Scope adjustment, multiproduct firms, and trade liberalization

Larry D. Qiu and Wen Zhou*

The University of Hong Kong

December 21, 2011

Abstract

A model of heterogeneous firms that produce multiple products is developed to study how such firms respond to trade liberalization through adjustments in both product scope (i.e., the number of varieties they produce, or the extensive margin) and production scale (the output of each variety, or the intensive margin). In contrast to all existing models, this study demonstrates that more-productive firms may expand their scope. The pattern of scope adjustments depends on the speed rate at which the cost of adding new varieties changes with scope. The industry-wide scope reductions predicted by other formulations are shown to be due to a special assumption about the cost.

Keywords: multiproduct firms, scope, scale, firm heterogeneity, trade liberalization

JEL Code: F12, F13, F15

1 Introduction

The study of firm heterogeneity in international trade started with single-product firms, but recently attention has turned to multiproduct firms. According to Bernard et al. (2009) and Bernard et al. (2010), 41

* *Correspondence*: larryqiu@hku.hk and wzhou@business.hku.hk. This is a substantially revised version of an earlier paper entitled "Globalization, Acquisitions and Endogenous Firm Structure". We thank Jiahua Che, Jee-Hyeong Park, Alan Spearot, Wing Suen, Shangjin Wei, Stephen Yeaple, and seminar participants at the Chinese University of Hong Kong, Fudan University, Hong Kong University of Science and Technology, Peking University, Seoul National University, Shanghai University of Finance and Economics, and the University of Hong Kong for their helpful comments. We also benefited from presentations at the 2008 International Industrial Organization Conference, the 2009 Asia-Pacific Trade Seminars, the 9th Conference of the Society for the Advancement of Economic Theory, the 2009 European Trade Study Group (ETSG) meeting, the 2009 Summer Workshop in Industrial Organization and Management Strategy, the 36th European Association for Research in Industrial Economics (EARIE) conference, and the Hitoshibashi Conference in 2009.

percent of U.S. manufacturing firms produce more than a single product and they account for 91 percent of U.S. manufacturing output and 94 percent of U.S. exports. Similar observations apply to other countries. How such firms respond to trade liberalization has been analyzed theoretically using a number of models. Since heterogeneous firms are expected to behave differently, it is surprising that all existing models, most notably those of Bernard et al. (2011) and Eckel and Neary (2010), predict that all firms, including the most productive ones, reduce their scope after bilateral trade liberalization.[1] Meanwhile, empirical findings are far from conclusive. In this study, we built a theoretical model of heterogeneous multiproduct firms with diseconomies of scope, which allows increasing cost for introducing product variety. The analysis demonstrates that more-productive firms may expand their product scope in response to trade liberalization, but the pattern of scope adjustment depends crucially on the cost structure for introducing new varieties. These results provide a more complete understanding of the behavior of multiproduct firms and offer a new guide for empirical investigation.

The previous findings that multiproduct firms always reduce scope can be understood through analogy with exit by single-product firms. In models with heterogeneous single-product firms, trade liberalization moves resources from less-productive firms to more-productive ones, and forces the least-productive firms to leave the market altogether. Similar resource reallocation will happen within a firm if it produces multiple products. Typically those products are produced with declining efficiency as they move farther away from the firm's core competence. Trade liberalization then induces a similar within-firm reallocation of resources. Like least-efficient firms in a single-product economy, each firm's least-efficient varieties are dropped. This leads to a smaller product range for each firm regardless of the firm's productivity, so all firms reduce their scope after trade liberalization.

This reasoning hinges on an implicit feature that all multiproduct firms' marginal varieties (i.e., the variety beyond which a firm stops further expanding its scope) are all very inefficient and they constitute the economy's marginal products. If firms differ in the efficiency of producing their marginal varieties, a very productive firm's marginal variety may turn out to be quite efficient in relation to all the varieties in the economy, and therefore may not be dropped after trade liberalization. In that case the firm will not reduce its product scope, and may even expand it. To see how this is possible, consider the following scenario. In addition to its core product, each firm may introduce new varieties at some fixed cost, referred to as the cost of scope. Suppose that the marginal cost of scope (i.e., the additional cost of adding one more product) increases with scope.[2] Then, even if the cost schedule is the same for all firms, a more-productive firm will pay a higher cost for introducing its marginal variety than will a less-productive one simply because it maintains a larger scope at equilibrium. Since a firm's marginal variety must be profitable enough to cover its introduction cost, the marginal variety of a more-productive firm must be produced more efficiently than that of a less-productive firm. That is, a firm's marginal variety's production efficiency must increase with

[1] There are two exceptions. Nocke and Yeaple (2008) found that smaller firms expand the number of product lines while larger firms reduce it. Feenstra and Ma (2008) showed numerically that, on average, firms expand product scope.

[2] The existence and properties of such cost have been treated in a number of papers and can be justified by a firm's limited internal resources in managing its scale and scope. Such internal resource is called knowledge capital by Klette and Kortum (2004), organizational ability by Maksimovic and Phillips (2001), and organizational capital by Santalo (2001) in the industrial organization literature and by Nocke and Yeaple (2008) in the trade literature.

the firm's productivity. Since a very productive firm's marginal variety can be substantially more efficient than the economy's marginal products, we can then demonstrate that more-productive firms may respond to trade liberalization by expanding their product scope. Such a pattern is absent from all existing models of multiproduct firms because they all assume a constant (in fact often zero) and identical marginal cost of scope. For example, Bernard et al. (2011) assumed a constant and positive entry cost for each variety, while Eckel and Neary (2010) assumed zero entry cost at the variety level.

So an increasing marginal cost of scope is necessary for scope expansion. That condition, however, is not sufficient. Scope expands if and only if the marginal cost of scope increases at a moderate rate. This can be understood by again inspecting how resources reallocate. Between firms a resource (say, labor) moves from less- to more-productive firms; within a firm the resource moves from less- to more-efficient varieties. Whether or not a firm shrinks its scope depends on whether the labor input to the marginal variety is reduced to zero. So if a firm is very productive, its total labor input will increase. But for any given amount of total labor input, the firm's less-efficient varieties will receive less labor. The net effect on the firm's marginal variety depends on how fast the marginal cost of scope increases. If it increases very fast, scope expansion will be very costly. Within-firm reallocation will be heavily unfavorable to inefficient varieties, so all firms including the economy's most productive firms will reduce their scope. Conversely, if the marginal cost increases only gradually, more-productive firms will expand their scope. In the latter case, the following sorting will emerge: After trade liberalization, more-productive firms expand both scope and scale (the output of each variety), those with intermediate productivity expand scale but shrink scope, less-productive firms shrink both scale and scope, and the least-productive firms exit.

Several recent papers have studied multiproduct firms' scope choices and within-firm resource reallocation. Bernard et al. (2011) assumed heterogeneous production efficiency both across firms and across varieties within each firm. Trade liberalization raises the wage rate, which squeezes least-efficient firms in the economy as well as the least-efficient varieties in each firm, so all firms reduce their scope. Eckel and Neary (2010) also found scope reduction by all firms, driven by cannibalization on the demand side and within-firm diminishing efficiency on the supply side.[3] Mayer et al. (2011) have explained how tougher competition induces firms to focus on more-efficient products, leading to a more skewed product mix as well as a smaller product scope.[4] They were mainly interested in the relationship between a firm's product mix and competitiveness of the market, rather than how scope changes in response to trade liberalization. A common feature of all three studies is within-firm heterogeneity in production efficiency. By contrast, other studies such as those by Baldwin and Gu (2009), Feenstra and Ma (2008) and Nocke and Neary (2008) have assumed that a firm produces all its varieties with equal efficiency. This study allowed for both possibilities and was able to demonstrate that whether or not production efficiency differs within a firm is not crucial for the pattern of scope adjustment. Rather, scope reduction is driven by a constant marginal cost of scope,

[3]Their main model assumes no entries, homogenous firms and Cournot competition, but they show that the result continues to hold with free entry and heterogeneous firms.

[4]Unlike Bernard et al. (2011) and Eckel and Neary (2010) who found scope reduction unconditional, Mayer et al. (2011) are more cautious: "We do not emphasize these results for the extensive margin, because they are quite sensitive to the specifications of fixed production and export costs."

which has been assumed in all the studies mentioned above.

Multiproduct firms' scope choices can be constrained either on the demand side through cannibalization, or on the supply side through diseconomies of scope. Baldwin and Gu (2009), Dhingra (2011), Eckel and Neary (2010) and Feenstra and Ma (2008) have all emphasized cannibalization. Although it is important in oligopolistic industries, cannibalization tends to produce a negative relationship between extensive and intensive margins, which seems at odds with empirical findings about most industries (e.g., Bernard et al. 2011; Iacovone and Javorcik 2008). Furthermore, scope reduction is found whether or not cannibalization is considered, and therefore does not seem to be driven by cannibalization. Partly for this reason, this study focused on the supply linkage by assuming away cannibalization, as Bernard et al. (2011), Eckel and Neary (2010) and Mayer et al. (2011) have done. The exact way diseconomies of scope were modeled in this study differs, however. Those three prior studies all assumed constant marginal cost of scope and declining production efficiency within a firm.[5] We also allowed within-firm heterogeneity, but the distinguishing feature of our model is that it considers the possibility of increasing marginal cost of scope.

Some scholars have previously considered supply linkage through cost of scope. Arkolakis and Muendler (2010) generated within-firm product heterogeneity through variable entry cost at the product level and allowed the fixed entry cost (corresponding to the marginal cost of scope) to vary with scope. But they did not investigate the impact of trade liberalization on a firm's scope choice. Bernard et al. (2011) considered fixed production costs for each product, rendering the marginal cost of scope constant. Dhingra (2011) showed that trade liberalization induces firms to reduce product R&D (so scope is reduced) and increase process R&D (so productivity is improved). In her model, product R&D was a marginal cost of scope and was assumed to be constant.

Nocke and Yeaple (2008) assumed that a firm produces all its varieties with equal efficiency, which declines with the firm's scope, but the speed of the decline depends on the firm's organizational capability. Such an assumption generates a negative correlation between intensive and extensive margins, a result which contradicts empirical findings and all other theoretical models of this phenomenon. They also predicted that some firms will expand their scope after trade liberalization. The difference between their analysis and the one we will introduce lies in which firms will expand. Due to the negative correlation between intensive and extensive margins, Nocke and Yeaple (2008) found that expanding firms are less-capable and therefore smaller with fewer product lines. By contrast, our model predicts a positive correlation between intensive and extensive margins, so the expanding firms are those with higher intrinsic productivity and larger scope.

Most empirical studies of multiproduct firms have found significant impacts of trade liberalization on firms' scope choices, but only one has directly confirmed that different firms respond differently. Berthou and Fontagne (2009) found that after the eurozone was established in 1999, the most-productive French firms increased the number of products they exported to eurozone destinations, while less-productive French firms concentrated their exports on a smaller range of product lines. This lends clear empirical support to

[5] Although Bernard et al. (2011) generated within-firm heterogeneity through variety-specific random shocks, firms produced more-efficient varieties first as the fixed cost of each variety was the same. As a result, varieties' productivity levels decreased as scope expanded.

our prediction.[6] Bernard et al. (2011) found that U.S. firms exposed to more tariff reductions under the Canada–U.S. Free Trade Agreement reduced the number of products they produce relative to firms exposed to fewer tariff reductions. This finding represents the average impact of trade liberalization on all firms in an industry, but it does not show the impact on each individual firm. For example, is the average impact due to reduction by all firms (the conventional prediction) or by some firms, which does not preclude some other firms' expanding their scope (the prediction of the present paper)?[7] Baldwin and Gu (2009) have shown that both exporters and non-exporters in Canada reduced product diversification from 1973 to 1997, but the exporting firms' reduction of product diversification does not seem to have been related to tariff cuts, even though Canada underwent two rounds of trade liberalization during that period. Iacovone and Javorcik (2008) have documented how a substantial number of Mexican firms developed new products for export as a response to improved access to foreign markets, which is inconsistent with the theoretical prediction that all firms reduce scope. Goldberg et al. (2010) have shown that during the 1989-2003 period when profound trade and other reforms took place in India, Indian firms added more products than they dropped. They were unable to establish any relationship between tariff reduction and a firm's product dropping.[8] Dhingra (2011) showed that in Thailand during 2003-2006, less export-oriented domestic firms increased their product lines in response to a unilateral tariff cut while more export-oriented domestic firms reduced their product lines. But that trade liberalization was unilateral, which can have impacts very different from those of bilateral trade liberalization. Clearly, the previous empirical findings have been far from complete or conclusive. This study points out the possibility of and the conditions for scope expansion, thus provides a useful guide for empirical investigations.[9]

2 An autarkic economy

2.1 Model setting

Imagine a world with two identical countries. Originally there is no trade between the two countries due to, say, prohibitively high trade costs. Each country is inhabited by L identical consumers who are also the only source of labor. Each consumer has a linear-quadratic preference (à la Melitz and Ottaviano, 2008) for

[6] Arkolakis and Muendler (2010) also considered the possibility of increasing marginal cost of scope, but their empirical study focused on the characteristics of multiproduct firms with respect to their scope, scale and export destinations, not the effects of trade liberalization on these margins.

[7] In a previous empirical study, Bernard et al. (2010) had demonstrated that high efficiency U.S. manufacturing firms increased scope between 1987 and 1997, while low efficiency firms reduced it. They attributed the finding to idiosyncratic shocks to individual product's productivity, but the pattern is certainly consistent with our theory if trade liberalization played an important role in affecting firms' scope choices during that period.

[8] They attributed the discrepancy between their findings and the predictions of prevailing theories to regulations in India which prevented optimal allocation of resources.

[9] For example, it cautions the interpretation of empirical findings using developing countries' data. Even if all exporters in a developing country reduced scope after trade liberalization, the pattern is still broadly consistent with the predictions of this model, as such exporters are likely to be less productive in the international market. Their scope reduction does not necessarily imply that their counterpart in a developed country, which are more productive, will do the same thing.

differentiated goods:[10]

$$U = \alpha \int_{i\in\Omega} q_i di - \frac{1}{2}\beta \left(\int_{i\in\Omega} q_i di \right)^2 - \frac{1}{2}\gamma \int_{i\in\Omega} q_i^2 di,$$

where $\alpha, \beta,$ and γ are positive constants, Ω is the set of all products sold in the market, and q_i is the consumption of product i. Consumption choice gives rise to the market demand (from all L consumers) for product i:[11]

$$p_i = \alpha - \frac{\beta}{L} \int_{j\in\Omega} q_j dj - \frac{\gamma}{L} q_i.$$

In each country, goods are produced by a continuum of firms using labor as the only input. Firms differ in their productivity, and each firm can produce multiple varieties of the goods, each counted as a distinct product in consumers' utility. A firm with productivity φ (≥ 0) has the following production function for its kth variety ($k \geq 0$ is the index of varieties and is treated as a continuous variable for mathematical convenience):

$$q_k = q(x_k) \equiv \frac{\varphi}{1+\lambda k} x_k, \tag{1}$$

where q_k is the variety's output, x_k is the labor input, and $\lambda \geq 0$ represents within-firm heterogeneity in the production of varieties. The production function yields a constant unit cost of production: $c(\varphi, k) \equiv \frac{w(1+\lambda k)}{\varphi}$ for the kth variety, where w is the wage rate which will be determined endogenously. If $\lambda > 0$, the firm's varieties are produced at different efficiencies. The variety with the lowest index, i.e., $k = 0$, is produced at the lowest cost and is referred to as the firm's core variety. A variety with higher k is farther away from the core variety and produced at higher unit cost. If $\lambda = 0$, the firm's varieties are homogeneous, all produced at the same unit cost.[12]

In addition to production costs, each firm incurs some cost to manage its varieties.[13] In particular, a firm needs

$$h(v) \equiv \rho v^\delta$$

units of labor to manage v varieties, where $\rho \geq 0$ and $\delta \geq 1$. Given the wage rate w, the management cost is $wh(v)$, which is the same for all firms. When $\delta = 1$, the cost of managing an extra variety, $w\rho$, is constant no matter how many varieties a firm already has. More importantly, this marginal cost of scope is identical

[10] The numeraire good has been removed so that the wage rate can be endogenized.

[11] Strictly speaking, the demand should be $\kappa p_i = \alpha - \frac{\beta}{L}\int_{j\in\Omega} q_j dj - \frac{\gamma}{L} q_i$, where κ is the consumer's marginal utility of income. Like Eckel and Neary (2010), we normalize κ to be 1 so all nominal variables should be interpreted as relative to the marginal utility of income.

[12] With the exception of Baldwin and Gu (2009), Feenstra and Ma (2008) and Nocke and Yeaple (2008), all previous studies of multiproduct firms have assumed $\lambda > 0$.

[13] This management cost can be interpreted as a firm-level entry cost (or $h'(v)$ as the variety-level entry cost) that depends on the product scope, v. Alternatively, it may represent the research and development (R&D) cost to expand product scope (Dhingra 2011; Klette and Kortum 2004).

across all firms, an assumption of all previous studies except that of Arkolakis and Muendler (2010). When $\delta > 1$, the management cost is strictly convex in the number of varieties, so it becomes increasingly costly to maintain additional varieties. In the extreme case where $\delta = \infty$, having any extra variety in addition to the core will be infinitely costly, so each firm will produce only a single variety ($v = 0$). Therefore, the model admits of single-product firms as a special case.

Firms engage in monopolistic competition, so the seller of variety i regards itself as a monopolist facing the following demand:

$$p_i = A - bq_i, \tag{2}$$

where p_i is the price of variety i, $b = \frac{\gamma}{L}$, and A indicates the degree of competition in the product market with

$$A = \frac{\alpha bL + \beta P}{\beta M + bL}, \tag{3}$$

where M is the measure of Ω, and $P = \int_{i\in\Omega} p_i di$ is the aggregate price of all varieties.

The sequence of moves is as follows. There is no fixed cost for firms to enter the industry. All firms randomly draw their productivity φ from a uniform distribution on $[0, 1]$ before making their production decisions. Treating the demand parameter A and wage rate w as given, each firm chooses the number of varieties and the labor input (or equivalently the output) of each variety. That is, a firm with productivity φ chooses $v \geq 0$ and $x_k \geq 0$ for all $k \in [0, v]$ to maximize its profit

$$\pi(\varphi) \equiv \int_0^v \{[A - bq(x_k)]q(x_k) - wx_k\} \,\mathrm{d}k - wh(v). \tag{4}$$

An equilibrium (in terms of A and w) is reached when the product and labor markets both clear. The product market clears when (3) holds (notice that M and P are functions of A and w). The labor market clears when the total demand for labor, consisting of the labor used in the production and management of all varieties, equals the total supply of labor, L.

We refer to a firm's output of each variety (q_k) as the variety's scale and a firm's total number of varieties (v) as its product scope. Labor must be used in both activities. In our setting, the marginal returns to scale (for each given variety) are constant, an assumption maintained by all published models of heterogenous firms producing single or multiple products. The marginal returns to scope, on the other hand, must decrease because otherwise every firm that produces anything will attempt to maintain infinite scope and in equilibrium all production will be concentrated in the most-productive firm. There are basically two ways to introduce diminishing returns to scope, or diseconomies of scope. One is to have declining profits for each additional variety,[14] an approach adopted by most published models of multiproduct firms. In the

[14]There may be several reasons why additional varieties bring declining profits. Increasing unit costs in producing each additional variety define the so-called core competency approach. The source of the heterogeneity can be explained by random draws, either at the beginning of the game (Eckel and Neary 2010; Mayer et al. 2011) or repeatedly (Bernard et al. 2011). A second mechanism could be identical unit cost (within each firm) that increases with the firm's scope (Nocke and Yeaple 2008). A third is when selling

framework of our model, this can easily be generated if each additional variety is produced at higher unit costs, corresponding to $\lambda > 0$. In that case the management cost can be linear ($\delta = 1$). We refer to the setting with $\lambda > 0$ and $\delta = 1$ as the *core competence* case. The second approach, a distinctive feature of the present model, is to allow increasing marginal cost of managing varieties ($\delta > 1$).[15] In that case, a firm can produce its varieties with equal efficiency ($\lambda = 0$). We will refer to this setting with $\lambda = 0$ and $\delta > 1$ as the *within-firm homogeneity* case. Note that the requirement of decreasing returns to scope precludes $\lambda = 0$ and $\delta = 1$ coexisting. Also, the restriction on ρ depends on the value of δ.[16]

2.2 Firm optimization

Suppose that a firm with productivity φ decides to produce v varieties. Given A and w, the firm chooses its labor input for variety $k \in [0, v]$ to maximize that variety's profit:

$$\max_{x_k \geq 0} \pi_k(\varphi) \equiv \left(A - b\frac{\varphi x_k}{1+\lambda k}\right)\left(\frac{\varphi x_k}{1+\lambda k}\right) - w x_k. \tag{5}$$

The optimal input for variety k is therefore

$$x_k(\varphi) = x(\varphi, k) = \begin{cases} 0 & \text{for } \lambda k > \frac{\varphi A}{w} - 1, \\ \frac{1+\lambda k}{2b\varphi^2}[A\varphi - w(1+\lambda k)] & \text{for } \lambda k \leq \frac{\varphi A}{w} - 1. \end{cases}$$

In what follows, it is convenient to define

$$y \equiv \frac{A}{w}$$

an additional variety depresses the demand for a firm's existing varieties. Such cannibalization can be modeled either as firms' optimization (Baldwin and Gu 2009; Eckel and Neary 2010; Feenstra and Ma 2008) or using a product substitution parameter in the demand function (Dhingra 2011).

[15] Arkolakis and Muendler (2010) introduced a firm-level entry cost that depends on the firm's product scope. Their marginal cost of scope can be increasing, constant, or even decreasing because the marginal benefit of variety diminishes ($\lambda > 0$).

[16] If $\delta > 1$, then implicitly $\rho > 0$. If $\delta = 1$, then the value of ρ is inconsequential, and we may even have $\rho = 0$. If the demand had been derived from a CES preference, ρ must be positive. This is because the vertical intercept of a CES demand approaches infinity at zero output, so production will be positive no matter how high the marginal cost of production. Each variety's profit will then be positive, and a firm will keep adding new varieties if $\rho = 0$, leading to infinite scope, which is impossible in equilibrium. In all existing models with core competency and linear demand (Eckel and Neary 2010; Mayer et al. 2011), $\rho = 0$. In models with CES-generated demand (Arkolakis and Muendler 2010; Bernard et al. 2010, 2011), $\rho > 0$ (and $\delta = 1$).

and use wy in place of A (so the two endogenous variables defining the equilibrium will be w and y instead of w and A). Then, variety k's optimal input is

$$x(\varphi, k) = \begin{cases} 0 & \text{for } \lambda k > k_0, \\ \frac{w(1+\lambda k)}{2b\varphi^2}(y\varphi - 1 - \lambda k) & \text{for } \lambda k \leq k_0, \end{cases}$$

where $k_0 = y\varphi - 1$. A necessary condition for a firm to produce any variety is that $\varphi \geq \varphi^0$, where

$$\varphi^0 \equiv \frac{1}{y}.$$

Firms with $\varphi < \varphi^0$ will not produce anything because the unit cost of producing even the core variety is greater than the demand intercept.[17] For $\varphi > \varphi^0$, only varieties sufficiently close to the core (i.e., $k \leq k_0$) are produced.

Accordingly, firm φ's optimal scale for variety k is

$$q_k(\varphi) = q(\varphi, k) = \begin{cases} 0 & \text{for } \lambda k > k_0, \\ \frac{w}{2b}\left(y - \frac{1+\lambda k}{\varphi}\right) & \text{for } \lambda k \leq k_0, \end{cases}$$

and its optimal profit from variety k is

$$\pi_k(\varphi) = \pi(\varphi, k) = \begin{cases} 0 & \text{for } \lambda k > k_0, \\ \frac{w^2}{4b}\left(y - \frac{1+\lambda k}{\varphi}\right)^2 & \text{for } \lambda k \leq k_0. \end{cases}$$

Clearly, $\frac{\partial q(\varphi,k)}{\partial \varphi} > 0$, $\frac{\partial q(\varphi,k)}{\partial k} < 0$, $\frac{\partial \pi(\varphi,k)}{\partial \varphi} > 0$, and $\frac{\partial \pi(\varphi,k)}{\partial k} < 0$ when $q(\varphi, k) > 0$.

The optimal scope, v, is given by

$$v(\varphi) \equiv \arg\max_{v \geq 0} \int_0^v \pi(\varphi, k)\mathrm{d}k - wh(v).$$

[17]CES preferences as used by Melitz (2003) give rise to an infinite demand intercept, so a fixed production cost is needed for very inefficient firms to exit the market.

So $v(\varphi)$ is implicitly defined by

$$\left(y - \frac{1+\lambda v}{\varphi}\right)^2 - \frac{4b\rho\delta}{w} v^{\delta-1} = 0. \tag{6}$$

It is clear from this expression that marginal returns to scope indeed must be diminishing: $v(\varphi)$ will be ill-defined if $\lambda = 0$ and $\delta = 1$, so the two cannot coexist.

The following properties of $v(\varphi)$ can be established:

Proposition 1 (product scope): *In any equilibrium (i.e., given w and y) there exists a unique optimal $v(\varphi)$ and a unique $\varphi^0 \in (0,1)$ defined by $v(\varphi^0) = 0$ such that $v(\varphi) = 0$ for all $\varphi < \varphi^0$, and $v(\varphi) > 0$ and $v'(\varphi) > 0$ for all $\varphi > \varphi^0$.*

The proposition says that a firm produces if and only if it is sufficiently efficient, and that more-productive firms produce more varieties. If a firm's productivity is below some threshold ($\varphi \leq \varphi^0$), it exits. Its cost of producing even the core variety is too high to make any profit. Among those firms that produce something, a firm chooses its scope such that the marginal cost of scope, $wh'(v)$, equals the marginal benefit, which is the profit of its least efficient variety, or the *marginal variety*, $\pi(\varphi, v)$. A comparison between the two terms is equivalent to comparing $h'(v)$ with $\frac{\pi(\varphi,v)}{w}$. In later discussion it will be useful to define $B(\varphi, k) \equiv \frac{\pi(\varphi,k)}{w}$ as the *normalized profit* of variety k. Similarly, $\frac{wh'(k)}{w} = h'(k)$ is the *normalized marginal cost* of variety management.The normalization, expressing the costs and benefits in units of labor rather than monetary terms, is helpful because it fixes the cost side of scope determination (w is an endogenous variable and will change after trade liberalization) so that any change in scope due to trade liberalization comes solely from changes in normalized profit. Because $h'(k)$ is the same for all firms while $B(\varphi, k)$ increases with φ, a firm's optimal scope is larger when φ is higher. That is, more-productive firms produce more varieties.

Figure 1 presents three representative cases for two firms with different productivity. $B_i \equiv B(\varphi_i, v_i(\varphi_i))$ for $i = H, L$ is each firm's normalized profit for its marginal variety. Case (i) is the general case with strictly convex management cost ($\delta > 1$) and within-firm heterogeneity ($\lambda > 0$); case (ii) shows within-firm homogeneity ($\lambda = 0$ and $\delta > 1$); case (iii) is the core competence situation ($\lambda > 0$ and $\delta = 1$). In all three cases, the more-productive firm has a larger scope ($v_H > v_L$). More importantly, the normalized profit of the marginal variety is larger for the more-productive firm ($B_H > B_L$) when the management cost is strictly convex (cases (i) and (ii)), but equal for the two firms ($B_H = B_L$) when the management cost is linear (case (iii)). Thus the production efficiency of a firm's marginal variety increases with the firm's productivity when the management cost is strictly convex, but is equal across all firms when the management cost is linear. This difference will play an important role in determining whether or not firms expand their scope in response to trade liberalization.

Given the optimal choice of $v(\varphi)$ and $q(\varphi, k)$, a firm's total output is $Q(\varphi) \equiv \int_0^{v(\varphi)} q(\varphi, k)\mathrm{d}k$. Then, $\frac{dQ}{d\varphi} = \int_0^{v(\varphi)} \frac{\partial q(\varphi,k)}{\partial \varphi}\mathrm{d}k + v'(\varphi) > 0$. Therefore, a firm's scope, variety-level scale, and firm-level scale all

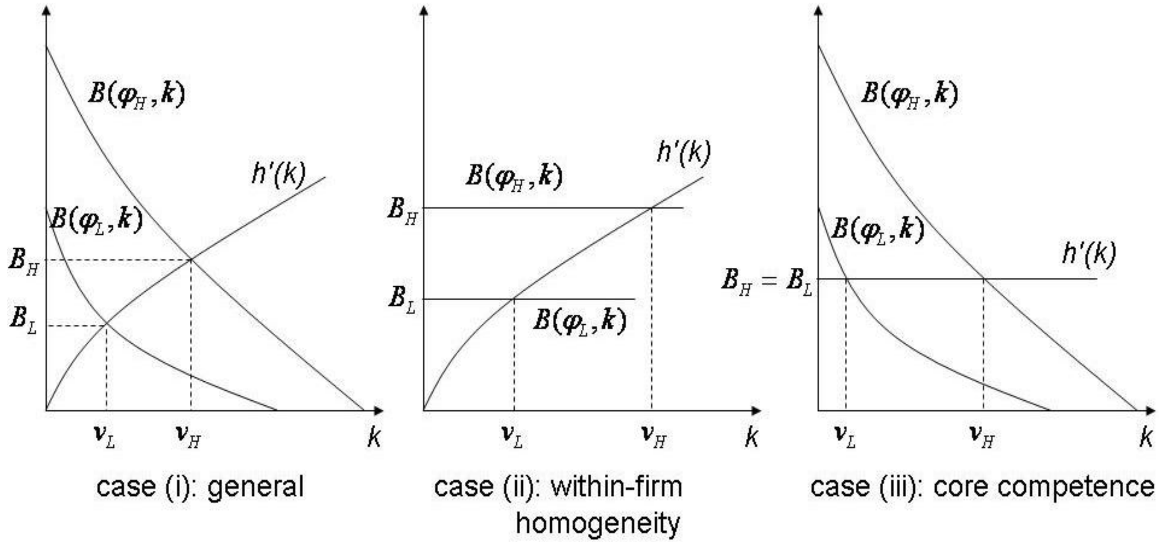

Figure 1: The determination of optimal scope

increase with the firm's productivity. As a result, intensive margins (i.e., variety-level or firm-level outputs) and extensive margins (i.e., the number of varieties of a firm) are positively correlated, a result commonly found in prior research and supported by empirical studies.

2.3 Autarkic equilibrium

The measure of varieties produced by all firms is $M(y,w) \equiv \int_{\varphi^0}^{1} v(\varphi)\mathrm{d}\varphi$. Given $q(\varphi,k)$, variety k's price is $p(\varphi,k) = \frac{w}{2}\left(y + \frac{1+\lambda k}{\varphi}\right)$, so the aggregate price is $P(y,w) \equiv \int_{\varphi^0}^{1}\int_{0}^{v(\varphi)} p(\varphi,k)\mathrm{d}k\mathrm{d}\varphi$. The total demand for labor, $L_d(y,w)$, consists of labor used in both the production and the management of varieties, so $L_d(y,w) = \int_{\varphi^0}^{1}\left[\int_{0}^{v(\varphi)} x(\varphi,k)\mathrm{d}k + h(v(\varphi))\right]\mathrm{d}\varphi$. The equilibrium in autarky, denoted by subscript a, is then defined by the market clearing conditions in the product and labor markets:

$$y_a w_a = \frac{\alpha bL + \beta P(y_a,w_a)}{\beta M(y_a,w_a) + bL}, \text{ and} \tag{7}$$

$$L = L_d(y_a,w_a). \tag{8}$$

These two equations lead to a unique solution for y_a and w_a. All the choice variables can then be calculated as in subsection 2.2.

3 The effects of trade liberalization

Now assume that trade liberalization completely removes trade costs. In particular, there are neither (Melitz type) fixed costs nor variable cost of exports (such as tariff and transport costs). Analytical comparison for the general case is complicated and challenging, so we will analyze three special cases before discussing the general setting. In the first, each firm produces a single product, and all the major forces affecting multiproduct firms' choices are already present. The next two special cases show that more-productive firms may expand their scope in the case of within-firm homogeneity, but all firms reduce their scope in the core competence case. Finally, we will demonstrate that scope expansion is driven by the convexity of management cost, not homogeneity of varieties within firms.

3.1 Equilibrium after trade liberalization

Trade liberalization gives each producer both the opportunity to export (a market expansion effect) and the challenge of increased competition in its domestic market (a competition effect). We are interested in how firms initially in autarkic equilibrium respond to the trade liberalization, in particular how different firms adjust their scopes differently. As the two countries are identical, we will focus on a symmetric equilibrium.

Since there is no trade cost, any variety a firm produces for its domestic market will also be exported to the foreign market. Therefore, unlike most models of firm heterogeneity, this model does not lead to a sorting between exporters and firms that produce only for the domestic market. Given A_t and w_t in each country (subscript t denotes variables under free trade), a firm's optimization problem is to choose $v_t \geq 0$ and $x_{k_t} \geq 0$ for all $k_t \in [0, v_t]$ to maximize its total profit from the two markets:

$$\begin{aligned}
\pi_t(\varphi) &\equiv \int_0^{v_t} \left[2\left(A_t - b\frac{q(x_t(k))}{2} \right) \left(\frac{q(x_t(k)}{2} \right) - w_t x_t(k) \right] \mathrm{d}k - w_t h(v_t) \\
&= \int_0^{v_t} \left[\left(A_t - \frac{b}{2} q(x_t(k)) \right) q(x_t(k)) - w_t x_t(k) \right] \mathrm{d}k - w_t h(v_t).
\end{aligned}$$

This problem is identical to that in the autarkic case, (4), except that the demand slope b is replaced by $\frac{b}{2}$, which represents the market expansion effect. The competition effect is embedded in the change in A. Therefore, if $y_a = y(b)$ and $w_a = w(b)$ represent the autarkic equilibrium, the trade equilibrium will be $y_t = y\left(\frac{b}{2}\right)$ and $w_t = w\left(\frac{b}{2}\right)$ and there is no need to derive the trade equilibrium separately. It is similar for the two major choices, scale $q_k(b)$ and scope $v(b)$.

3.2 Single-product firms

Our multiproduct model is general enough to admit of single-product firms as a special case with $\delta = \infty$. This special case is useful because it highlights all the major forces that are important for understanding multiproduct firms' responses. Based on the analysis in Section 2, the autarkic equilibrium can be derived as $v_a(\varphi) = 0$ (i.e., each firm produces only its core variety), $x_a(\varphi) = x_a(\varphi, 0) = \frac{w_a(y_a\varphi - 1)}{2b\varphi^2}$, and $\varphi_a^0 = \frac{1}{y_a}$, where y_a and w_a are solved from equations (7) and (8):

$$\begin{aligned} \alpha &= w_a y_a + \frac{w_a \beta}{2bL}\left(y_a - 1 - \ln y_a\right), \text{ and} \\ L &= \frac{w_a}{2b}\left(1 - y_a + y_a \ln y_a\right). \end{aligned}$$

The two equations then combine to yield

$$b = \frac{1}{2Ly_a}\left[\alpha\left(1 - y_a + y_a \ln y_a\right) + \beta\left(1 - y_a + \ln y_a\right)\right], \tag{9}$$

from which y_a can be determined uniquely. The equilibrium under free trade is obtained by replacing b with $\frac{b}{2}$ in the above expression. We prove in the Appendix the following proposition comparing the two equilibria:[18]

Proposition 2 (single-product firms): *Suppose that each firm produces a single product. Then, $\varphi_t^0 > \varphi_a^0$, and there exists a unique $\varphi^* \in (\varphi_t^0, 1)$ such that $q_t(\varphi) > q_a(\varphi)$ if and only if $\varphi > \varphi^*$.*

After trade liberalization, the cutoff productivity for active firms rises ($\varphi_t^0 > \varphi_a^0$), so the least productive firms (i.e., those with $\varphi \in [\varphi_a^0, \varphi_t^0)$) exit. Among those that remain in the market, less-productive firms ($\varphi < \varphi^*$) reduce their scale while more-productive firms ($\varphi > \varphi^*$) expand it. These responses can be understood as follows. Trade liberalization brings both a competition effect (in the form of a downward shift of the linear demand curve) and a market expansion effect (in the form of a counter-clockwise rotation of the demand curve around the vertical intercept). The competition effect is uniform for all firms, but the market expansion effect is larger for the more productive ones. So the competition effect tends to dominate for low-productivity firms, while the market expansion effect tends to dominate for those with high productivity. Put another way, firms that can barely survive under autarky find that intensified competition completely wipes out their profits in any market; expanding the market does not make things any better. Those firms will exit. For firms with small profits under autarky, the competition effect dominates. Doubling a small profit does not help much, so they reduce their scale. The opposite is true for more-productive firms, who will expand their scale.

As a result of these responses, labor redistributes towards more-productive firms. Such resource reallocation is well understood in markets where trade liberalization works by intensifying competition in either the labor market (Melitz 2003), or the product market (Melitz and Ottaviano 2008), or both (Eckel and Neary 2010; and this study). Our emphasis here is on how trade liberalization affects firm performance (in terms

[18] Throughout this discussion, statements such as "$q_t(\varphi) > q_a(\varphi)$ if and only if $\varphi > \varphi^*$" mean the following: $q_t(\varphi) > q_a(\varphi)$ if $\varphi > \varphi^*$; $q_t(\varphi) = q_a(\varphi)$ if $\varphi = \varphi^*$; and $q_t(\varphi) < q_a(\varphi)$ if $\varphi < \varphi^*$.

of normalized profits), which is important because normalized profits are directly related to multiproduct firms' scope choices.

Consider a single-product firm's performance after trade liberalization.

Lemma (normalized profits): *For single-product firms and after trade liberalization,*
(i) $\frac{B(\varphi')}{B(\varphi)}$ *is larger for any* $\varphi' > \varphi$*;*
(ii) $\frac{B(\varphi)}{q(\varphi)}$ *is smaller.*

Since each firm produces a single product, $B(\varphi)$ denotes $\frac{\pi(\varphi,0)}{w}$. Part (i) of the lemma says that, in terms of normalized profits, more-productive firms/products benefit more from trade liberalization. This is hardly surprising given the previous explanation that more-productive firms are more capable of taking advantage of trade opportunities, but the important point here is that more-productive firms are more likely to survive after trade liberalization than are less-productive ones. If trade liberalization eliminates any products, they must be the economy's *marginal products*, those with the lowest production efficiencies. As will be clear later, a particular multiproduct firm's marginal varieties may or may not constitute the economy's marginal products depending on the convexity of the management cost, and so may or may not be eliminated by trade liberalization.

Part (ii) of the lemma reveals the impacts of trade liberalization on normalized profit relative to scale. The market expansion effect doubles both $B(\varphi)$ and $q(\varphi)$ and so does not affect the ratio between the two. The competition effect reduces the normalized profit of every unit of output,[19] so $\frac{B(\varphi)}{q(\varphi)}$ drops for each firm. So trade liberalization reduces each unit's profitability. As a result, if a product's (normalized) profit has increased, then its output must have increased. But the converse is not true: if a firm's output has increased, its profit may still drop.

3.3 Within-firm homogeneity

We now consider multiproduct firms where each firm produces its varieties with equal efficiency, i.e., $\lambda = 0$. Models of this type include those of Baldwin and Gu (2009), Feenstra and Ma (2008) and Nocke and Yeaple (2010), and the equal efficiency is sometimes referred to as product homogeneity or symmetric products. Given $\lambda = 0$, we must have $\delta > 1$ and $\rho > 0$. From (6), the optimal scope in autarky can be solved explicitly:

$$v_a(\varphi) = \left[\frac{w_a}{4b\delta\rho} \left(y_a - \frac{1}{\varphi} \right)^2 \right]^{\frac{1}{\delta-1}},$$

and $\varphi_a^0 = \frac{1}{y_a}$. Since integration of the power functions leads to different functional forms depending on the value of δ, no general form of the solution can be found. We will solve the equilibrium for various values of

[19] This can be seen from the fact that the normalized demand intercept, $\frac{A}{w}$, drops after trade liberalization, as $\frac{A}{w} \equiv y$ and y drops. It must be emphasized that the drop is unambiguous only after normalization, as the equilibrium A and the absolute profit $\pi(\varphi)$ may increase or decrease depending on w and hence on the parameters.

δ, from which a general conclusion can then be established.

Let $\delta = \frac{3}{2}$. The optimal scope and scale (which is the same for every variety produced by the same firm) under autarky are then

$$v_a(\varphi) = \left(\frac{w_a}{6b\rho}\right)^2 \left(y_a - \frac{1}{\varphi}\right)^4, \text{ and} \quad (10)$$

$$q_a(\varphi) = \frac{w_a}{2b}\left(y_a - \frac{1}{\varphi}\right) \text{ for } k \in [0, v_a(\varphi)]. \quad (11)$$

Under free trade, $v_t(\varphi)$ and $q_t(\varphi)$ can be obtained from (10) and (11) by substituting b, w_a and y_a with $\frac{b}{2}$, w_t and y_t. Note that $q_t(\varphi)$ is the total output of each variety sold in both countries. In the Appendix we prove that y drops after trade liberalization ($y_t < y_a$) and that the following results hold:

Proposition 3 (within-firm homogeneity, small δ). *Suppose $\lambda = 0$ and $\delta = \frac{3}{2}$. Then,*

(i) Scale: there exists a unique $\varphi_q^* \in (\varphi_t^0, 1)$ such that $q_t(\varphi) > q_a(\varphi)$ if and only if $\varphi > \varphi_q^*$.

(ii) Scope: there exists a unique $\varphi_v^* \in (\varphi_t^0, 1)$ such that $v_t(\varphi) > v_a(\varphi)$ if and only if $\varphi > \varphi_v^*$.

(iii) Cutoff points: $\varphi_t^0 > \varphi_a^0$ and $\varphi_v^* > \varphi_q^*$.

The proposition indicates the following sorting: In response to trade liberalization, more-productive firms ($\varphi > \varphi_v^*$) increase both scale and scope, intermediate firms ($\varphi \in (\varphi_q^*, \varphi_v^*)$) increase their scale but reduce their scope, less-productive firms ($\varphi \in (\varphi_t^0, \varphi_q^*)$) reduce both scale and scope, and the least-productive firms ($\varphi \in (\varphi_a^0, \varphi_t^0)$) exit the market altogether. These results are illustrated in Figure 2, where the solid curves represent autarky and the dotted curves free trade.

Consistent with the single-product case, when firms produce multiple products, more-productive firms expand their scale (of every variety they produce), less-productive firms reduce scale, and the least-productive firms exit (Proposition 3(i) and (iii)). In fact, fixing each firm's scope, the multiproduct-firm economy can be regarded as a single-product-firm economy modified in the following way: Each individual variety of a multiproduct firm is produced by a single-product firm with the corresponding productivity, so all v varieties of multiproduct firm φ are produced by v distinct single-product firms that just happen to have the same productivity φ. Although the distribution of φ has to be modified, Proposition 2 should still hold, so more-productive firms expand the scale of all their products.

When firms produce multiple products, in addition to adjusting the scale of each variety, there is an extra channel through which a firm may respond to trade liberalization—adjusting its product scope. Recall that a multiproduct firm's scope is directly related to the normalized profit of its marginal variety. In the case of within-firm homogeneity, a firm's marginal variety is as efficient as any other the firm produces, so the sorting of the economy's varieties by efficiency corresponds perfectly to the sorting of firms (see case (ii) of Figure 1). According to Lemma (i), more-efficient products benefit more from trade liberalization in terms of their normalized profits, so if a multiproduct firm increases its scope (meaning that each variety's normalized profit has increased), all firms with higher productivity will also increase their scope, because with higher productivity their varieties must have larger normalized profits.

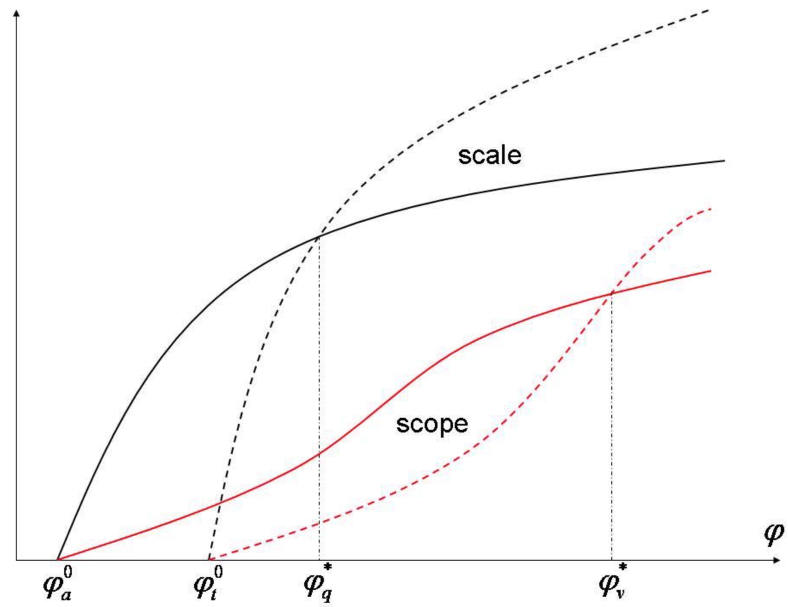

Figure 2: Scale and scope in the within-firm homogeneity case

Notice that although scope expansion has been confirmed by Proposition 3(ii), the intuition discussed here is incomplete. All it has established is that if some firms expand their scope they must be among the more productive. But there is an extra condition for these firms to actually expand their scopes: The marginal cost of scope cannot increase too fast (which is satisfied when $\delta = \frac{3}{2}$ as in Proposition 3).

Although the general pattern indicates that very productive firms expand both scale and scope, there is a difference between the two choices. Proposition 3(iii) says that the cutoff point for scope expansion is greater than that for scale expansion ($\varphi_v^* > \varphi_q^*$), so for any given firm, scope expansion implies scale expansion, but not vice versa. This result follows from Lemma (ii), which says that trade liberalization reduces each surviving product's profitability per unit of output. If a firm produces more varieties after trade liberalization (implying that each existing variety generates larger normalized profits), it must also expand the production of each variety. Conversely, if a firm reduces each variety's scale, it must shrink its product scope. There are also some firms which produce fewer varieties but expand the production of each remaining variety.

The difference between scale and scope expansions manifests a within-firm rationalization of labor in favor of scale (i.e., expanding existing varieties) over scope (i.e., adding new varieties). Note that $r(\varphi) \equiv \frac{h(v(\varphi))}{x(\varphi)v(\varphi)}$ is the ratio of labor between scope and scale. In equilibrium, $r(\varphi) = \frac{y\varphi-1}{2\delta}$ (for any δ). Since y drops after trade liberalization, $r(\varphi)$ also drops, so each firm allocates relatively more input to scale than to scope. Such reallocation of labor improves a firm's productivity. To see this, note that at the variety level, productivity as defined by $\frac{q_k(\varphi)}{x_k(\varphi)}$ (which equals φ) remains constant after trade liberalization. But at the firm level the productivity is $\omega(\varphi) \equiv \frac{Q(\varphi)}{X(\varphi)}$, where $X(\varphi) = \int_0^{v(\varphi)} x(\varphi, k)\mathrm{d}k + h(v(\varphi))$ is the firm's total labor input. It can be shown that $\omega(\varphi) = \frac{\varphi}{1+r(\varphi)}$ (again for any δ). Because $r(\varphi)$ drops after trade liberalization, firm-level productivity improves, and more so for more-productive firms. The intuition is the following. A firm's total output comes from the production of each variety, not from the management of these varieties. Nevertheless, the firm is willing to incur the management cost in order to introduce new varieties because existing varieties face declining profitability when they expand their scales along the linear demand curve. Trade liberalization doubles the market's size, so there is new room for expansion by existing varieties. This would tip the tradeoff within each firm between expanding existing varieties and managing new varieties in favor of the former. Since the cost incurred for managing varieties does not contribute directly to the firm's output, such reallocation improves productivity.

For the entire economy, productivity improves after trade liberalization because labor is rationalized both across firms (moving from less- to more-productive firms) and within firms (by focusing on expanding existing varieties rather than managing new varieties). The first effect is found in all models with heterogeneous firms, but the second effect is present only when firms produce multiple products.

Proposition 3 assumed $\delta = \frac{3}{2}$. The results also hold for other values of δ provided that they are not too large.

Proposition 4 (within-firm homogeneity, general δ). *Suppose $\lambda = 0$. Proposition 3(i) then holds for all δ and Proposition 3(ii) holds for all $\delta < 2$. If $\delta > 2$, $v_t(\varphi) < v_a(\varphi)$ for all φ.*

Proposition 4 says that more-productive firms always expand their scale regardless of the value of δ, but they expand their scope only when $\delta < 2$. Recall that $\frac{B(\varphi)}{q(\varphi)}$ decreases after trade liberalization. Thus if the most productive firm in the economy reduces the scale of its products, it immediately implies that the firm must also reduce its scope, and all other firms must reduce both scale and scope. The economy's aggregate labor demand will then decrease, which cannot happen in equilibrium because the economy's aggregate labor supply is fixed. So more-productive firms must increase their scales.

The same cannot be said about scope expansion. Because trade liberalization reduces a product's profitability per unit of output, there is a tendency for firms to shrink their scope even though their scale may have increased. Recall that trade liberalization reallocates labor along two dimensions: across firms labor moves from less-productive to more-productive firms, but within a firm relatively more labor is spent on building scale than on scope. Consider the most productive firm, which faces the following opposite effects: By the first reallocation, its total labor input will increase, which tends to increase both scale and scope. By the second reallocation, for any given total amount of labor, more will be spent on scale, so scope may shrink. The net effect on scope depends on δ, which parameterizes the cost of scope expansion. If δ is not very large (< 2), scope expansion is not very costly and Proposition 4 establishes that indeed more-productive firms will expand their scope. By contrast, when δ is large (> 2), scope expansion will be very costly and all firms reduce their scope.

Nocke and Yeaple (2008) similarly assumed within-firm homogeneity across varieties, but due to the exogenous correlation that they imposed between scope and unit costs, they reached the opposite conclusion:

High-capability firms reduce scope in response to trade liberalization.[20]

3.4 Core competence

Many previous studies have assumed that each multiproduct firm produces its varieties with decreasing efficiency and manages (or introduces) the varieties at constant marginal cost. They have then concluded that all firms reduce scope after trade liberalization (Bernard et al. 2011; Dhingra 2011; Eckel and Neary 2010). Their setting corresponds to $\lambda > 0$ and $\delta = 1$ in this model, i.e., the core competence case. To better understand why their conclusions differ from ours as drawn in the within-firm homogeneity case, we will now analyze the core competence case in detail. A comparison will demonstrate that scope expansion is driven by increasing marginal cost of scope, not within-firm homogeneity.

As it turns out, when $\delta = 1$, the value of ρ is inconsequential,[21] so for simplicity assume $\rho = 0$. Then,

$$\begin{aligned} v_a(\varphi) &= \frac{\varphi y_a - 1}{\lambda}, \text{ and} \\ q_a(\varphi, k) &= \frac{w_a}{2b}\left(y_a - \frac{1+\lambda k}{\varphi}\right) \text{ for } k \in [0, v_a(\varphi)], \end{aligned}$$

with $\varphi_a^0 = \frac{1}{y_a}$ and $q_a(\varphi, v_a(\varphi)) = 0$. The appendix shows that $\varphi_t^0 > \varphi_a^0$, so firms with $\varphi \in [\varphi_a^0, \varphi_a^0)$ exit. Furthermore, the following results can be established:

Proposition 5 (core competence). *Suppose that $\lambda > 0$ and $\rho = 0$. Then,*

(i) Scope: $v_t(\varphi) < v_a(\varphi)$ for all $\varphi \in [\varphi_t^0, 1]$ and $v_a(\varphi) - v_t(\varphi)$ increases with φ.

(ii) Core variety's scale: there exists a unique $\varphi_q^* \in (\varphi_t^0, 1)$ such that $q_t(\varphi, 0) > q_a(\varphi, 0)$ if and only if $\varphi > \varphi_q^*$.

(iii) Other varieties' scales: for $\varphi \in (\varphi_t^0, \varphi_q^*)$, $q_t(k) < q_a(k)$ for any $k \in [0, v_t(\varphi)]$. For $\varphi \in (\varphi_q^*, 1)$, there exists a unique $k^*(\varphi) \in [0, v_t(\varphi)]$ such that $q_t(k) > q_a(k)$ if and only if $k \in [0, k^*(\varphi)]$.

The key message is that all firms reduce their scope after trade liberalization, with more-productive firms reducing more (Proposition 5(i), and panel (a) of Figure 3). With regard to scale, as stated in (ii) and (iii) and demonstrated in panel (b) of Figure 3, a firm expands the scale of its core variety if and only if the firm is sufficiently productive. If a firm shrinks its core variety's scale ($\varphi < \varphi_q^*$) it also shrinks the scale of all its other varieties (panel (c)). If a firm expands its core variety's scale ($\varphi > \varphi_q^*$) it will expand the scale of those varieties that are close to the core ($k < k^*(\varphi)$), but shrink the scale of the varieties that are farther away ($k > k^*(\varphi)$) (panel (d)). That last result represents changes in the product mix (Mayer et al. 2010).

[20] More-capable firms maintain larger scopes, which by assumption immediately implies that their unit costs are higher. In other words, the least efficient varieties are produced by the most capable firms; the ranking of varieties' production efficiencies is exactly the opposite of the ranking of firms' productivity. After trade liberalization, the least efficient varieties are dropped, but because these varieties are produced by the most capable firms, the more-capable firms reduce their scope.

[21] When $\rho > 0$ the equilibrium conditions and expressions are not as neat as when $\rho = 0$, but Proposition 5 still holds. This should be expected, as none of the intuitions depend on the value of ρ.

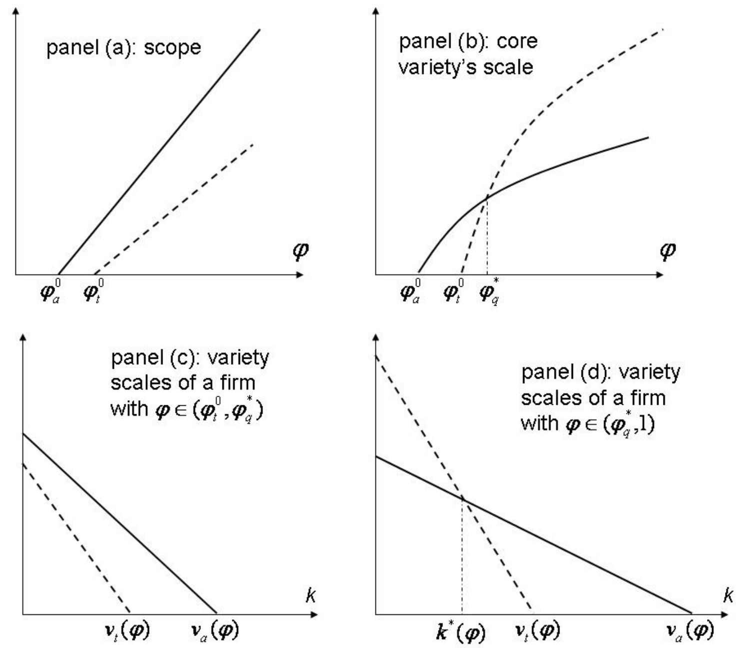

Figure 3: Scale and scope in the core-competence case

When $\delta = 1$, a firm chooses its scope such that the normalized profit of its marginal variety equals ρ. As $B(\varphi, v(\varphi)) = \frac{w}{4b}\left(y - \frac{1+\lambda v(\varphi)}{\varphi}\right)^2$, the marginal variety's unit cost of production is $\frac{w[1+\lambda v(\varphi)]}{\varphi} = wy - \sqrt{4bw\rho}$. Importantly, this unit cost is independent of φ, so it is the same for all firms, and it is the highest unit cost in the economy. That is, when the marginal cost of scope is constant, *all multiproduct firms' marginal varieties are equally inefficient, and they all constitute the economy's marginal products.* Given this observation, the effects of trade liberalization on scale and scope can again be understood using the analogy with single-product firms. Fixing the scope of every multiproduct firm, each individual variety can be regarded as being produced by a single-product firm with the corresponding productivity. As established earlier, trade liberalization eliminates the least efficient products including the marginal products, so every multiproduct firm will drop its least efficient varieties including its marginal variety. This immediately implies that all firms reduce their scope.

Now turn to the changes in scale. In the single-product firm case, firms close to the marginal firm reduce their production scale. Because every multiproduct firm has varieties close to the economy's marginal product, they all reduce the scale of their least-efficient varieties. Finally, in the single-product case, high-productivity firms expand their scale. With multiproduct firms, if a firm's φ is low, even its core variety is not efficient enough to qualify as one of the economy's "more-efficient" products, so it reduces the scale of all its varieties (panel (c)). The core variety of a very productive firm, by contrast, and varieties close by are efficient enough to count among the economy's efficient products, so their scales expand (panel (d)).

Labor redistributes both among and within firms, and both improve efficiency. The redistribution patterns are similar. Labor moves from less-productive to more-productive firms, and from less-efficient to more-efficient varieties within a firm. This similarity is not surprising, as varieties by multiproduct firms can be regarded as products by single-product firms, and we have already established that in the latter case, labor moves from less-efficient to more-efficient products. These two dimensions through which resource reallocation improves efficiency have been emphasized by Bernard et al. (2011) and others scholars.

3.5 Discussion

So far we have examined how multiproduct firms adjust their scope in two special cases and arrived at different conclusions. In the within-firm homogeneity case, more-productive firms expand their scope if $1 < \delta < 2$, and all firms reduce their scope if $\delta > 2$. In the core-competence case, the response is unanimous: Conditional on $\delta = 1$, all firms reduce their scope. These two cases differ mainly in how the benefits and costs of scope expansion are modeled. In the within-firm homogeneity case, the marginal benefit of scope is constant (i.e., $\lambda = 0$) while its marginal cost is increasing (i.e., $\delta > 1$). In the core-competence case, the marginal benefit decreases ($\lambda > 0$) while the marginal cost is constant ($\delta = 1$). Which, λ or δ or both, is driving the different results?

The general model is too complicated to yield the answer directly, but two numerical examples suffice to provide a satisfactory understanding. Both examples are general in that both assume decreasing marginal benefits of scope ($\lambda > 0$) and increasing marginal costs of scope ($\delta > 1$), but they differ in the assumed value of δ. Let $\alpha = \beta = \gamma = L = \rho = 1$ (so $b = \frac{\gamma}{L} = 1$) and $\lambda = \frac{1}{10}$ in both examples.

Example 1: $\delta = \frac{3}{2}$. Under autarky, $y_a = 10.7$ and $w_a = 0.0722$. The scale is $q_a(\varphi) = 0.386 - \frac{0.0036}{\varphi}(10 + k)$, and the scope $v_a(\varphi)$ is solved implicitly from $\left(y_a - \frac{10+v_a}{10\varphi}\right)^2 = \frac{6\sqrt{v_a}}{w_a}$. After trade liberalization, $y_t = 6.9$, $w_t = 0.101$, $q_t(\varphi) = 0.697 - \frac{0.01}{\varphi}(10+k)$, and $v_t(\varphi)$ is solved implicitly from $\left(y_t - \frac{10+v_t}{10\varphi}\right)^2 = \frac{3\sqrt{v_t}}{w_t}$. Then, $v_t(\varphi) > v_a(\varphi)$ if and only if $\varphi > \varphi_v^* = 0.892$, and $q_t(\varphi, 0) > q_a(\varphi, 0)$ if and only if $\varphi > \varphi_q^* = 0.209$.

Example 2: $\delta = 2$. Under autarky, $y_a = 11.8$ and $w_a = 0.0699$. The scale is $q_a(\varphi) = 0.412 - \frac{0.0035}{\varphi}(10 + k)$, and the scope $v_a(\varphi)$ is solved implicitly from $\left(y_a - \frac{10+v_a}{10\varphi}\right)^2 = \frac{8v_a}{w_a}$. After trade liberalization, $y_t = 7.85$, $w_t = 0.0869$, $q_t(\varphi) = 0.682 - \frac{0.0087}{\varphi}(10 + k)$, and $v_t(\varphi)$ is solved implicitly from $\left(y_t - \frac{10+v_t}{10\varphi}\right)^2 = \frac{4v_t}{w_t}$. Then, $v_t(\varphi) < v_a(\varphi)$ for all φ, and $q_t(\varphi, 0) > q_a(\varphi, 0)$ if and only if $\varphi > \varphi_q^* = 0.193$.

So when $\delta = \frac{3}{2}$, more-productive firms expand their scope; when $\delta = 2$, all firms reduce their scope. Figure 4 illustrates what happens in the two cases, where the lower right corner is Example 1 with $\delta = \frac{3}{2}$, while the upper left corner is Example 2 with $\delta = 2$. Each case shows the normalized marginal cost of variety management, $h'(k)$, which does not change after trade liberalization, and the normalized profits of the most productive firm in the economy (i.e., $\varphi = 1$) both before and after trade liberalization (solid and dotted lines, respectively). In both cases, trade liberalization increases the normalized profits of the firm's more-efficient varieties and decreases the normalized profits of its less-efficient ones. In other words, it

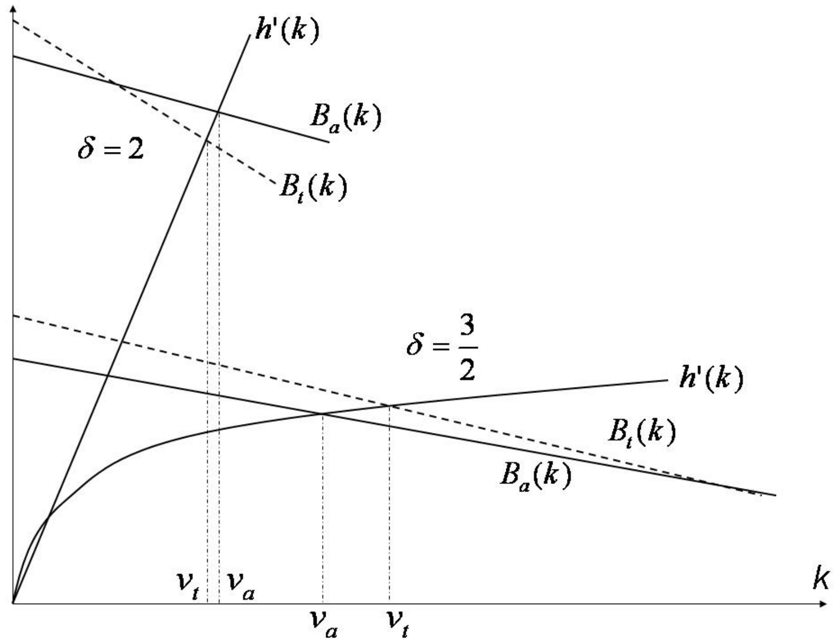

Figure 4: The most-productive firm's scope in the two examples

rotates the $B(k)$ curve clockwise, making it steeper. This is a combined effect of increased skewness of varieties' outputs within a firm and reduced profitability per unit of output.[22] The difference between the two cases is that when δ is larger the rotation is more dramatic, so the $B_t(k)$ curve crosses the $B_a(k)$ curve at a much smaller k. As a result, the $h'(k)$ curve is more likely to cut the two $B(k)$ curves to the right of the crossing point, meaning that v_t is more likely to be smaller than v_a for this firm.

To summarize, in the general case with $\lambda > 0$ and $\delta > 1$, more-productive firms expand their scale in response to trade liberalization, and less productive firms shrink it. Whether or not firms expand their scope depends crucially on the value of δ. If δ is not very large, more-productive firms will expand their scope, as demonstrated in Example 1 with $\delta = \frac{3}{2}$. If δ is large, all firms will reduce scope, as demonstrated in Example 2 with $\delta = 2$. Therefore, scope contraction has nothing to do with within-firm heterogeneity and has everything to do with the convexity of the management cost curve. More-productive firms expand their scope if management cost is mildly convex, but all firms shrink it if management costs are either linear or very convex.

[22]When δ is very large it is possible that the entire dotted line (i.e., the normalized profit after trade liberalization) lies below the solid line.

4 Concluding Remarks

This paper is intended to clarify how multiproduct firms respond to trade liberalization by changing their product line and adjusting the output of their existing products. We have shown that, contrary to the conclusions of most previous studies, trade liberalization may lead more-productive firms to expand their scope, and the condition for such expansion is a marginal cost of scope which increases only gradually.

The major novelty of this study is the introduction of a management cost curve that may take different forms. It clarifies the fact that a necessary condition for some firms to expand their scope is that firms' marginal varieties are not all equally inefficient. In particular, the production efficiency of a firm's marginal variety must increase with the firm's intrinsic productivity. Such a feature is most conveniently generated in this sort of analysis by assuming an increasing marginal cost of scope.

The discussion has used within-firm homogeneity to show analytically the possibility of scope expansion, and then demonstrated using examples that the driving force for scope expansion is increasing marginal cost of scope, not within-firm homogeneity. From a theoretical point of view, if we insist that all varieties compete in a single industry as differentiated and yet closely related products, within-firm homogeneity seems appropriate. For example, an automobile manufacturer probably produces its horizontally differentiated models with equal efficiency. If the firms are conglomerates that produce remotely related products, then within-firm heterogeneity is probably more appropriate. In any case, the key question for modeling multiproduct firms' scope choices is why a firm does not keep adding new varieties. Most likely the answer has something to do with the increasing cost of managing larger organizations. Our management cost approach is then highly appropriate. Ultimately, how the management cost changes with scope is an empirical question. More studies of the connection between scope change and the underlying cost structure are needed, for which our analysis may provide a useful guide.

We have assumed linear demand following the lead of Melitz and Ottaviano (2008) and have allowed the wage rate to be determined endogenously. Other scholars have used nested CES preferences and/or an exogenous wage rate. To test the robustness of our conclusions, we have tried fixing either the demand intercept A (to mimic the fixed and exogenous markup derived from CES demand) or the wage rate w. The qualitative results did not change. This is hardly surprising, as none of the intuitions depend on the endogeneity of either markup or the wage rate.

Appendix: Proofs

Proof of Proposition 1 (product scope)

Let $F(v)$ denote the function on the left hand side of (6). If $\delta > 1$, then $\frac{\partial F}{\partial v} < 0$, $F(0) > 0$ and $F\left(\frac{k_0}{\lambda}\right) < 0$, so there exists a unique $v \in \left(0, \frac{k_0}{\lambda}\right)$ such that $F(v(\varphi)) = 0$. Further, $x(\varphi, k) > 0$ for any $k \in [0, v(\varphi)]$. Let $\varphi^0 = \frac{1}{y}$. Then $v(\varphi^0) = 0$. $v'(\varphi) = -\frac{\partial F}{\partial v} \big/ \frac{\partial F}{\partial \varphi} > 0$ as $\frac{\partial F}{\partial v} < 0$ while $\frac{\partial F}{\partial \varphi} > 0$. Therefore, for all $\varphi \leq \varphi^0$, $v(\varphi) = x(\varphi, 0) = 0$. For all $\varphi > \varphi^0$, $v(\varphi) > 0$ and $x(\varphi, k) > 0$ for any $k \in [0, v(\varphi))$.

If $\delta = 1$ (and consequently $\lambda > 0$), then $v(\varphi) = \frac{1}{\lambda}\left[\varphi\left(y - \sqrt{\frac{4b\rho}{w}}\right) - 1\right]$. Note that y and w are endogenous, and it will be shown later that $y - \sqrt{\frac{4b\rho}{w}} > 1$ in equilibrium. For any given y and w, it is straightforward to see that $v'(\varphi) > 0$. Let $\varphi^0 = \frac{1}{y - \sqrt{\frac{4b\rho}{w}}} \in (0, 1)$. Then, $v(\varphi^0) = 0$ while $x(\varphi^0, 0) > 0$ if $\rho > 0$ and $x(\varphi^0, 0) = 0$ if $\rho = 0$. Given that $v'(\varphi) > 0$, for all $\varphi < \varphi^0$, $v(\varphi) = x(\varphi, 0) = 0$. For all $\varphi > \varphi^0$, $v(\varphi) > 0$ and $x(\varphi, k) > 0$ for any $k \in [0, v(\varphi))$. Note that $v(\varphi) \leq \frac{k_0}{\lambda}$. *Q.E.D.*

Proof of Proposition 2 (single-product firms)

The equilibrium y is solved from (9). Because $b > 0$, we have $\frac{\alpha}{\beta} > \frac{y-1-\ln y}{1-y+y\ln y}$, which immediately implies that $b'(y) = \frac{\alpha(y-1)-\beta \ln y}{2Ly^2} > 0$. Note that the right hand side of (9) equals 0 at $y = 1$ but infinity at $y = \infty$. Given that $b'(y) > 0$, (9) has a unique solution for y on $(1, \infty)$. Because $\frac{dy}{db} = \frac{1}{b'(y)} > 0$ and b decreases to $\frac{b}{2}$ after trade liberalization, we conclude that y drops: $y_t < y_a$. As a result, $\varphi_t^0 = \frac{1}{y_t} > \frac{1}{y_a} = \varphi_a^0$.

The scale is $q(\varphi) = \frac{L}{\varphi}\frac{y\varphi-1}{1-y+y\ln y}$. Because $\frac{y}{1-y+y\ln y}$ is a decreasing function of y, $q_t(\varphi) > q_a(\varphi)$ if and only if $\varphi > \varphi^*$ for some φ^*. Because $q(1) = \frac{L(y-1)}{1-y+y\ln y}$ decreases in y, we have $q_t(1) > q_a(1)$, which implies that $\varphi^* < 1$. Because $\varphi_t^0 > \varphi_a^0$, we have $q_a(\varphi_t^0) > 0 = q_t(\varphi_t^0)$, so $\varphi^* > \varphi_t^0$. *Q.E.D.*

Proof of the lemma (normalized profits)

The profit $\pi(\varphi) = \frac{w^2(y\varphi-1)^2}{4b\varphi^2}$, so the normalized profit is $B(\varphi) = \frac{\pi(\varphi)}{w} = \frac{w}{b}\frac{(y\varphi-1)^2}{4\varphi^2}$. The labor market clearing condition leads to $\frac{w}{b} = \frac{2L}{y\ln y-y+1}$, so $B(\varphi) = \frac{L(y\varphi-1)^2}{2\varphi^2(y\ln y-y+1)}$. Note that this expression does not contain b, so it is the same for both the autarkic and trade conditions.

(i) $\frac{B(\varphi')}{B(\varphi)} = \frac{\left(y-\frac{1}{\varphi'}\right)^2}{\left(y-\frac{1}{\varphi}\right)^2}$, which increases after trade liberalization because $\varphi' > \varphi$ while y has dropped.

(ii) $q(\varphi) = \frac{w(y\varphi-1)}{2b\varphi}$, so $\frac{B(\varphi)}{q(\varphi)} = \frac{y\varphi-1}{2\varphi}$, which decreases after trade liberalization because y has dropped. *Q.E.D.*

Proof of Proposition 3 (within-firm homogeneity, small δ)

Given firm choices as specified in (10) and (11), the equilibrium conditions (7) and (8) become:

$$\alpha = yw + \frac{\beta w^3}{b^3\rho^2 L}\theta(y), \text{ and} \tag{12}$$

$$L = \frac{w^3}{b^3\rho^2}\mu(y), \tag{13}$$

where $\theta(y) = \frac{1}{864}\left[12y^5 + (65 - 60\ln(y))y^4 - 120y^3 + 60y^2 - 45y + 3\right]$ and $\mu(y) = \frac{1}{4320}\left[20y^6 + (17 - 60\ln(y))y^5 - 100y^3 + 100y^2 - 20y + 8\right]$. It can be verified that $\mu(y) > \theta(y) > 0$ and $\mu'(y) > \theta'(y) > 0$. (12) and (13) combine to yield:

$$Lb^3\rho^2 = \left[\alpha - \beta\frac{\theta(y)}{\mu(y)}\right]^3 \frac{\mu(y)}{y^3},$$

from which a unique solution for y can be found on $(1, \infty)$. Furthermore, it can be verified that $\alpha - \beta\frac{\theta(y)}{\mu(y)}$

and $\frac{\mu(y)}{y^3}$ both increase in y. Therefore, $\frac{dy}{db} > 0$ and so y drops after trade liberalization: $y_t < y_a$. As a result, $\varphi_t^0 = \frac{1}{y_t} > \frac{1}{y_a} = \varphi_a^0$.

(i) Scale: From (13) we can solve $w = b\left(L\rho^2\mu^{-1}\right)^{\frac{1}{3}}$, where $\mu \equiv \mu(y)$. Plug this into $q(\varphi)$ to get $q(\varphi) = \frac{(L\rho^2)^{\frac{1}{3}}}{2\varphi}(y\varphi - 1)\mu^{-\frac{1}{3}}$. Because $y\mu^{-\frac{1}{3}}$ decreases in y and therefore $y_t\mu_t^{-\frac{1}{3}} > y_a\mu_a^{-\frac{1}{3}}$, we have $q_t(\varphi) > q_a(\varphi)$ if and only if $\varphi > \varphi_q^*$ for some φ_q^*. Because $q(1) = \frac{(L\rho^2)^{\frac{1}{3}}}{2}(y-1)\mu^{-\frac{1}{3}}$ decreases in y, $q_t(1) > q_a(1)$. Therefore, $\varphi_q^* < 1$. Because $\varphi_t^0 > \varphi_a^0$, we have $q_a(\varphi_t^0) > 0 = q_a(\varphi_a^0)$, so $\varphi_q^* > \varphi_t^0$.

(ii) Scope: Using the same w expression for scope, we have $v(\varphi) = \frac{1}{36\varphi^4}\left(\frac{L}{\rho}\right)^{\frac{2}{3}}\left[(y\varphi-1)\mu^{-\frac{1}{6}}\right]^4$. Because $y\mu^{-\frac{1}{6}}$ and $(y-1)\mu^{-\frac{1}{6}}$ both decrease in y, using the same method as in the case of scale we can prove that $v_t(\varphi) > v_a(\varphi)$ if and only if $\varphi > \varphi_v^*$ for some $\varphi_v^* \in (\varphi_t^0, 1)$.

(iii) Cutoff points: It can be verified that $\frac{\sqrt{v(\varphi)}}{q(\varphi)} = \frac{y\varphi-1}{3\rho\varphi}$, which decreases after trade liberalization. Therefore, when $v_t(\varphi_v^*) = v_a(\varphi_v^*)$, we must have $q_t(\varphi_v^*) > q_a(\varphi_v^*)$. In view of (i), this immediately implies that $\varphi_v^* > \varphi_q^*$.

In fact, the same pattern can be established for other variables: $x(\varphi) = \frac{(L\rho^2)^{\frac{1}{3}}}{2\varphi^2}(y\varphi-1)\mu^{-\frac{1}{3}}$ for each variety's input, $X(\varphi) = vx + h(v) = \frac{L}{216\varphi^6}(y\varphi-1)^5(y\varphi+2)\mu^{-1}$ for the firm's total labor input, and $Q(\varphi) = q(\varphi)v(\varphi) = \frac{L}{72\varphi^5}(y\varphi-1)^5\mu^{-1}$ for the firm's total output. We can show that for $Z = x, X, Q$, $Z_t(\varphi) > Z_a(\varphi)$ if and only if $\varphi > \varphi_Z^*$ for some $\varphi_Z^* \in (\varphi_t^0, 1)$. Furthermore, it can be shown that

$$1 > \varphi_v^* > \varphi_X^* > \varphi_Q^* > \varphi_x^* = \varphi_q^* > \varphi_t^0.$$

Q.E.D.

Proof of Proposition 4 (within-firm homogeneity, general δ)

The integration of the power functions gives different expressions depending on the value of δ, so we calculate the equilibrium separately for each individual value of δ using the same method as in the case of $\delta = \frac{3}{2}$. For $\delta < 2$, we have tried $\delta = 1 + \frac{1}{n}$ for $n = 3, 4, \cdots, 25$. For $\delta \geq 2$, we have tried $\delta = 2, 3, \cdots, 20$. In every case, y was found to drop after trade liberalization.

From $L = L_d(y, w)$ we can express $\frac{w}{b} = L^{\frac{\delta-1}{\delta}}\rho^{\frac{1}{\delta}}\mu_\delta^{\frac{1-\delta}{\delta}}$, where μ_δ is a function containing only y and δ; its functional form depends on δ, and μ_δ always increases in y. Then, $q(\varphi) = \frac{1}{2}\frac{w}{b}\frac{y\varphi-1}{\varphi} = \left(\frac{1}{2}L^{\frac{\delta-1}{\delta}}\rho^{\frac{1}{\delta}}\right)\mu_\delta^{\frac{1-\delta}{\delta}}\frac{y\varphi-1}{\varphi}$ and $v(\varphi) = \left[\frac{1}{4\delta\rho}\frac{w}{b}\frac{(y\varphi-1)^2}{\varphi^2}\right]^{\frac{1}{\delta-1}} = \left[(4\delta)^{\frac{1}{1-\delta}}\left(\frac{L}{\rho}\right)^{\frac{1}{\delta}}\right]\left(\mu_\delta^{\frac{1-\delta}{2\delta}}\frac{y\varphi-1}{\varphi}\right)^{\frac{2}{\delta-1}}$. We can verify that for every value of δ, $\mu_\delta^{\frac{1-\delta}{\delta}}y$ decreases in y, so $\mu_\delta^{\frac{1-\delta}{\delta}}y$ increases after trade liberalization, and therefore $q(\varphi)$ rises if and only if $\varphi > \varphi_q^*$ for some φ_q^*. Similarly, $\mu_\delta^{\frac{1-\delta}{2\delta}}y$ also decreases in y, so $v(\varphi)$ rises if and only if $\varphi > \varphi_v^*$ for some φ_v^*. The remaining question is whether φ_q^* and φ_v^* are smaller than 1.

Scale: $q(1) = \left(\frac{1}{2}L^{\frac{\delta-1}{\delta}}\rho^{\frac{1}{\delta}}\right)\mu_\delta^{\frac{1-\delta}{\delta}}(y-1)$. For any value of δ, $\mu_\delta^{\frac{1-\delta}{\delta}}(y-1)$ always decreases in y, so $q_t(1) > q_a(1)$ and therefore $\varphi_q^* < 1$.

Scope: $v(1) = \left[(4\delta)^{\frac{1}{1-\delta}}\left(\frac{L}{\rho}\right)^{\frac{1}{\delta}}\right]\left[\mu_\delta^{\frac{1-\delta}{2\delta}}(y-1)\right]^{\frac{2}{\delta-1}}$. When $\delta < 2$, $\mu_\delta^{\frac{1-\delta}{2\delta}}(y-1)$ always decreases in y, so $v_t(1) > v_a(1)$ and therefore $\varphi_v^* < 1$. When $\delta = 2$, $\mu_2 = (y-1)^4$, so $\mu_\delta^{\frac{1-\delta}{2\delta}}(y-1) = \mu_2^{-\frac{1}{4}}(y-1) = 1$, which is independent of y. This means that $v_t(1) = v_a(1)$ and so $\varphi_v^* = 1$. When $\delta = 3$, $\mu_3 = 2y^3 + 6y^2\ln y - 15y^2 + 18y - 5$, and so $\mu_\delta^{\frac{1-\delta}{2\delta}}(y-1) = \mu_3^{-\frac{1}{3}}(y-1)$, which increases in y. Therefore, $v_t(1) < v_a(1)$ and so $\varphi_v^* > 1$. For $\delta = 4, 5, \cdots, 20$, we find the same pattern: $\mu_\delta^{\frac{1-\delta}{2\delta}}(y-1)$ increases in y and therefore $\varphi_v^* > 1$.

In fact we can prove that even when $v(\varphi)$ drops after trade liberalization, $v_a(\varphi) - v_t(\varphi)$ decreases when φ increases, which is the opposite of Proposition 5 about core competence. From the expression for $v(\varphi, y)$ it is clear that the sign of $\frac{\partial v^2(\varphi,y)}{\partial y \partial \varphi}$ is the same as the sign of $\frac{\partial z^2(\varphi,y)}{\partial y \partial \varphi}$, where $z(\varphi, y) = \mu_\delta^{\frac{1-\delta}{2\delta}}\left(y - \frac{1}{\varphi}\right)$. But $\frac{\partial z^2(\varphi,y)}{\partial y \partial \varphi} = \frac{1}{\varphi^2}\frac{d\mu_\delta^{\frac{1-\delta}{2\delta}}}{dy}$. Because μ_δ increases in y and $\frac{1-\delta}{2\delta} < 0$, we have $\frac{\partial z^2(\varphi,y)}{\partial y \partial \varphi} < 0$ and therefore $\frac{\partial v^2(\varphi,y)}{\partial y \partial \varphi} < 0$. When $\varphi_v^* > 1$, $v_a(\varphi) - v_t(\varphi) > 0$ for all $\varphi \leq 1$. Because $v_a(\varphi) - v_t(\varphi) = -\frac{\partial v^2(\varphi,y)}{\partial y \partial \varphi} dy d\varphi$ and $dy < 0$, we have shown that $v_a(\varphi) - v_t(\varphi)$ decreases when φ increases. *Q.E.D.*

Proof of Proposition 5 (core competence)

Given the expressions for $v(\varphi)$ and $x(\varphi, k)$, equations (7) and (8) become

$$\alpha = wy + \frac{w\beta}{8bL\lambda}\left(y^2 - 4y + 2\ln y + 3\right), \text{ and} \tag{14}$$

$$L = \frac{w}{24b\lambda}\left(y^3 + 3y - 6y\ln y - 4\right). \tag{15}$$

As a result, y is uniquely determined from

$$b = \frac{1}{24L\lambda y}\left[\alpha\left(y^3 + 3y - 6y\ln y - 4\right) - 3\beta\left(y^2 - 4y + 2\ln y + 3\right)\right]. \tag{16}$$

The sign of $\frac{dy}{db}$ is the same as that of $b'(y) = \frac{1}{24L\lambda y^2}\left[2\alpha(y-1)^2(y+2) - 3\beta(y^2 - 1 - 2\ln y)\right]$. Because $b > 0$, we have $\frac{\alpha}{\beta} > \frac{3\left(y^2-4y+2\ln y+3\right)}{y^3+3y-6y\ln y-4}$, which immediately leads to $b'(y) > 0$ for all $y > 1$. Because $\frac{dy}{db} > 0$ and b decreases to $\frac{b}{2}$ after trade liberalization, we conclude that y drops. Then $\varphi_t^0 = \frac{1}{y_t} > \frac{1}{y_a} = \varphi_a^0$.

(i) Scope: $v(\varphi) = \frac{\varphi y - 1}{\lambda}$. Given that $y_t < y_a$, the conclusions are straightforward.

(ii) Core variety's scale: $q(\varphi, 0) = \frac{w(y\varphi-1)}{2b\varphi}$. From (15) we have $\frac{w}{b} = \frac{24L\lambda}{\mu}$, where $\mu \equiv y^3 + 3y - 6y\ln y - 4$, so $q(\varphi, 0) = \frac{12L\lambda}{\varphi}\frac{y\varphi-1}{\mu}$. Since $\frac{y}{\mu}$ decreases in y, there exists a φ_q^* cutoff point such that $q_t(\varphi, 0) > q_a(\varphi, 0)$ if and only if $\varphi > \varphi_q^*$. Since $q(1, 0) = \frac{24L\lambda(y-1)}{\mu}$ also decreases in y, $q_t(1, 0) > q_a(1, 0)$ and therefore $\varphi_q^* < 1$.

(iii) Other varieties' scales: $q(\varphi, k) = \frac{w}{2b}\left(y - \frac{1+\lambda k}{\varphi}\right) = \frac{12L\lambda}{\varphi}\frac{y\varphi-1-\lambda k}{\mu}$. Then $q_t(\varphi, k) > q_a(\varphi, k)$ if and only if $\frac{y_t\varphi-1-\lambda k}{\mu_t} > \frac{y_a\varphi-1-\lambda k}{\mu_a}$. Because $\frac{1}{\mu}$ decreases in y, $\frac{1}{\mu_t} > \frac{1}{\mu_a}$ so $q_t(\varphi, k) > q_a(\varphi, k)$ if and only if $k < k^*(\varphi)$ for some $k^*(\varphi)$. When $\varphi \in (\varphi_t^0, \varphi_q^*)$, $q_t(\varphi, 0) < q_a(\varphi, 0)$, so $q_t(\varphi, k) < q_a(\varphi, k)$ for any $k \in$

$[0, v_t(\varphi)]$. When $\varphi \in (\varphi_q^*, 1)$, $q_t(\varphi, 0) > q_a(\varphi, 0)$ so $k^*(\varphi) > 0$. Because $q_t(\varphi, v_t(\varphi)) = 0 < q_a(\varphi, v_t(\varphi))$, $k^*(\varphi) < v_t(\varphi)$. *Q.E.D.*

Computation of the equilibrium for the numerical examples

$v(\varphi)$ is solved from (6) only implicitly. This poses difficulties in the calculation. Since v and φ are both firm level characteristics, and since there is a one-to-one mapping between the two variables, we can express φ as a function of v by solving (6):

$$\varphi(v) = \frac{1 + \lambda v}{y - 2\sqrt{\frac{b\rho\delta v^{\delta-1}}{w}}}.$$

Then x, p and other firm-level choice variables are all expressed as functions of y, w, and v. So $M(y, w, v_1) \equiv \int_0^{v_1} v\varphi'(v)dv$, $P(y, w, v_1) \equiv \int_0^{v_1} \left(\int_0^v p(\varphi(v), k, y, w)dk \right) \varphi'(v)dv$, and $L_d(y, w, v_1) \equiv \int_0^{v_1} \left(\int_0^v x(\varphi(v), k, y, w)dk + h(v) \right) \varphi'(v)dv$, where v_1 is the scope chosen by the most productive firm ($\varphi = 1$). Then, when φ increases from $\varphi^0 = \frac{1}{y}$ to 1, v will increase correspondingly from 0 to v_1. Because v_1 satisfies $\varphi(v_1) = 1$, we have

$$w(y, v_1) = \frac{4b\rho\delta v_1^{\delta-1}}{(y - 1 - \lambda v_1)^2}.$$

After the three integrations, plug $w(y, v_1)$ into the two equilibrium conditions (7) and (8), which now contain only two unknowns, y and v_1, and the equilibrium can then be calculated. *Q.E.D.*

References

Arkolakis, C. and M. Muendler, 2010, “The extensive margin of exporting goods: A firm-level analysis”, Yale University, mimeograph.

Baldwin, J. and W. Gu, 2009, “The impact of trade on plant scale, production-run length and diversification”, in *Producer Dynamics: New Evidence from Micro Data*, ed. T. Dunne, J. B. Jensen and M. J. Roberts (Chicago: University of Chicago Press).

Bernard, A.B., J.B. Jensen and F.K. Schott, 2009, “Importers, exporters, and multinationals”, in *Producer Dynamics: New Evidence from Micro Data*, ed. T. Dunne, J. B. Jensen and M. J. Roberts (Chicago: University of Chicago Press).

Bernard, A.B., S.J. Redding and F.K. Schott, 2010, “Multi-product firms and product switching”, *American Economic Review*, 100:1, 70-97.

Bernard, A.B., S.J. Redding and F.K. Schott, 2011, “Multiproduct firms and trade liberalization”, *Quarterly Journal of Economics*, 126, 1271-1318.

Berthou, A. and L. Fontagne, 2009, “How do multi-product exporters react to a change in trade costs?”, EFIGE Working Paper 16.

Dhingra, S., 2011, “Trading away wide brands for cheap brands”, Princeton University, mimeograph.

Eckel, C. and J.P. Neary, 2010, “Multi-product firms and flexible manufacturing in the global economy”, *Review of Economic Studies*, 77, 188-217.

Feenstra, R. and H. Ma, 2008, “Optimal choice of product scope for multiproduct firms”, in *The Organization of Firms in a Global Economy*, ed. E. Helpman, D. Marin and T. Verdier (Harvard University Press).

Goldberg, P.K., A. Khandelwal, N. Pavcnik and P. Topalova, 2009, “Multi-product firms and product turnover in the developing world: Evidence from India”, *Review of Economics and Statistics*, 92:4, 1042-1049.

Greenaway, D., J. Gullstrand and R. Kneller, 2008, “Surviving globalization”, *Journal of International Economics*, 74, 264-277.

Helpman, E., 1985, “Multinational corporations and trade structure”, *Review of Economic Studies*, 52:3, 443-457.

Iacovone, L. and B.S. Javorcik, 2008, “Multi-product exporters: Diversification and micro-level dynamics”, World Bank Policy Research working paper no.4723.

Klette, J. and S. Kortum, 2004, “Innovating firms and aggregate innovation”, *Journal of Political Economy*, 112:5, 986-1018.

Maksimovic, V. and G. Phillips, 2001, “The market for corporate assets: Who engages in mergers and asset sales and are there efficiency gains?”, *Journal of Finance*, 55:6, 2019-2065.

Mayer, T., M. Melitz and G. Ottaviano, 2011, “Market size, competition and the product mix of exporters”, mimeograph.

Melitz, M., 2003, “The impact of trade on intra-industry reallocations and aggregate industry productivity”, *Econometrica*, 71:6, 1695-1725.

Melitz, M. and G. Ottaviano, 2008, “Market size, trade and productivity”, *Review of Economic Studies*, 75, 295-316.

Nocke, V. and S. Yeaple, 2008, “Globalization and the size distribution of multiproduct firms”, CEPR Discussion Paper No. DP6948.

Santalo, Juan, 2001, “Organizational capital and the existence of a diversification and size ‘discount’”, University of Chicago, mimeograph.

UNCTAD, 2000, *World Investment Report 2000: Cross-border Mergers and Acquisitions and Development*, (New York: United Nations).