factor prices may drop.

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The study of heterogeneous firms started with single product firms, but attention has recently turned to multiproduct firms. According to Bernard, Jensen and Schott (2005) and Bernard, Redding and Schott (2008), only 41 percent of U.S. manufacturing firms produce more than a single product, but these firms account for 91 percent of U.S. manufacturing output and 95 percent of U.S. exports. Existing researches have taken two approaches in modelling multiproduct firms. The first is to extend heterogeneity to products within a firm (Bernard, Redding and Schott, 2009; Eckel and Neary, 2010; Mayer, Melitz and Ottaviano, 2009). Trade liberalization bids up the factor price, forcing each firm to drop its marginal products. As a result, all firms reduce scope. The second approach assumes symmetric products within firms. To explain why the market is not taken over by the most efficient firm, Nocke and Yeaple (2006) imposed an exogenous tradeoff between product scope and plant-level productivity. They predicted that high efficiency firms trim product lines after trade liberalization.

We model multiproduct firms differently and reach different conclusions. Like Nocke and Yeaple (2006), we assume within-firm symmetry, but constrain product scope in a different way: Managing varieties within each firm is increasingly costly. Trade liberalization will then lead high efficiency firms to expand their scope and low efficiency firms to shrink the scope.

In all three approaches, trade liberalization favors firms with lower marginal costs, which in fact is true in all firm heterogeneity models, whether single or multiple products. However, firm scope is affected differently depending on the relation between scope and marginal cost. Nocke and Yeaple (2006) imposed an exogenous tradeoff between the two: A firm choosing more varieties is assumed to have higher marginal cost in producing each variety, implying a negative relation between extensive and intensive margins. As a result, lower efficiency firms, which have lower marginal costs, expand product lines after trade liberalization. By contrast, the other two approaches assume that all firms face the same fixed cost (Bernard, Redding and Schott, 2009) or management cost (the present study) in managing multiple products. A more efficient firm which have lower marginal cost will therefore choose to produce more varieties, leading to a positive relation between extensive and intensive margins. In Bernard, Redding and Schott (2009), products are asymmetric within a firm. Trade liberalization favors not only the more efficient firms within the industry, but also the more efficient products within each firm. The equilibrium spectrum of product varieties within each firm will be reduced, in much the same way as trade liberalization reduces the spectrum of firms within the industry. In our model, by contrast, products are symmetric within firms. Trade liberalization favors more efficient firms by increasing (under some conditions) the profits of each product that these firms produce. As a result, high efficiency firms produce more varieties.

Empirical studies of multiproduct firms have turned up mixed results about product scope. Although Bernard, Redding and Schott’s (2009) theory predicts uniform reduction in scope, their own (2008) empirical investigation of U.S. manufacturing firms between 1987 and 1997 had different findings: High efficiency firms increased scope while low efficiency firms reduced scope, which is consistent with our theory. Baldwin and Gu (2006) found that both exporters and non-exporters in Canada reduced product diversification from 1973 to 1997, but the reduction does not seem to have been related to tariff cuts even though Canada underwent two rounds of trade liberalization during that period. Iacovone and Javorcik (2008) found that Mexican firms developed new products for export as a response to trade liberalization. Goldberg et al (2009)
found that Indian firms did not reduce their product scopes during the 1989-2003 period when profound trade and other reforms took place.\textsuperscript{1} This research therefore contributes to the study of multiproduct firms by showing the possibility of and the conditions for expansions in scope after trade liberalization.

The second extension of our research is to allow trade liberalization to affect firm choices through both product and factor markets, while individual papers of existing studies have focused on only one of them. In his seminal paper, Melitz (2003) demonstrated how, by bidding up the wage rate, trade liberalization forces the least efficient firms to quit. The intensified competition in the product market, however, does not play any role, as a CES preference was used.\textsuperscript{2} This is changed in Melitz and Ottaviano (2008), who used a linear demand function and showed that exit of the least efficient firms was caused by tougher competition in the product market. But in that model, the factor market was dormant because the wage rate was assumed to be fixed.

This research adopts Melitz and Ottaviano’s (2008) linear demand model so the markup is endogenous and variable. At the same time, the price of capital, an input whose supply is fixed at the industry level, is endogenized as firms trade capital before production takes place. Trade liberalization will then disturb both product and capital markets. We found that intensified competition in the product market may reduce capital price, which never happens in models where only one market is at work. Exporting means firms need more capital to produce for the foreign market, but importing intensifies domestic product market competition, reducing the demand for each product and hence the demand for capital. We show that when the products are close substitutes, the second effect may dominate, leading to lower capital prices.

Studying product and factor markets jointly helps clarify the real impacts of trade liberalization. When factor market is the only channel, trade liberalization bids up factor price (Melitz, 2003). When product market is the only channel, trade liberalization reduces the demand for each single product (Melitz and Ottaviano, 2008). When both markets are at work, we demonstrate that factor price increases relative to demand. Since demand is lower, trade liberalization may lower the absolute level of factor prices. The clarification is important for at least two reasons. First, it clarifies what empirical evidence can refute firm heterogeneity theory. In models where trade liberalization works through the factor market, rising factor price is an unambiguous prediction. In fact, it is the driving force behind resource reallocation. The whole theory may then seem in danger if empirical evidence found otherwise. Our model demonstrates that such worries are unnecessary.

Second, conclusions about factor prices would have implications for factor employment within the industry and, in the case of multiple factors, substitutions between factors. To demonstrate the last point, we have incorporated into our model two production inputs, labor and capital. While labor is readily available at an exogenous wage rate, industry-specific capital is in fixed supply and the endogenized capital price determines firms choices. We show that, not surprisingly, trade liberalization moves the two inputs to more efficient firms, improving overall productivity. Independent of this resource reallocation effect, however,

\textsuperscript{1}Goldberg et al (2009) attributed the discrepancy between their findings and the predictions of prevailing theories to regulations in India, which prevented the optimal allocation of resources.

\textsuperscript{2}“In the isoelastic case, the demand level has no effect on the cutoff because any shift in demand is offset by entry, in contrast to the case of linear demand.” (Helpman, 2006, p.603)
there is substitution between the two inputs, and labor productivity may rise or drop depending on the ratio
between capital price and wage rate. Such considerations, although straightforward and of secondary con-
cern to the present paper, clarify that there are different kinds of productivity and not all of them will be
improved by trade liberalization.

Capital trading was modeled explicitly in this study. Such trading enables firms to add or delete produc-
tion facilities, and thus can be regarded as a partial merger or acquisition. Mergers and acquisitions (M&As)
constitute a major method of industrial restructuring (UNCTAD, 2000) and are the quickest and least costly
way to respond to external shocks such as trade liberalization. Waves of mergers have been documented
as a consequence of trade liberalization and other such shocks (Mitchell and Mulherin, 1996). Breinlich
(2008) found that the Canada-United States Free Trade Agreement of 1989 increased domestic Canadian
M&A activity by over 70%. Using data on Swedish firms for the period 1980-1996, Greenaway, Gullstrand
and Kneller (2008) have shown that intensified international competition induced M&As. Maksimovic and
Phillips (2001) contended that “industry shocks alter the value of the assets and create incentives for trans-
fers to more productive uses”, and they showed that productive assets tend to move from less efficient to
more efficient firms when an industry undergoes a positive demand shock. Our analysis has now provided a
theoretical explanation for these empirical findings.

We will first present the model and analyze the autarkic equilibrium. The equilibrium after trade liberal-
alization will then be found and compared with the autarky case. All the proofs are collected in Appendix.

2 Autarky

2.1 Production and inputs

Consider a world with two identical countries, between which there initially is no trade due to prohibitively
high trade costs. In each country, consumption consists of a numeraire good and differentiated products
produced by a continuum of firms in an industry. From a uniform distribution on $[0, 1]$, each firm draws its
efficiency, $\varphi$, which is the only parameter that distinguishes firms.

A firm can produce multiple products by incurring a management cost, $mv^2$, where $m > 0$ and $v$ is the
number of varieties that the firm produces. Production of the numeraire good requires two units of labor
for one unit of output. Labor supply to this industry is perfectly elastic, so the wage rate, $w$, is equal to $\frac{1}{2}$.

Producing differentiated products requires both labor and capital as inputs. The production function of a
single variety is

$$q = (\varphi x l)^{\frac{1}{2}},$$

in which $q$ is the output of the variety, $x$ is its capital input and $l$ is its labor input. In the short run, a plant’s
capital is fixed, and thus its variable cost is $\min_x (wl)$ subject to $q = (\varphi x l)^{\frac{1}{2}}$, which gives rise to a labor cost
of $\frac{\phi x}{q^2}$, or

$$c(q|\varphi, x) = \frac{q^2}{2\varphi x}.$$  

Each firm is endowed with one unit of capital. With this endowment and knowing their efficiency levels, firms play a two-stage game. In the first stage (the acquisition stage), firms buy and sell capital in a perfectly competitive acquisition market. If a firm sells all of its capital, it becomes inactive in the product market. An inactive firm still keeps its $\varphi$ and may choose to re-enter the product market later by buying capital in the acquisition market. Entry or exit incurs no extra cost. After the capital trading, every active firm chooses its number of varieties and the allocation of capital to each plant. A firm’s structure therefore consists of its product scope (the number of varieties) and plant-level capital scale (the amount of capital used in each plant). For expositional convenience, scale and scope will be treated as continuous variables. Capital is assumed to be perfectly divisible, and there is no minimum capital requirement for running a plant. In the second stage (the production stage), production is carried out and all varieties compete in a monopolistically competitive product market.3

In this model, capital represents industry-specific physical assets that are needed in production. Unlike financial assets, after the initial trading round, capital must be acquired through acquisition or merger rather than from financial institutions. This implies that the supply of capital is perfectly inelastic at the industry level. This is justified if the total amount of physical assets cannot quickly be increased, but the results still hold even if new capital can be generated in response to an increase in the capital price. Assuming a perfectly competitive acquisition market implies that capital is homogeneous and acquisitions can be partial. That is, instead of acquiring a stand-alone target firm, the acquirer buys some productive assets from other firms and uses them along with its own assets. Jovanovic and Rousseau (2002) have argued that transactions in the used-capital market work just like those in a M&A market. Maksimovic and Phillips (2001) report that about half of U.S. M&A transactions are partial acquisitions or divestitures by multi-product conglomerates.

Consider first the product market, then the acquisition market (also referred to as the factor market), and finally the industry equilibrium.

2.2 The product market

Assume $L$ identical consumers in each country. Each consumer has a quasi-linear preference (à la Melitz and Ottaviano, 2008) for the numeraire good and all varieties in the industry:

$$U = Q_0 + \alpha \int_{i \in \Omega} q_i di - \frac{1}{2} \beta \left( \int_{i \in \Omega} q_i di \right)^2 - \frac{1}{2} \gamma \int_{i \in \Omega} q_i^2 di,$$

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3We assume away any cannibalization producing one variety has on the profitability of other varieties. Schwartz and Thompson (1986) and also Baye, Crocker and Ju (1996) have shown that, in an effort to gain market share from competing companies or forestall entry, many companies instruct their divisions to act as independent firms despite cannibalization. If this assumption is relaxed to accommodate cannibalization, our major conclusions still hold. Feenstra and Ma (2008) have demonstrated that cannibalization would not change the major findings of Melitz (2003).
where $\alpha, \beta, \gamma > 0$, $Q_0$ is the consumption of the numeraire good, $\Omega$ is the set of all varieties, and $q_i$ is the consumption of variety $i$. A consumer maximizes his utility subject to a budget constraint. As a result, the market demand for variety $i$ from all $L$ consumers is $p_i = \alpha - \beta \int_{j \in \Omega} q_j dj - \frac{\gamma}{L} q_i$. For a given $\gamma$, when $\beta$ is larger, other varieties’ outputs reduce the demand for variety $i$ by a larger amount, meaning that the substitution between varieties is stronger. Therefore, $\beta$ measures substitutability between varieties: larger $\beta$ means stronger substitution.

In monopolistic competition, the seller of variety $i$ regards itself as a monopolist, and the competition between products is captured only in the vertical intercept of the demand function. In equilibrium, the demand function for variety $i$ is:

$$p_i = A - bq_i, \text{ where } A = \frac{\alpha \gamma + \beta P}{\beta M + \gamma} \text{ and } b = \frac{\gamma}{L}. \quad (1)$$

In this demand function, $p_i$ is the price of variety $i$, $M$ is the measure of $\Omega$, and $P = \int_{i \in \Omega} p_i di$ is the aggregate price of all varieties. The slope $b$ is exogenous, but the intercept $A$ is endogenous, depending on both the degree of product substitution (captured in $\beta$) and the degree of product market competition (captured in the endogenous $P$ and $M$).

Each firm takes $A$ as given when choosing its output. If a firm with efficiency $\varphi$ has amount $x_i$ of capital in its plant to produce variety $i$, it chooses output $q_i$ to maximize its profit from this variety:

$$\max_{q_i \geq 0} \pi_i \equiv (A - bq_i)q_i - \frac{q_i^2}{2\varphi x_i}. \quad (2)$$

The resulting quantity, price and profit for this variety are, respectively,

$$q_i(x_i) = \frac{\varphi x_i A}{2\varphi bx_i + 1}, \quad p_i(x_i) = \frac{(\varphi bx_i + 1)A}{2\varphi bx_i + 1}, \quad \text{and } \pi_i(x_i) = \frac{\varphi x_i A^2}{2(2\varphi bx_i + 1)}. \quad (3)$$

Greater demand (i.e., a larger $A$) leads to more output, a higher price and more profit for each variety. Since a variety’s profit, $\pi_i(x_i)$, is increasing and concave in $x_i$, a firm will always allocate its total capital among its varieties equally. Consequently, the subscript $i$ can be dropped in (3).

### 2.3 The acquisition market

Let $R$ be the market price of capital. If a firm chooses scope $v$ and scale $x$, its capital cost is $(vx - 1)R$, where $vx - 1$ is the firm’s net demand for capital, which can be negative (meaning that the firm is selling...
capital). The firm’s optimization problem in the acquisition market is

$$
\max_{x \geq 0, v \geq 0} \Pi(v, x) \equiv v\pi(x) - (vx - 1)R - mv^2 = v\tilde{\pi}(x) + R - mv^2,
$$

where $\tilde{\pi}(x) \equiv \pi(x) - xR$ is the profit of each single plant, taking into account the capital cost but not the management cost. Given the expression for $\pi(x)$, a plant’s profit is

$$
\tilde{\pi}(x) = \frac{\varphi x A^2}{2(2\varphi bx + 1)} - xR.
$$

It is as if the firm first sells its endowment of unit capital and then chooses how much capital to buy for each of its plants. Since there is no transaction cost, selling and buying capital is fully reversible.

Given the above decomposition of the profit function, a firm’s optimization problem can be solved in two steps: The optimal scale is $x^* = \arg\max_x \tilde{\pi}(x)$, which is independent of the choice of $v$, and the optimal scope is then $v^* = \arg\max_v \Pi(v, x^*)$.

### 2.3.1 Plant-level capital scale

Define

$$
y \equiv \frac{A}{\sqrt{2R}}
$$

The variable $y$ reflects the value of capital (in the product market) relative to the cost of capital (in the factor market), as $\pi(x)$ is proportional to $\frac{A^2}{x}$. Equivalently, $\frac{1}{y^2}$ represents capital price relative to demand.

The first-order condition $\frac{\partial \tilde{\pi}}{\partial x} = 0$ leads to the optimal scale for each plant (the asterisk is dropped for simplicity of notation):

$$
x = \begin{cases} 
0 & \text{for } \varphi \leq \varphi^0, \\
\frac{y\sqrt{\varphi - 1}}{2\varphi b} & \text{for } \varphi > \varphi^0,
\end{cases}
$$

where $\varphi^0 \equiv \frac{1}{y^2}$. The second-order condition is always satisfied. It will later become clear that $y > 1$ in equilibrium and therefore $\varphi^0 < 1$. Expression (5) says that very inefficient firms will not operate in the product market. When $\varphi$ is small, a plant’s profit from the product market will also be small, and the firm will earn a better payoff by selling all its capital and discontinuing production.\footnote{In many formulations, least efficient firms exit due to fixed production costs (e.g., Melitz, 2003). Without fixed costs, exit happens with a linear demand function (Melitz and Ottaviano, 2008), as the constant marginal costs that some firms draw turn out to be larger than the intercept of the demand. In our model, demand is linear as in Melitz and Ottaviano (2008) and there is no fixed production cost. A firm’s marginal cost can be arbitrary small (when its output is close to zero) even if it is very inefficient. It will always generate some profit, however small, in the product market. A firm exits only because it can earn a better payoff by selling.}

Note that the cutoff point
for exit, $\varphi^0$, depends solely on $y$. When products are more profitable relative to the capital price, even a low efficiency firm will choose to operate, so more firms will be active. For an active plant, the scale increases with $y$: When products become more profitable relative to capital, each plant will operate on a larger scale. The reason is straightforward: $\frac{4\varphi}{2}$ represents the marginal benefit of capital while $R$ represents the marginal cost of acquiring capital.

Interestingly, more efficient firms do not necessarily have larger scale:

$$\frac{\partial x}{\partial \varphi} = \frac{2 - y\sqrt{\varphi}}{4\varphi^2 b} \begin{cases} > 0 & \text{for } \varphi \in (\varphi^0, 4\varphi^0), \\ < 0 & \text{for } \varphi > 4\varphi^0. \end{cases}$$

Plant scale increases with a firm’s efficiency when the efficiency is low, but if $4\varphi^0 < 1$ (or equivalently, $y > 2$), the scale decreases with efficiency when the efficiency is sufficiently high, generating an inverse U-shape. The reason is that when $\varphi$ is small, each variety is sold at a high price, where demand is highly elastic. Increasing $x$ will bring a large benefit. But when $\varphi$ is large, each variety is already sold at a low price, where the demand is barely elastic. Increasing $x$ will not bring much benefit. Note that the inverse U relationship does not exist with CES preferences.

### 2.3.2 Product scope

Given the optimal scale, a firm’s profit from each variety (after paying for capital) is $	ilde{\pi} = \frac{R(y\sqrt{\varphi} - 1)^2}{2\varphi b}$ for $\varphi > \varphi^0$, which is strictly increasing in $\varphi$. Given $	ilde{\pi}$, the first-order condition for optimal scope $\frac{\partial \Pi}{\partial v} = \tilde{\pi} - 2mv = 0$ leads to $v = \frac{\tilde{\pi}}{2m}$ or

$$v = \begin{cases} 0 & \text{for } \varphi \leq \varphi^0, \\ \frac{R(y\sqrt{\varphi} - 1)^2}{4\varphi b m} & \text{for } \varphi > \varphi^0. \end{cases}$$

The second-order condition is always satisfied. Notice that

$$\frac{\partial^2 v}{\partial \varphi} > 0 \text{ for all } \varphi > \varphi^0.$$

Thus, more-efficient firms maintain larger scopes. The marginal benefit from adding a variety is $\tilde{\pi}$, while the marginal cost is $2vm$. Since the marginal benefit increases with $\varphi$, the optimal scope should be larger when $\varphi$ is higher.

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its capital in the factor market. The tradeoff between product and factor markets is the driving force here.
For $\varphi > \varphi^0$, the scope can be rewritten as

$$v = \frac{(A\sqrt{\varphi} - \sqrt{2R})^2}{8\varphi bm}.$$  \hfill (7)

The numerator relates a firm’s product scope to the product and acquisition markets, or more precisely to the value of capital in the product market vis-a-vis the cost of capital in the acquisition market. The value of capital is positively related to $\varphi$, while the cost of capital is independent of $\varphi$. Therefore, if there is any change to the two markets due to, say, trade liberalization, firms will respond differently depending on their efficiencies even though they face the same changes in $A$ and $R$. In particular, high efficiency firms are affected mainly by the value of capital and thus by changes in the product market, while low efficiency firms are affected mainly by the cost of capital and thus by changes in the acquisition market.

### 2.3.3 Firm-level capital

A firm’s capital is $xv$. For an active firm,

$$\frac{\partial xv}{\partial \varphi} = \frac{R(y\sqrt{\varphi} - 1)^2}{16\varphi^3 b^2 m}(4 - y\sqrt{\varphi}) \begin{cases} > 0 & \text{for } \varphi < 16\varphi^0, \\ < 0 & \text{for } \varphi > 16\varphi^0. \end{cases}$$

Because scope increases with $\varphi$ while scale is inverse U-shaped, firm capital is also inverse U-shaped with efficiency. Note that the turning point for firm capital (at $16\varphi^0$) is larger than that for plant scale (at $4\varphi^0$). In any case, the important message is that more efficient firms do not necessarily require more capital. Also note that the envelope theorem indicates that a firm’s profit always increases with $\varphi$ even if its capital scale does not.

### 2.3.4 Outputs

With the optimal choice of $x$, a plant’s output is

$$q = \frac{A}{2b} \left( 1 - \frac{1}{y\sqrt{\varphi}} \right),$$

while a firm’s total output is

$$qv = \frac{A^3}{16b^2 m} \left( 1 - \frac{1}{y\sqrt{\varphi}} \right)^3.$$
Both outputs increase with efficiency.

### 2.3.5 Productivity

Three measures of productivity will be calculated: labor productivity, total factor productivity (or TFP) of labor and capital, and overall productivity that takes into consideration management costs. Labor productivity is the ratio between output and labor input. At the plant level, labor input is \( l = \frac{q^2}{\varphi x} \). Given the optimal choices of \( x \) and \( q \), the plant-level labor productivity is:

\[
\lambda^p \equiv \frac{q}{l} = \sqrt{\frac{\varphi}{2R}}.
\]

Since all plants of a firm are identical, the firm-level labor productivity is the same. Capital price \( R \) matters for labor productivity because it affects the relative price between labor and capital and thus the optimal combination of the two inputs in production.

In this model, both labor and capital are used in production, so we need a measure that relates outputs to the two inputs. Given the production function at the plant level, \( q = \sqrt{\varphi xl} \), define total factor productivity as

\[
\omega^p \equiv \frac{q}{\sqrt{xl}} = \sqrt{\varphi},
\]

which is the same at both the plant and firm levels. Unlike labor productivity, TFP does not depend on market conditions such as input prices, substitutability between products, or degree of competition. To the extent that firm-level productivity should reflect technologies only and should be independent of changes in market conditions, TFP is a more useful measure than labor productivity. Although fixed at the firm level, TFP at the industry level will change as labor and capital is redistributed among firms, thus providing a measure of the impact of trade liberalization.

The two productivity measures defined above ignore the management costs of maintaining multiple products.\(^5\) To take that into consideration, define overall productivity as the reciprocal of average cost. Plant-level cost consists of labor and capital costs, and the overall productivity is

\[
\mu^p \equiv \frac{q}{wl + xR} = \sqrt{\frac{\varphi}{2R}},
\]

which equals the plant-level labor productivity.\(^6\) At the firm level, management cost is included and the

\(^5\)We view the management cost as a labor cost, but this labor is skilled labor, unlike the (unskilled) labor that is an input to production. More specifically, \( v^2 \) can be viewed as the physical units of skilled labor needed to manage \( v \) varieties, and \( m \) is the exogenous wage rate for skilled labor. Throughout this discussion, labor refers to unskilled labor.

\(^6\)This is not an coincidence. Given the Cobb-Douglas production function, the expenditure on the two inputs will be equalized when labor and capital are chosen optimally. Then \( wl + xR = 2wl = l \) given that \( w = \frac{1}{2} \).
The overall productivity is
\[ \mu_\varphi = \frac{vq}{vwl + vxR + mv^2} = \frac{4}{A \left( 1 + \frac{3}{y\sqrt{\varphi}} \right)}. \]

The overall productivity increases with \( \varphi \) at both the plant and firm levels, and the difference between the two reflects the impact of management cost.

To summarize:

**Lemma 1.** In equilibrium, capital scale at both the plant and firm levels increases with a firm’s efficiency \( \varphi \) when \( \varphi \) is small, but may decrease with \( \varphi \) when \( \varphi \) is large. Scope, output and productivity all increase with \( \varphi \).

Since a firm’s scope and each plant’s output are both increasing in \( \varphi \), the two are positively correlated. That is, intensive margins (i.e., each variety’s output) and extensive margins (i.e., the number of varieties) are positively correlated, confirming what Bernard, Redding and Schott (2009) have found.

### 2.3.6 Capital price

Now to determine the equilibrium capital price \( R \) for a given \( A \). Market clearing in the acquisition market requires total capital supply to equal capital demand at the industry level. Since each firm is initially endowed with one unit of capital, the total capital supply in the industry is \( \int_0^1 d\varphi = 1 \). The total capital demand is

\[ K \equiv \int_0^1 xvd\varphi = \frac{R}{8b^2m}\rho, \quad \text{where} \quad \rho = 2y^3 + 3y^2(1 - 2 \ln y) - 6y + 1. \]

Given \( A \), it can be proved (see Appendix I) that there exists a unique equilibrium capital price \( R \) satisfying the equilibrium condition \( K(R) = 1 \), from which \( R \) can be derived as a function of \( y \):

\[ R(y) = \frac{8b^2m}{\rho}. \] (8)

Note that this expression is not a reduced form solution for \( R \), as \( y \) is defined on \( R \).

### 2.4 Industry equilibrium

The measure of varieties is

\[ M \equiv \int_{\varphi^0}^1 vd\varphi = \frac{R}{4bm}\psi, \quad \text{where} \quad \psi = y^2 - 4y + 3 + 2 \ln y. \] (9)
Given the choice of \( x \), each single plant’s price and quantity are:

\[
p = \frac{A}{2} \left( 1 + \frac{1}{y\sqrt{\varphi}} \right) \quad \text{and} \quad q = \frac{A}{2b} \left( 1 - \frac{1}{y\sqrt{\varphi}} \right)
\]

for all \( \varphi > \varphi^0 \). Therefore, the aggregate price is

\[
P \equiv \int_{\varphi_0}^{1} vpd\varphi = \frac{AR \phi}{8bm y}, \quad \text{where} \quad \phi = y^3 - 2y^2 - 2 + 3y - 2y \ln y.
\]

By the definition of \( A \) in (1): \( A = \frac{\alpha \gamma + \beta P}{\beta M + \gamma} \), and using (9) and (10) we have

\[
A = \frac{\alpha}{1 + \beta \frac{\eta y}{\varphi y}},
\]

where \( \eta = y^3 - 6y^2 + 2 + 3y + 6y \ln y \). The three equilibrium values \( A, R \), and \( y \) are then jointly determined by equations (4), (8) and (11). Use (8) and (11) in (4) to yield the following equation expressed in terms of \( y \) only:

\[
Z(y) = 0, \quad \text{where} \quad Z(y) \equiv \frac{\rho^3}{\left( y\rho + \frac{\beta y}{\varphi y} \right)^2} - \frac{16b^2m \alpha^2}{\alpha^2}.
\]

It can be shown that \( Z'(y) > 0, Z(1) < 0 \), and \( Z(y) > 0 \) when \( y \) is sufficiently large. Therefore, there exists a unique \( y > 1 \) satisfying (12). Once \( y \) is determined, \( R \) is determined from (8) and \( A \) is determined from (4) or (11). As a result, a unique equilibrium exists in autarky.

3 Trade Liberalization: The Impacts of Imports and Exports

Now suppose that trade liberalization reduces all trade costs (fixed and variable) to zero.\(^7\) Such a trade liberalization gives producers in each country both the opportunity to export and the challenge of intensified competition. We are interested in how firms respond to trade liberalization by adjusting their capital structure, and how productivity changes as a result of resource redistribution. Cross-border M&A is excluded.\(^8\)

\(^7\)If trade costs are positive but small enough to allow for trade, the qualitative results will not change.

\(^8\)Because the two countries are symmetrical, cross-border trading of capital will not happen even if it is allowed. Cross-border M&A will matter if the two countries are asymmetric or if the trading in the product market is one directional.
3.1 Equilibrium after trade liberalization

Since the two countries are symmetrical, there must be a symmetric equilibrium. Consider one of the countries and define

\[ y_t = \frac{A_t}{\sqrt{2R_t}} \text{ and } \varphi_t^0 = \frac{1}{y_t^2}, \]

where the subscript \( t \) stands for trade. Variables in autarky are denoted by subscript \( a \). In what follows, we will use \( f_i \) to denote \( f(y_i) \) for any given function \( f(y) \), where \( i = a, t \).

Given \( A_t \) in each country, a firm chooses the quantity of each variety, \( q_t \), which is allocated equally to the domestic and export markets to maximize the variety’s total profit from the two markets:

\[
\max_{q_t \geq 0} \pi_t = \frac{2}{2} \left( A_t - \frac{bq_t}{2} \right) \frac{q_t^2}{2} \varphi x_t
\]

\[
= \left( A_t - \frac{b}{2}q_t \right) q_t - \frac{q_t^2}{2} \varphi x_t. \tag{13}
\]

Problem (14) is identical to that in the autarky case (2) except that the demand slope \( b \) is replaced by \( \frac{b}{2} \). Therefore, if \( p(A, \varphi, x, b) \), \( q(A, \varphi, x, b) \) and \( \pi(A, \varphi, x, b) \) are solutions in autarky, \( p_t = p(A, \varphi, x, \frac{b}{2}) \), \( q_t = q(A, \varphi, x, \frac{b}{2}) \) and \( \pi_t = \pi(A, \varphi, x, \frac{b}{2}) \) will be the corresponding solutions in trade liberalization. Consequently, if \( y_a = y(b) \), \( A_a = A(b) \) and \( R_a = R(b) \) are the equilibria in autarky, the trade equilibria will be \( y_t = y \left( \frac{b}{2} \right) \), \( A_t = A \left( \frac{b}{2} \right) \), and \( R_t = R \left( \frac{b}{2} \right) \).

3.2 Equilibrium comparison

The following two propositions summarize the major impacts of trade liberalization.

**Proposition 1.** Trade liberalization brings the following changes to the product and acquisition markets:

1. **Product market:** the demand in each country drops: \( A_t \in (\frac{A_a}{2}, A_a) \).
2. **Acquisition market:** capital price may increase or decrease. Capital price increases when \( \frac{\beta}{\alpha} \) is small, and decreases when \( \frac{\beta}{\alpha} \) is large while \( \frac{2m}{\alpha \gamma} \) is small.
3. **Capital price always increases relative to demand** \( (y_t < y_a) \).

**Proof.** See Appendix.

After trade liberalization, more products are sold in each country, so the demand for each variety, represented by \( A \), drops, meaning that product market competition is intensified. It is a little surprising that capital price is not always bid up by trade liberalization as predicted by all existing models in which factor price is endogenized. The secret lies in the interaction between product and factor market competition.

Recall that the demand for each variety in autarky is \( p_i = A - bq_i \), where \( A = \frac{\alpha \gamma + \beta P}{\beta M + \gamma} \) and \( b = \frac{\gamma}{\alpha} \). Trade liberalization brings two changes to product demand: \( A \) drops, meaning that competition in the product
market is intensified; and $b$ is halved, meaning that the market size for each product is doubled. If $\beta$ is very small, for example, imagine $\beta = 0$ (each variety is a monopoly), then $A = \alpha$, and trade liberalization will not reduce $A$. In that case, every firm produces for a larger market with no more competition. The extra production raises demand for capital, bidding up the capital price. Conversely, when $\beta$ is large, products are close substitutes. Because competition between products is strong, trade liberalization will intensify competition greatly, leading to a much smaller $A$. This may dominate the double-market-size effect and lead many firms to sell capital, in which case the capital price declines.

Note that for the capital price to drop, in addition to a large $\beta$, another condition is a small $\frac{\gamma^2 m}{L\alpha^2}$, which implies that $y$ must be small both before and after trade liberalization. That is, capital must be scarce. Also note that, in this model, the only use of capital is for production, so the capital price relative to product demand can be a measure of capital scarcity. On that measure, the effect of trade liberalization is unambiguous: capital becomes more expensive relative to product demand as $y = \frac{4}{\sqrt{2mL}}$ decreases.

The finding that factor prices may drop in response to trade liberalization is a surprise. In all previous studies in which factor price was not fixed (Melitz, 2003; Bernard, Redding and Schott, 2009), trade liberalization was invariably found to raise factor prices. The major reason is that, all existing models in which factor price is endogenous happen to use CES preference, which leads to a constant and exogenous markup. Free entry (i.e., repeated drawing of efficiencies) then means that any disturbance to the market demand was exactly offset by entry. In this analysis, by contrast, demand is linear so trade liberalization will intensify competition in the product market, which is a necessary condition for factor price to decrease. Note that even when capital price drops, the least efficient firms still exit, which was impossible in previous models.

The change in factor prices will have implications for factor employment and factor substitution. If capital price is lower, firms will substitute capital for labor, so the industry will employ less labor. This in turn may lead to unemployment in a more general economy with multiple industries. The reverse is true if capital price is higher: the industry will employ more labor, which may bid up wage rate in a general equilibrium framework.

Proposition 1 focuses on the two markets through which trade liberalization exerts its impacts. The next proposition summarizes how firms adjust their choices in the face of trade liberalization.

**Proposition 2.** After trade liberalization, the following changes happen:

1. Capital redistribution: capital moves from low to high efficiency firms; the least efficient firms exit ($\phi_t^0 > \phi_a^0$);
2. Firm structure: low efficiency firms reduce scope and scale, while high efficiency firms expand scope and/or scale;
3. Aggregate scope: the total number of varieties consumed in each country increases ($2M_t > M_a$);
4. Outputs: low efficiency firms reduce plant and firm outputs, while high efficiency firms increase these outputs; industry output increases.

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9Recall that $y_a$ is solved from equation (12) and $y_t$ is solved from the same equation with $b$ being replaced with $\frac{b}{2}$. In both cases, $y$ is increasing in $\frac{\gamma^2 m}{L\alpha^2}$. 

14
Some of the changes are illustrated in Figure 1, where the solid lines represent autarky and the dotted lines represent the situation after liberalization. Plant capital scale is affected by trade liberalization through two effects. First, because product sales in each country become less profitable relative to capital prices, each plant will reduce its scale (recall that scale increases with \( y \)). Second, because each variety is sold in both countries, plant scale will be doubled. Low efficiency firms have small scales to begin with; doubling the scale will not add much. The first effect dominates and they reduce their scales after trade liberalization. Firms with very low efficiency will sell their capital and exit. For high efficiency firms, the second effect dominates, so they expand their scale. Depending on the parameters, it is possible that all firms reduce scale, but if a firm expands its scale, all firms with higher efficiency will also expand their scale.

Product scope is affected by trade liberalization in exactly the same way as scale. In terms of capital redistribution, low efficiency firms reduce both scale and scope, so they must sell capital; the least efficient firms sell all of their capital and exit. High efficiency firms, by contrast, expand scale and/or scope, and they buy capital. Therefore, inputs and output redistribute from low efficiency firms to high efficiency ones, which is not surprising. Interestingly, it is possible that all firms reduce scale, or all firms reduce scope (but not both at the same time). This is because each firm has two choice variables, scale and scope. Given the redistribution of capital across firms, there is still room for high efficiency firms to reduce one variable. Of course, because high efficiency firms increase their capital holding, which is scale times scope, when one variable decreases, the other must increase.\(^{10}\)

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\(^{10}\)When \( \frac{2}{\varphi} \) is larger or \( \frac{\gamma}{\eta m} \) is smaller, the cutoff point of \( \varphi \) for scale moves to the left (i.e., more firms increase scale) while the cutoff point of \( \varphi \) for scope moves to the right (i.e., fewer firms expand scope).
The effect of trade liberalization on the aggregate scope (i.e., the total number of varieties) produced in each country is ambiguous, as low efficiency firms reduce scope while high efficiency firms may or may not expand it. Nevertheless, the aggregate scope consumed in each country, produced by both countries, is higher after trade liberalization. The effect on outputs is not surprising: low efficiency firms reduce plant and firm outputs, while high efficiency firms increase these outputs even when they reduce scale or scope. Total industry output increases.

A prediction of many previous multiproduct models has been that all firms reduce product scope after trade liberalization (Bernard, Redding and Schott, 2009; Eckel and Neary, 2009; Mayer, Melitz and Ottaviano, 2009). In those models, products are heterogeneous within a firm. Trade liberalization intensifies the competition in either the product or the factor market, forcing each firm to drop its marginal products, just as trade liberalization forces marginal firms to exit in single product models. This study assumes products are symmetric within a firm. Trade liberalization favors more efficient firms and may increase the profit of each product that these firms produce. If that is the case, high efficiency firms will expand their scope. Our model helps identify the conditions for this to happen: Scope expansion is more likely when products are less valuable (small $\alpha$) or more differentiated (small $\beta$), the market size is small (small $L$), or managing varieties is more costly (large $m$).

Like the present study, Nocke and Yeaple (2006) have also assumed within-firm symmetry across products, but due to the exogenous tradeoff that they imposed between scope and marginal costs, they reached the opposite conclusion: High efficiency firms reduce scope in response to trade liberalization. Given different predictions, it is then an empirical question as to whether or not high efficiency firms expand their scope and, if so, under what conditions. As mentioned in the Introduction, empirical investigations so far have turned up mixed results.

A major purpose of studying heterogeneous firms is to investigate how trade liberalization changes industry productivity. Three measures of productivity have been defined in the present study.

Lemma 2. After trade liberalization,

1. Total factor productivity improves: $\omega_t > \omega_a$;
2. Labor productivity has the following properties: $\frac{\lambda_t}{\lambda_a} > \sqrt{\frac{R_a}{R_t}}$;
3. Overall productivity has the following properties: $\mu_t = \frac{\mu_t^p}{\mu_t^\phi} > \frac{\mu_a}{R_t} > \frac{\mu_a}{R_a} > \frac{\mu_a^p}{\mu_a^\phi}$; $\mu_t^\phi$ increases with $\phi$.

Proof. See Appendix.

TFP is fixed at $\sqrt{\phi}$ at both the plant and firm level, and therefore does not change after trade liberalization. However, because capital, labor and output redistribute to more efficient firms, the industry’s TFP improves.

Labor productivity at the plant and firm level is inversely related to the capital price—trade liberalization improves labor productivity if and only if capital becomes cheaper. This is easy to understand. When capital is cheaper, firms will substitute capital for labor, so for the same output, the labor input will decrease, improving labor productivity. Note that the proportional change in labor productivity is the same for all
firms. This is because, in this two-stage game, all firms choose the optimal combinations of labor and capital. Because they face the same wage rate and capital price, the combination is the same for all firms. On the industry level, because labor moves to more efficient firms, the improvement of labor productivity is greater than at the individual firm level.

Overall productivity at the plant, firm and industry levels are all inversely related to the capital price. That is simply because capital is part of the cost, so more expensive capital affects overall productivity inversely. At the plant level, the proportional change in overall productivity depends only on the proportional change in the capital price and is therefore the same for all firms. The improvement in firm productivity is greater than that at the plant level ($\frac{\mu_F^{\prime}}{\mu_F} > \frac{\mu_P^{\prime}}{\mu_P}$), indicating rationalization within firms. Because the two measures of overall productivity differ only in the management cost, the rationalization indicates that trade liberalization is conducive to the management of multiple products. Furthermore, the rationalization is greater for more efficient firms ($\frac{\mu_F^{\prime}}{\mu_F}$ increases with $\varphi$). If overall productivity improves for firm $\varphi'$, then it also improves for all firms with $\varphi > \varphi'$. The improvement in overall industry productivity is also greater than that at the plant level.

Of the three measurements, TFP best captures the redistribution of resources among firms, labor productivity reflects substitution between labor and capital, and overall productivity captures the joint effectiveness of managing varieties and the production of each variety. In the latter two cases, although all the measures depend on the capital price, which may rise or drop, rationalization is still evident in comparisons between the measures. Therefore, the general conclusion is that trade liberalization induces more efficient use of resources.

Combining Proposition 1 and Lemma 2, we have

**Corollary:**

1. When $\beta$ is large while $\frac{\gamma^m}{\alpha^m}$ is small, labor productivity and overall productivity improve at all levels, plant, firm and industry.
2. When $\frac{\alpha}{\beta}$ is small, labor productivity drops at the plant and firm levels, but may rise at the industry level. Overall productivity drops at the plant level, but may rise at the firm and industry levels.

As has been discussed, trade liberalization always improves industry TFP, as resources move from low to higher efficiency firms. Other measurements depend on the capital price. The corollary says that when the capital price declines, all efficiency measures improve, indicating a more efficient distribution of resources both across and within firms. Even when the capital price increases, rationalization is evident in the fact that the improvement at the firm and industry levels is greater than at the plant level.

### 4 Concluding Remarks

This paper presents a study of how trade liberalization affects multiproduct firms through both product and factor markets. Consistent with prior work, we have shown that trade liberalization improves industry productivity by redistributing resources toward more efficient firms. However, the joint consideration of
product and factor markets reveals the possibility of lower factor prices caused by trade liberalization, which is impossible if the two markets are considered separately. We have also shown that, contrary to other multiproduct models, trade liberalization may lead high efficiency firms to expand their product scope.

To allow the product market to affect firms’ choices, we have followed Melitz and Ottaviano (2008) in assuming a linear demand function. An alternative is the CES demands assumed by Melitz (2003) and most subsequent studies. A CES demand has the peculiar feature of exogenous and constant markups that are independent of costs, demands or market sizes. This is not only unrealistic (Helpman, 2006), but also leads to the special property that firm choices depend only on the ratio between demand and the factor price, leading to uniform impacts of trade liberalization even when product and factor markets are both considered. With linear demand, firms’ choices will depend on both demand and the factor price rather than the ratio between the two. Trade liberalization causes non-uniform responses from firms because only the product market impact depends on a firm’s efficiency. Admittedly, a linear demand function is highly specific, but a CES function is equally so. Our model demonstrates that some of the fine details of the implications drawn from previous models based on CES functions will not survive under alternative forms of demand.

We have assumed that production requires two inputs, labor and capital, which were treated asymmetrically: Labor is supplied perfectly elastically at a fixed wage rate, while capital is supplied perfectly inelastically at a price that is determined endogenously. Our qualitative results will not change if the wage rate is also endogenized, or if the supply of capital is elastic, as long as capital price is endogenous.

Most prior work has assumed a single production factor such as labor, and hence has focused on labor productivity. Firms in our model use two inputs, and the analysis demonstrates that labor productivity depends crucially on the substitution between labor and capital and hence on the relative factor prices, which in turn depends on the degree of product substitution in consumers’ demand function. Since two inputs are used, it seems that rationalization between the two inputs may provide another channel for trade liberalization to improve productivity. However, this turns out not to be the case in this formulation. The substitution between the two inputs in the production function is assumed to be the same for all firms. Given the optimal choice of capital and labor in the two-stage game and the same wage rate and capital price faced by all firms, the labor/capital ratio is uniform across firms. In fact, a composite input (at the optimal labor/capital ratio) can be used in place of the two inputs to generate the same results. Although trade liberalization changes the capital price and thus the labor/capital ratio, the same ratio applies to all firms, so there is no rationalization between labor and capital. Nevertheless, such a rationalization will be possible if firms differ in the labor/capital substitutability of their production functions. This is a refinement worth further investigation.

We have assumed away the possibility of cross-border capital movement. When the two countries are symmetric, relaxing this assumption will not change any result for bilateral trade liberalization, as capital

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11CES preference has also been used by Bernard, Redding and Schott (2009) and Feenstra and Ma (2008), while the linear demand was used by Baldwin and Gu (2005), Eckel and Neary (2009), and Nocke and Yeaple (2006).

12An exception is the work of Bernard, Redding and Schott (2007), who investigated the impact of trade liberalization in a general equilibrium framework with different relative factor abundance across countries. But they used CES demand functions and therefore trade liberalization affected firms’ choices only through the factor markets.
price will be the same in any case. If the countries are asymmetric, or if trade liberalization is unilateral, cross-border M&A will allow an extra channel through which resources are rationalized. Such a setting will be particularly suitable for investigating trade liberalization between countries that are asymmetric in size, preferences, or labor/capital endowments, and represents a second direction for future work.

Appendix: Proofs

The existence and uniqueness of $R$ for given $A$

Since $R = \frac{A^2}{2y}$, we have $K = \frac{A^2}{16b^2m} \xi(y)$, where $\xi(y) = \frac{\rho(y)}{y^2}$ and $\xi'(y) > 0$. Hence,

$$\frac{\partial K}{\partial R} = \frac{A^2}{16b^2m} \xi'(y) \frac{\partial y}{\partial R} < 0.$$ 

When $R = \frac{A^2}{2y}$, we have $y = 1$, $\xi = 0$ and so $K = 0$. When $R$ is sufficiently close to zero, $y$ is large, which means $\xi$ can be large enough that $K > 1$. Thus, given $A$, there exists a unique equilibrium capital price, $R$, satisfying $K(R) = 1$.

Direct calculation of the equilibrium under trade liberalization

A firm’s quantity choice given $A_t$ in each country is described by the optimization problem (13). The optimal output, price and profit are, respectively,

$$q_t = \frac{\varphi x_t A_t}{\varphi bx_t + 1}, \quad p_t = \frac{(\varphi bx_t + 2) A_t}{2(\varphi bx_t + 1)}, \quad \text{and} \quad \pi_t = \frac{\varphi x_t A_t^2}{2(\varphi bx_t + 1)}.$$ 

The firm chooses its optimal scale for each plant to maximize the plant-level profit $\tilde{\pi}_t = \pi_t - x_t R_t$. The solution is

$$x_t = \begin{cases} 
0, & \text{if } \varphi \leq \varphi^0_t, \\
\frac{y_t \sqrt{\varphi - 1}}{\varphi^0_t}, & \text{if } \varphi > \varphi^0_t.
\end{cases}$$

The firm chooses its scope, $v_t$, to maximize its total profit $v_t \tilde{\pi}_t(x_t) - m v_t^2 + R_t$, yielding the following optimal scope

$$v_t = \begin{cases} 
0, & \text{if } \varphi \leq \varphi^0_t, \\
\frac{R_t(y_t \sqrt{\varphi - 1})^2}{2m \varphi^0_t}, & \text{if } \varphi > \varphi^0_t.
\end{cases}$$

The aggregate demand for capital is $K_t \equiv \int_{\varphi^0_t}^{1} x_t v_t d\varphi = \frac{R_t}{2b^2 m} p_t$. Market clearing in the acquisition
market, $K_t = 1$, leads to

$$R_t(y_t) = \frac{2b^2m}{\rho_t}. \quad (15)$$

The aggregate variety produced in a country is $M_t \equiv \int_0^1 \nu_t d\varphi = \frac{R_t\varphi_0}{2\alpha^2}$, while the aggregate variety consumed in each country is $2M_t$. The aggregate price is $P_t \equiv 2\int_0^1 \nu_t p_t d\varphi = \frac{A_t R_t \varphi_0}{2\alpha\gamma m y_t}$. Plug $P_t$, $M_t$ and $R_t$ into $A_t = \frac{\alpha \gamma + \beta P_t}{2\beta M_t + \gamma}$ to yield

$$A_t(y_t) = \frac{\alpha}{1 + \frac{\beta}{\gamma} \frac{m}{y_t}}. \quad (16)$$

Equations (15) and (16) and the definition of $y_t$ jointly determine the three unknowns $A_t$, $R_t$ and $y_t$ as the equilibrium after trade liberalization.

The direct calculation above is mathematically equivalent to the indirect derivation in the text based on the autarky equilibrium. These two approaches are also economically isomorphic. Without any trade cost, a firm views the two product markets (domestic and export) as identical. At the same time, a consumer is assumed not to treat domestic varieties differently from imported varieties. Compared to autarky, therefore, a firm’s demand under trade liberalization is doubled (population increases from $L$ to $2L$), which is captured by the change of the slope from $b$ to $\frac{b}{2}$ (because $b = \frac{b}{2}$). From the firm’s point of view, for given $A$ and $R$, the change in the slope of the demand curve is the only difference between autarky and liberalized trade, so its optimal scale and scope are doubled after trade. Of course, $A$, $R$ and consequently $y$ will be different from those in autarky.

**Proof of Proposition 1**

We first prove $y_t < y_a$ (conclusion (3)). $y_a$ is solved from equation (12): $Z_a(y_a) \equiv \frac{\rho^2}{(y_{\alpha\rho} + \frac{1}{\gamma} y_{\alpha})^2} - \frac{4b^2m}{\gamma^2} = 0$. $y_t$ is solved from $Z_t(y_t) \equiv \frac{\rho^2}{(y_{\phi} + \frac{1}{\gamma} y_{\phi})^2} - \frac{4b^2m}{\gamma^2} = 0$. Obviously, $Z_t(y) > Z_a(y)$. Then $Z_t(y_a) > Z_a(y_a) = 0 = Z_t(y_t)$. Because $Z_t(\cdot) > 0$, we have $y_a > y_t$.

Product market: $A(y) = \frac{\alpha}{1 + \frac{\beta}{\gamma} \frac{m}{y}}$. It is straightforward to verify that $\frac{\gamma}{\gamma} \frac{y}{y_t}$ decreases in $y$. Therefore, $A_t = A(y_t) < A(y_a) = A_a$. On the other hand, $A = A(b, y) = 4b\sqrt{m \cdot \frac{y}{\gamma}}$, so $A_a = A(b, y_a)$ while $A_t = A(b, y_t) = \frac{A(b, y_a)}{2}$. Then $2A_t > A(b, y_t)$.

Acquisition market: Capital price $R(y) = \frac{A^2}{2y^p} = \frac{\alpha^2}{2(y + \frac{\beta}{\gamma} \frac{m}{y})^2}$. It can be easily verified that $y + \frac{\beta}{\gamma} \frac{m}{y}$ increases with $y$ when $\frac{\beta}{\gamma}$ is small, but decreases with $y$ when $\frac{\beta}{\gamma}$ is sufficiently large and $y$ is small, which happens when $\frac{\gamma^2}{2m}$ is small. Here is a numerical example: When $\alpha = \beta = \gamma = L = m = 1$, $R_a = 0.0019$ and $R_t = 0.0079 > R_a$. When $\alpha = 20$, $\beta = 100$, and other parameters remain the same, $R_a = 0.034$ and $R_t = 0.032 < R_a$. Q.E.D.

**Proof of Proposition 2**

As in the proof of Proposition 1, to compare a particular variable before and after trade liberalization, we
will express the variable as a function of $y$ independent of $b$. To do so, make use of the relations $R = \frac{8b^2m}{p}$, $A = \frac{\alpha}{1 + \frac{\alpha}{\sqrt[3]{y^2}}}$ (which does not contain $b$) or $A = y\sqrt{2R} = 4b\sqrt{m} - \frac{y}{\sqrt{p}}$ (which is linear in $b$). A comparison is possible because the expression of a particular variable differs between the two cases only in the value of $b$. With an expression independent of $b$, we need only determine whether the expression is increasing or decreasing in $y$, as we already know that $y_t < y_a$.

(1) Capital redistribution:

Since $y_t < y_a$, we have $\phi^0_t > \phi^0_a$. Therefore, for $\phi \in [\phi^0_a, \phi^0_t]$, $x_a v_a > 0$ while $x_t v_t = 0$, i.e., these firms sell capital.

A firm buys capital, i.e., $x_t(\phi) v_t(\phi) > x_a(\phi) v_a(\phi)$, if and only if $\frac{y_t}{y_a} \phi^0 = \left( \frac{R_t}{4M_t} \right)^{\frac{1}{3}}$. The left hand side of this inequality is increasing in $\phi$ because $y_t < y_a$, while the right hand side is independent of $\phi$. Therefore, if the inequality holds for $\phi'$, it also holds for all $\phi > \phi'$. Because some firms (at least those who exit) sell capital, there must be some firms who buy capital. Thus, the threshold of $\phi$ must be strictly smaller than 1.

(2) Firm structure:

Scope: $v_t(\phi) > v_a(\phi)$ if and only if $\frac{y_t}{y_a} \phi^0 > \sqrt{\frac{R_t}{2M_t}}$. Thus, a firm expands scope if and only if its $\phi$ exceeds some threshold.

Scale: $x_t(\phi) > x_a(\phi)$ if and only if $(2y_t - y_a) \sqrt{\phi} > 1$. If $2y_t < y_a$, all firms reduce scale. If $2y_t > y_a$, a plant expands scale if and only if its $\phi$ exceeds some threshold.

Note that the thresholds for $x$, $v$ and $xv$ are in general different, and that the thresholds for $x$ and $v$ may be 1, i.e., all plants reduce scale, or all firms reduce scope, but the two will not happen at the same time, as all firms would reduce capital, which cannot occur in equilibrium.

(3) Aggregate scope:

$M = M(b, y) = \frac{2b^2}{p}$, so $M_a = M(b, y_a)$ and $M_t = M(\frac{b}{2}, y_t)$, which means $2M_t = M(b, y_t)$. Because $\frac{d(q)}{dy} < 0$ and $y_t < y_a$, we have $2M_t > M_a$.

(4) Outputs:

Plant level: $q = \frac{A^3}{16b^2m} \left( 1 - \frac{1}{y^{\alpha}} \right) = 2\sqrt[m]{\left( \frac{y^{\alpha}}{y^{\alpha} - \frac{1}{y^{\alpha}} \phi^0} \right)}$. Then $q_t > q_a$ if and only if $\left( \frac{1}{\sqrt[m]{y}} - \frac{1}{\sqrt[m]{y_a}} \right) > \frac{\mu_t}{\sqrt[m]{y}} - \frac{\mu_a}{\sqrt[m]{y_a}}$. Because $\frac{dp}{dy} > 0$, $\frac{d}{y} < 0$, and $y_t < y_a$, we have $\frac{1}{\sqrt[m]{y_a}} > \frac{1}{\sqrt[m]{y_t}}$ and $\frac{\mu_t}{\sqrt[m]{y}} > \frac{\mu_a}{\sqrt[m]{y_a}}$. Then $q_t > q_a$ if and only if $\phi$ exceeds some threshold. Because firms with $\phi \in [\phi^0_a, \phi^0_t]$ exit and therefore reduce their outputs, the threshold is greater than $\phi^0_t$ (i.e., the least efficient firms that survive must reduce plant-level output). For the most efficient firm, $\phi = 1$ and therefore $q = 2\sqrt[m]{\frac{y^{\alpha} - 1}{y^{\alpha} - \phi^0}}$. Because $\frac{d}{y} < 0$, we have $q_t > q_a$ for $\phi = 1$, i.e., the most efficient firm must increase plant-level output.

Firm level: $qv = \frac{A^3}{16b^2m} \left( 1 - \frac{1}{y^{\alpha}} \right)^3 = \left( \frac{y^{\alpha} - 1}{y^{\alpha} - \frac{1}{y^{\alpha}}} \right)^3 \frac{\alpha}{y^{\alpha} + \frac{1}{y^{\alpha}}}. Then, if $q_t v_t > q_a v_a$ for $\phi'$, it must be true for all $\phi > \phi'$. So firm-level output increases if and only if $\phi$ exceeds some threshold. The least efficient firms that survive must reduce output. For the most efficient firm, $\phi = 1$ and therefore $qv = \frac{\alpha(y-1)^3}{y^{\alpha} + \frac{1}{y^{\alpha}}}$. It can be shown that $\frac{(y-1)^3}{y^{\alpha} + \frac{1}{y^{\alpha}}}$ decreases with $y$ for any positive value of $\frac{\alpha}{\sqrt[m]{y}}$. Thus, firm-level output increases for the most efficient firm.
Industry level output: $Q = \frac{AR \eta}{8b^2m} = \frac{\alpha}{\eta + \frac{\eta}{\rho}}$. Because $\frac{\eta}{\rho}$ increases with $y$, industry output increases. 

Q.E.D.

Proof of Lemma 2

Industry-level productivity are calculated as follows. The industry’s total output is

$$Q \equiv \int_{\phi_0}^{1} v q d\phi = \frac{AR \eta}{8b^2m},$$

and the industry’s total labor input is

$$L_b \equiv \int_{\phi_0}^{1} v^2 \varphi dx = \frac{A^2 R \rho}{8b^2m y^2}.$$

So the industry’s labor productivity is

$$\lambda \equiv \frac{Q}{L_b} = \frac{y \eta}{A \rho},$$

and industry TFP is

$$\omega \equiv \frac{Q}{\sqrt{KL_b}} = \frac{y \eta}{\rho^2},$$

where $K = 1$ is the industry’s aggregate input of capital. Overall productivity at the industry level is

$$\mu \equiv \frac{Q}{R + \int_{\phi_0}^{1} (vwL)d\phi + \int_{\phi_0}^{1} mv^2d\phi} = \frac{4y \eta}{A \theta},$$

where $\theta = y^4 + 12y^2(1 - \ln y) - 16y + 3$.

1. TFP: $\omega = \frac{y \eta}{\rho^2}$, which decreases with $y$. Because $y_t < y_a$, we have $\omega_t > \omega_a$.

2. Labor productivity:
   
   Plant and firm level: $\lambda^p = \sqrt{\frac{\eta}{\rho^2}}$, so $\frac{\lambda^p}{\lambda_a} = \sqrt{\frac{R_a}{R_t}}$.
   
   Industry level: $\lambda = \frac{m^p}{\rho^p} = \frac{n}{p} \sqrt{\frac{\eta}{\rho^2}}$. Because $\frac{d(\frac{n}{p})}{dy} < 0$, we have $\frac{n}{p_t} > \frac{n}{p_a}$. Then, $\frac{\lambda}{\lambda_a} = \frac{n}{p_t} / \frac{n}{p_a} \sqrt{\frac{R_a}{R_t}} > \sqrt{\frac{R_a}{R_t}} = \frac{\lambda^p}{\lambda_a}$.
   
   (3) Overall productivity:
   
   Plant level, $\mu^p = \sqrt{\frac{\eta}{\rho^2}}$, so $\frac{\mu^p}{\mu_a}^p = \sqrt{\frac{R_a}{R_t}}$.
   
   Firm level, $\mu^p = \frac{4}{A(1 + \frac{\eta}{\rho^2})} = \frac{4}{\sqrt{2}R(y + \frac{\eta}{\rho^2})}$. Then, $\frac{\mu^p}{\mu_a}^p = \sqrt{\frac{R_a}{R_t} \frac{\eta}{y(\eta + \frac{\eta}{\rho^2})}} > \sqrt{\frac{R_a}{R_t}} = \frac{\mu^p}{\mu_a}$. Because
   
   $\frac{\mu^p}{\mu_a} = \sqrt{\frac{R_a}{R_t} \frac{\eta}{y(\eta + \frac{\eta}{\rho^2})}}$ and $y_t < y_a$, $\frac{\mu^p}{\mu_a}$ increases with $\phi$.
   
   Industry level, $\mu = \frac{4y \eta}{A \theta} = \frac{4}{\sqrt{2}R \theta}$. Because $\frac{d(\frac{1}{\theta})}{dy} < 0$, we have $\frac{1}{\theta_t} > \frac{1}{\theta_a}$. Then, $\frac{\mu}{\mu_a} > \sqrt{\frac{R_a}{R_t}} = \frac{\mu^p}{\mu_a}$. 

22
References


