On the Dynamic Efficiency of Bertrand and Cournot Equilibria*

by

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Abstract

This paper compares Bertrand and Cournot equilibria in a differentiated duopoly with R&D (research and development) competition. Cournot competition is shown to induce more R&D effort than Bertrand competition. However, the price is lower and output is higher in Bertrand than in Cournot competition. Furthermore, the Bertrand equilibrium is more efficient than the Cournot equilibrium if either R&D productivity is low, spillovers are weak, or products are very different. If R&D productivity is high, spillovers are strong and goods are close substitutes, the Bertrand equilibrium becomes *less* efficient than the Cournot equilibrium.

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1. Introduction

It is well understood that (i) equilibrium prices are lower and outputs are higher in Bertrand (price) competition than in Cournot (quantity) competition, and (ii) the Bertrand equilibrium is more efficient than the Cournot equilibrium, in terms of greater consumer surplus and welfare (see Singh and Vives [8], Cheng [3], and Vives [10]). These traditional results are obtained under the assumption that firms face the *same* demand and cost structure in both types of competition. To make the comparison meaningful, such a static assumption seems reasonable and desirable. However, this assumption may not be innocuous because firms often compete against each other by investing in research and development (R&D) to improve in product quality (in the case of product R&D) and/or to reduce production cost (in the case of process R&D). As a result, the structure of market demand and cost of production may change. In such a *dynamic* environment, if R&D investments are different in Bertrand and Cournot competition, the post-innovation demand and cost structures will be different even though they were identical before the R&D competition. The ensuing important question is whether the traditional result still holds will be affected in any way.

The present study focuses on cost-reducing R&D and re-examines the relative efficiency of Cournot and Bertrand equilibria. The case for cost-reducing R&D is of particular interest. As I shall show, firms invest more in R&D in Cournot competition than in Bertrand competition. With lower pre- competition production costs in Cournot competition, do the firms still charge lower prices and produce more in Bertrand competition? Even if the answer is yes, it does not always yield the traditional welfare results since the cost of production is now larger in Bertrand than in Cournot competition. In other words, if the market with Bertrand competition enjoys higher *static* efficiency, will it necessarily lead to higher *dynamic* efficiency?

In this paper, I first introduce the basic model in Section 2, which is more complete than those used by Singh and Vives [8] and others. I then derive the equilibrium when the market is characterized by a Cournot game. In Section 3, I repeat the same exercise in a Bertrand game.

In Sections 4 and 5, I show that although firms conduct more R&D and therefore have lower post-innovation costs in Cournot competition,[1] they still charge higher prices, produce less, and generate a smaller consumer surplus than those in Bertrand competition. If R&D productivity is low, spillovers are weak, or goods are not close substitutes, the Bertrand equilibrium is shown to be more efficient, in terms of larger total surplus, than the Cournot equilibrium; but the *opposite* holds when R&D productivity is high, spillovers are strong and product differentiation is low.

2. The Basic Model and Cournot Equilibrium

Consider a non-cooperative two-stage game with two firms producing differentiated goods. In the first stage (R&D stage), each firm independently undertakes cost-reducing R&D. In the second stage (market stage), both firms produce and sell their products to the market. I first consider Cournot (quantity) competition in this section. The case of Bertrand (price) competition is relegated to the next section.

Following Singh and Vives [8], I assume that the representative consumer's utility function is $U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2)$, where q_i is the quantity of the good produced by firm i, $\alpha > 0$, and $\gamma \in (0, 1)$. The degree of product differentiation decreases with the parameter γ. The resulting market demands are linear and given by

$$p_i = \alpha - q_i - \gamma q_j, \qquad i, j = 1, 2, \quad i \neq j. \tag{1}$$

The two firms start with the same constant marginal cost $c(< \alpha)$. If only firm i does R&D, then by spending $V(x_i)$ on R&D it can lower its marginal cost by x_i. It is commonly assumed that R&D investment has diminishing returns and that the R&D expenditure function is quadratic (e.g., d'Aspremont and Jacquemin [4]): $V(x_i) = \frac{1}{2}vx_i^2$, where the parameter v relates to the efficiency or productivity of the R&D technology (higher v means lower efficiency). Assume

A1: $v > \alpha/c$.

[1]Brander and Spencer [2] analyze R&D incentives in a Cournot model and Okuno-Fujiwara and Suzumura [5] examine them in a Bertrand model. Bester and Petrakis [1] have also compared R&D incentives in Bertrand and Cournot competition but reached a different conclusion. An explanation of reasons behind the differences will be given in Section 4.

It will be shown below that A1 guarantees positive post-innovation costs of production in both price and quantity competition. A1 is also needed for the second-order and stability conditions. To understand A1, note that if the demand is very strong (i.e., α is large) and the pre-innovation cost of production c is not too high, then the R&D technology should not be too efficient or otherwise the firms will invest a lot in R&D, resulting in zero or even negative post-innovation costs, which is unrealistic.

To model the spillover effects of R&D, I assume that the firms' marginal costs are

$$c_i = c - x_i - \theta x_j, \qquad i, j = 1, 2, \text{ and } i \neq j,$$

should they spend $V(x_1)$ and $V(x_2)$ on R&D, respectively. The parameter $\theta \in [0, 1]$ captures the extent of spillovers.

I use the backward induction approach to derive sub-game perfect equilibria. Let π_i denote firm i's market profit (profit excluding R&D costs). Then given any first-stage R&D outcome (x_1, x_2), $\pi_i(q_1, q_2; x_1, x_2) = p_i q_i - (c - x_i - \theta x_j)q_i$. The firms choose output to maximize their respective market profits and the Cournot-Nash equilibrium is

$$q_i^*(x_1, x_2) = \frac{1}{4-\gamma^2}[(\alpha - c)(2-\gamma) + (2-\theta\gamma)x_i + (2\theta - \gamma)x_j]. \tag{2}$$

I now turn to the R&D stage. Each firm chooses an R&D level to maximize its overall profit, $\Pi_i(x_1, x_2) = \pi_i(q_1^*, q_2^*;\ x_1, x_2) - \frac{1}{2}vx_i^2$. The (symmetric) equilibrium R&D level of each firm is[2]

$$x_C = \frac{2(2-\theta\gamma)(\alpha - c)}{\Delta_C}, \tag{3}$$

where $\Delta_C \equiv v(2+\gamma)(4-\gamma^2) - 2(2-\theta\gamma)(1+\theta)$. It is straightforward to verify the following lemma (the proof is omitted).

Lemma 1. *For all θ and γ, A1 is sufficient but not necessary for $v(4-\gamma^2)^2 - 2(2-\theta\gamma)^2 > 0$ (the second-order condition) and $\Delta_C > 0$ (the stability condition); A1 is both necessary and sufficient for $c - (1+\theta)x_C > 0$ (positive post-innovation costs).*

[2] The second-order and the Routh-Hurwitz stability conditions are, respectively,

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} < 0 \text{ and } \frac{\partial^2 \Pi_1}{\partial x_1^2}\frac{\partial^2 \Pi_2}{\partial x_2^2} - \frac{\partial^2 \Pi_1}{\partial x_2 \partial x_1}\frac{\partial^2 \Pi_2}{\partial x_1 \partial x_2} > 0.$$

Substituting x_C into (1) and (2) gives the (symmetric) equilibrium output and price:

$$q_C = \frac{1}{\Delta_C} v(\alpha - c)(4 - \gamma^2) \qquad \text{and} \qquad p_C = \alpha - (1+\gamma)q_C. \tag{4}$$

Finally, I obtain the equilibrium consumer surplus $[CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2]$, producer profits $(\Pi = \Pi_1 + \Pi_2)$, and welfare (total surplus) in Cournot competition as follows:

$$CS_C = (1+\gamma)q_C^2, \qquad \Pi_C = 2q_C^2 - vx_C^2, \tag{5}$$

$$W_C = CS_C + \Pi_C = \frac{1}{\Delta_C^2} v(\alpha - c)^2 [v(3+\gamma)(4-\gamma^2)^2 - 4(2-\theta\gamma)^2]. \tag{6}$$

3. Bertrand Equilibrium

Suppose now the product market involves Bertrand competition. Rewrite (1) as

$$q_i = \frac{1}{1-\gamma^2}[\alpha(1-\gamma) - p_i + \gamma p_j], \qquad i, j = 1, 2, \ \ i \neq j. \tag{7}$$

Given any first-stage R&D outcome (x_1, x_2), firm i in the market stage chooses p_i to maximize its market profit. The resulting Bertrand-Nash equilibrium is

$$p_i^*(x_1, x_2) = \frac{1}{4-\gamma^2}[(2+\gamma)(\alpha - \alpha\gamma + c) - (2+\theta\gamma)x_i - (2\theta + \gamma)x_j]. \tag{8}$$

In the R&D stage, each firm chooses its R&D level to maximize its overall profit. In the (symmetric) equilibrium, both firms choose

$$x_B = \frac{2}{\Delta_B}(\alpha - c)(2 - \theta\gamma - \gamma^2), \tag{9}$$

where $\Delta_B \equiv v(1+\gamma)(2-\gamma)(4-\gamma^2) - 2(1+\theta)(2-\theta\gamma-\gamma^2)$.

It is now possible to check the following lemma (the proof is straightforward and omitted).

Lemma 2. *For all θ and γ, A1 is sufficient but not necessary for $\Delta_B > 0$ (the stability condition), and it is both necessary and sufficient for $c - (1+\theta)x_B > 0$ (positive post-innovation costs). If $\theta = 1$, the second-order condition for optimal x_B is $v(1+\gamma)(2-\gamma)^2 - 2(1-\gamma) > 0$, which is ensured by A1; but if $\theta \neq 1$, the second-order condition is*

$$v > \frac{2(2-\theta\gamma-\gamma^2)^2}{(1-\gamma^2)(4-\gamma^2)^2}, \tag{10}$$

and A1 is no longer sufficient to ensure (10) *for all* γ *and* θ.

A brief discussion of Lemma 2 regarding the second-order condition is in order. According to (10), if θ is not equal to 1 and γ is very close to one, v must be large in order to satisfy the second-order condition for an interior solution of optimal R&D. When the two goods are close substitutes, competition is fierce as both firms cut prices to seize market shares. The ability of the firms to lower prices, however, is limited by their marginal costs. Suppose v is small, i.e., it is not too costly to do R&D. In this case, a firm is always willing to invest a lot to create a cost advantage over its rival so as to win the price war. That means, v has to be large to avoid a corner solution which results from an extremely large R&D investment. When $\theta = 1$, such a restriction on v is not needed. With perfect spillovers, no pre-competition cost advantages could occur and so this motive of R&D investment goes away. In fact, as γ approaches one, x_B becomes zero [see (9)].

Using (9) in (7) and (8), I obtain the market equilibrium:

$$p_B = \frac{1}{\Delta_B}[v(\alpha - \alpha\gamma + c)(1+\gamma)(4-\gamma^2) - 2\alpha(1+\theta)(2-\theta\gamma-\gamma^2)], \quad q_B = \frac{1}{\Delta_B}v(\alpha - c)(4-\gamma^2). \quad (11)$$

Finally, I calculate the equilibrium consumer surplus, producer profits, and welfare (total surplus) in Bertrand competition:

$$CS_B = (1+\gamma)q_B^2, \qquad \Pi_B = 2(1-\gamma^2)q_B^2 - vx_B^2, \tag{12}$$

$$W_B = CS_B + \Pi_B = \frac{1}{\Delta_B^2}v(\alpha - c)^2[v(3+\gamma-2\gamma^2)(4-\gamma^2)^2 - 4(2-\theta\gamma-\gamma^2)^2]. \tag{13}$$

4. Comparison I: R&D Incentives

Before comparing the two equilibria, I first derive the socially optimal (or the first-best) allocation as a benchmark. The social planner's problem is[3]

$$\max_{x_1,x_2,q_1,q_2} W_S = U(q_1,q_2) - \sum_{i=1}^{2}(c - x_i - \theta x_j)q_i - \frac{1}{2}v\sum_{i=1}^{2}x_i^2.$$

Assume

[3]In general, the degree of cross-firm R&D spillover is jointly determined by many factors including the nature of technologies, the legal framework, and the information control of the firms. I consider the case in which even the social planner cannot affect θ.

A2: $v > 4\alpha/c$.

This assumption is necessary and sufficient for the optimal post-innovation costs to be positive, and is sufficient but not necessary to ensure the second-order condition for the social planner's problem.

It is easily verified that the social planner's optimal decision is symmetric and given by

$$x_i = x_S = \frac{(1+\theta)(\alpha - c)}{(1+\gamma)v - (1+\theta)^2} \quad \text{and} \quad q_i = q_S = \frac{(\alpha - c)v}{(1+\gamma)v - (1+\theta)^2}. \tag{14}$$

Would the firms invest more in R&D when the product market involves Cournot competition than when it involves Bertrand competition? How are these investment levels compared to the first-best levels. What are the comparisons between the Bertrand and Cournot outputs and prices when the pre-competition production costs are endogenously determined by the first-stage R&D efforts? These are the questions that beg for answers. A simple and direct comparison based on (3), (9) and (14) immediately yields the following proposition.

Proposition 1. *For any given $\gamma \in (0,1)$ and $\theta \in [0,1]$, $x_S > x_C > x_B$ under A2 and (10).*

In words, Proposition 1 states that the firms always invest more in R&D if the product market involves Cournot competition than if it involves Bertrand competition, but in both cases they invest less than the social optimum. The key to the understanding of this result is to carefully examine the factors that induce the firms to undertake R&D and their interactions in different competition modes. Even with general demand and R&D cost functions, I can decompose each firm's R&D effect into four parts (see Appendix for derivation). In Cournot competition,

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial x_i} &= [-\frac{1}{\Psi_C}\frac{\partial \pi_i}{\partial q_j}\frac{\partial^2 \pi_i}{\partial q_i^2}\theta] + [\frac{1}{\Psi_C}\frac{\partial \pi_i}{\partial q_j}\frac{\partial^2 \pi_j}{\partial q_i \partial q_j}] + q_i + [-V_i'(x_i)] \\
&= \underbrace{\text{spillover effect}}_{(-)} + \underbrace{\text{strategic effect}}_{(+)} + \underbrace{\text{size effect}}_{(+)} + \underbrace{\text{cost effect}}_{(-)}.
\end{aligned}$$

In Bertrand competition,

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial x_i} &= [-\frac{1}{\Psi_B}\frac{\partial \pi_i}{\partial p_j}\frac{\partial^2 \pi_i}{\partial p_i^2}\frac{\partial q_j}{\partial p_j}\theta] + [\frac{1}{\Psi_B}\frac{\partial \pi_i}{\partial p_j}\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\frac{\partial q_i}{\partial p_i}] + q_i + [-V_i'(x_i)] \\
&= \underbrace{\text{spillover effect}}_{(-)} + \underbrace{\text{strategic effect}}_{(-)} + \underbrace{\text{size effect}}_{(+)} + \underbrace{\text{cost effect}}_{(-)}.
\end{aligned}$$

First, the firm's R&D lowers the unit cost of production. For a given cost reduction, *ceteris paribus*, the more it produces, the more it benefits. Thus, the size effect is always positive (i.e., it gives the firm incentives to do R&D) in both Cournot and Bertrand competition. Second, the firm's R&D also lowers its rival's cost, which in turn is detrimental to the R&D-taking firm. Because of this, the spillover effect is always negative (i.e., it gives the firm disincentives to do R&D) in both Cournot and Bertrand competition. Third, R&D activity is costly, implying that the cost effect is negative in both types of competition. Finally, the firm's R&D lowers its production cost and so affects the rival firm's output or price decision. This is the strategic effect and, unlike the other effects, is positive in Cournot competition but negative in Bertrand competition. In the Cournot case, by doing more R&D and thereby lowering its cost, the firm is tougher in the market and thus discourages its rival's sale, which in turn benefits itself.[4] In contrast, in the Bertrand case, the firm's R&D lowers its cost and induces its rival to cut price, which in turn hurts itself.[5]

In the central planning case, both the spillover and strategic effects are internalized. The spillover effect becomes positive[6] and the strategic effect vanishes. Table 1 summarizes the above discussion.

Table 1. Decomposition of R&D Incentives

	spillover effect	strategic effect	size effect
Central planning	+	nil	+
Cournot	−	+	+
Bertrand	−	−	+

Based on Proposition **??** to be stated below, the size effect is strongest in the central planning case and weakest in the Cournot case. This, together with Table 1, clearly shows that $x_S > x_B$. Other comparisons hinge on the relative importance of the three effects that gives rise to the ranking reported in Proposition 1.[7]

[4]This positive strategic effect leads to over- investment in the absence of spillovers (see Brander and Spencer [2]).

[5]This negative strategic effect leads to under-investment in the absence of spillovers (see Okuno-Fujiwara and Suzumura [5]).

[6]This is because $\partial x_S/\partial \theta > 0$ from (14).

[7]For more detailed discussions on these three effects, see Qiu [6], [7].

It is worth pointing out that while I obtain a unique R&D investment ranking for all possible values of θ and γ, Bester and Petrakis [1], who do not consider spillovers, show that all types of rankings are possible, depending on γ and the degree of cost-reduction. To understand the differences, it suffices to discuss just one of their results, $x_C < x_B$ for a large γ (and of course $\theta = 0$), and compare it with Proposition 1. Recall that Bester and Petrakis allow only one of the two firms to invest in R&D. The innovating firm ends up enjoying of a large cost advantage over its competitor, which results in different market shares of the R&D firm and the non-R&D firm. This asymmetry helps the R&D firm but hurts the non-R&D firm. As a result, the R&D firm's output in this asymmetry case substantially exceeds that in the symmetric case. In other words, the size effect is much stronger in Bester and Petrakis' model (the asymmetric case) than in the present one (the symmetric case), and it may become sufficiently strong to dominate the strategic effect. When the size effect prevails and products are alike, a firm will have incentives to do more R&D in Bertrand competition than in Cournot competition. This follows because a Bertrand firm always produces more (see Proposition **??** below) and thus faces a stronger size effect than a Cournot firm. The R&D game in the present model, in contrast, is symmetric. Both firms conduct R&D and the amounts of cost reduction are optimally chosen by each firm in a non-cooperative manner. If pre-innovation conditions are symmetric between the two firms, the post-innovation equilibrium is also symmetric. Therefore, the size effect is less important than that in Bester and Petrakis.

I now turn to the quantity and price comparison. Directly comparing (4), (5), (11), (12) and (14) establishes

Proposition 2. *For any given $\gamma \in (0,1)$ and $\theta \in [0,1]$, $q_S > q_B > q_C$, $p_B < p_C$ under A2 and* (10). *Consequently, $CS_B > CS_C$ under the same conditions.*

Propositions 1 and 2 together convey a *new message*: Although Cournot firms make more R&D investments and therefore have lower costs than Bertrand firms, the Bertrand firms still charge lower prices and produce more than the Cournot firms. However, greater outputs and lower prices do not ensure larger total surplus as costs of production in Bertrand competition are higher than those in Cournot competition. This warrants a careful scrutiny of the efficiency issue which is the focus of the next section.

5. Comparison II: Welfare and Efficiency

Obviously, welfare is highest in the case of social planning. However, it is not so clear for the Cournot and Bertrand equilibria. In what follows, I first examine the case of perfect spillovers, then the case of no spillovers, and finally the case of $\theta \in (0, 1)$.

For $\theta = 1$, the welfare functions in Cournot competition can be simplified to

$$W_{C1} = \frac{v(\alpha - c)^2}{\Delta_{C1}^2}[v(3+\gamma)(2+\gamma)^2 - 4],$$

where $\Delta_{C1} \equiv v(2+\gamma)^2 - 4$, and in Bertrand competition to

$$W_{B1} = \frac{v(\alpha - c)^2}{\Delta_{B1}^2}[v(1+\gamma)(3-2\gamma)(2-\gamma)^2 - 4(1-\gamma)^2],$$

where $\Delta_{B1} \equiv v(1+\gamma)(2-\gamma)^2 - 4(1-\gamma)$. Note that when v is large, R&D investment will be small in both types of competition. Consider the more interesting case wherein R&D could be large. Succinctly, assume that A1 is not too restrictive for v in the sense that

A3: $v^* \equiv (32 + 4\sqrt{55})/9 > \alpha/c$.

Proposition 3. *Suppose that $\theta = 1$, and both A1 and A3 hold.*

(i) *If $v \geq v^*$, then $W_{B1} > W_{C1}$ $\forall\, \gamma \in (0, 1)$; and*

(ii) *if $v < v^*$, then there exists $\overline{\gamma}(v) \in (0, 1)$ such that*

$$W_{B1} - W_{C1} \begin{cases} > 0 & \forall\ \gamma < \overline{\gamma}(v) \\ = 0 & \text{if } \gamma = \overline{\gamma}(v) \\ < 0 & \forall\ \gamma > \overline{\gamma}(v). \end{cases}$$

Proof. See Appendix.

The main message contained in Proposition 3 is that the Bertrand equilibrium does not always have higher dynamic efficiency than the Cournot equilibrium. This is in contrast to the traditional result. The underlying intuition is as follows. It is well known (Singh and Vives [8]) that, in the absence of R&D investment, consumer surplus is larger in the Bertrand equilibrium, but the Bertrand profits are lower. Given the opportunity to reduce pre-competition production cost, Proposition 1 shows that Cournot firms do more R&D than Bertrand firms, but due to spillovers, even the Cournot firms do not over-invest. Thus, both consumers and producers benefit from R&D and they benefit more in Cournot competition. However, if R&D is very costly [case (i) of Proposition 3], even Cournot firms will only do minimal amounts of R&D. That means, the extra social benefit from R&D in Cournot competition no longer outweighs the static efficiency of Bertrand competition. As a result, the traditional result remains valid. If, however, the R&D cost is moderate [case (ii) of Proposition 3], it is possible that the cost discrepancy between Cournot and Bertrand firms is large enough to generate sufficiently high dynamic efficiency in Cournot competition to dominate the static efficiency of Bertrand competition, especially for closely substitutable goods.

The above discussion also implies that the traditional result should continue to hold in the absence of spillovers. This is stated in Proposition 4 below and proved in the appendix.

Proposition 4. *Suppose that* $\theta = 0$. *Then,* $W_B > W_C$ *for all* γ *so long as A1 and* (10) *are satisfied.*

Proof. See Appendix.

Propositions 3 and 4 indicate that in the efficiency comparison, the degrees of spillover and product differentiation play a very critical role. Our final proposition (Proposition **??** below) concludes that the traditional result holds for small θ or γ, and the opposite result holds for large θ and γ. The explanation for this claim is as follows. Although spillovers adversely affect individual firms' incentives to invest in R&D, they enhance welfare from the social point of view.[8] Since firms do more R&D in Cournot than in Bertrand competition, the welfare increase from a larger spillover is greater in Cournot than in Bertrand competition. Thus, for given v and γ, it is more likely that the Cournot equilibrium has greater efficiency than the Bertrand equilibrium as θ increases, and it is less likely that the Cournot equilibrium gives larger welfare as θ decreases.

Proposition 5. *Suppose that* $\theta \in (0, 1)$, *and both A1 and A3 hold. Also suppose that for any given* $\gamma \in (0, 1)$, *inequality* (10) *holds and in particular* v^* *satisfies* (10). *Then, given* γ, *either*

[8] To see this, simply apply the envelope theorem to W_S.

(i) $W_B > W_C$, $\forall v$ *and* θ, *or*

(ii) *there exists a unique* $\bar{v}(\gamma)(> 1)$ *such that*

(a) $W_B > W_C$, $\forall\, v \geq \bar{v}(\gamma)$ *and* $\theta \in (0,1)$, *and*

(b) *for* $v < \bar{v}(\gamma)$, *there exists* $\overline{\theta}(v) \in (0,1)$ *such that*

$$W_B - W_C \begin{cases} > 0 & \forall\ \theta < \overline{\theta}(v) \\ = 0 & \text{if } \theta = \overline{\theta}(v) \\ < 0 & \forall\ \theta > \overline{\theta}(v). \end{cases}$$

Furthermore, for γ *close to 0, outcome (i) prevails. For* γ *close to 1, outcome (ii) prevails.*

Proof. See Appendix.

6. Concluding Remarks

The main finding of the present study is that, in contrast to the conventional wisdom, the Cournot equilibrium can be more efficient than the Bertrand equilibrium. First, when R&D is not too costly, firms in Cournot competition invest much more in R&D and thus have much lower production costs than they do in Bertrand competition. Second, when spillovers are sufficiently high, the resulting welfare increase from more R&D investment is drastic.

Except in the case of perfect spillovers, the strongest condition imposed for most of the results to hold is condition (10), which ensures interior solutions of optimal R&D investment. It might be of interest to carry out further study on the corner-solution equilibrium once condition (10) is relaxed.

Government intervention in R&D investment, especially for the type of R&D with strong spillovers, is a world-wide common practice . When R&D investment is controlled through either subsidization or taxation, will the welfare superiority of Bertrand competition be reinstated? Elsewhere ([7]), I have shown that in the case of Cournot competition, R&D subsidy is optimal, i.e., subsidy is preferred to laissez faire and tax. Since under-investment is more serious in Bertrand competition than in Cournot competition as evident from Proposition 1, a larger subsidy is required in Bertrand competition. If that is the case, the R&D differential between Cournot and Bertrand competition would be reduced. Recall that a larger R&D investment in Cournot competition is the reason for the welfare of Cournot competition being higher than that of Bertrand competition. Also note that the subsidy per se is simply a transfer from the government to the firms, and so it is cancelled out in the welfare function (it does affect the equilibrium welfare though). Therefore, the possibility that the Cournot equilibrium dominates the Bertrand equilibrium in welfare term would be lower in the presence of government intervention than laissez faire. Whether the Cournot equilibrium can dominate the Bertrand equilibrium with government intervention deserves a closer scrutiny.

Appendix

A. Decomposition of R&D incentives.

Consider general market demand $p_i = p_i(q_1, q_2)$, $\partial p_i/\partial q_j < 0$, and R&D cost $V_i(x_i)$, $V_i'(\cdot) > 0$, $V_i''(\cdot) > 0$. The first-order conditions for Cournot-Nash outputs are,

$$\frac{\partial \pi_i}{\partial q_i} = p_i + q_i \frac{\partial p_i}{\partial q_i} - (c - x_i - \theta x_j) = 0.$$

Assume that both the second-order conditions and the stability condition are satisfied, i.e.,

$$\frac{\partial^2 \pi_i}{\partial q_i^2} \leq 0, \qquad \Psi_C \equiv \frac{\partial^2 \pi_1}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_1}{\partial q_2 \partial q_1}\frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} > 0.$$

Differentiating the first-order conditions with respect to x_i yields

$$\begin{pmatrix} \partial^2 \pi_i/\partial q_i^2 & \partial^2 \pi_i/\partial q_j \partial q_i \\ \partial^2 \pi_j/\partial q_i \partial q_j & \partial^2 \pi_j/\partial q_j^2 \end{pmatrix} \begin{pmatrix} \partial q_i/\partial x_i \\ \partial q_j/\partial x_i \end{pmatrix} = \begin{pmatrix} -1 \\ -\theta \end{pmatrix},$$

from which I obtain

$$\frac{\partial q_j}{\partial x_i} = \frac{1}{\Psi_C}\left(\frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \frac{\partial^2 \pi_i}{\partial q_i^2}\theta\right).$$

Note also $\partial \pi_i/\partial q_j = q_i(\partial p_i/\partial q_j) < 0$ for substitute products, and $\partial \pi_i/\partial x_i = q_i > 0$. Thus,

$$\begin{aligned} \frac{\partial \Pi_i}{\partial x_i} &= \frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j}{\partial x_i} + \frac{\partial \pi_i}{\partial x_i} - V_i'(x_i) \\ &= \left[-\frac{1}{\Psi_C}\frac{\partial \pi_i}{\partial q_j}\frac{\partial^2 \pi_i}{\partial q_i^2}\theta\right] + \left[\frac{1}{\Psi_C}\frac{\partial \pi_i}{\partial q_j}\frac{\partial^2 \pi_j}{\partial q_i \partial q_j}\right] + q_i + [-V_i'(x_i)] \\ &= \underbrace{\text{spillover effect}}_{(-)} + \underbrace{\text{strategic effect}}_{(+)} + \underbrace{\text{size effect}}_{(+)} + \underbrace{\text{cost effect}}_{(-)}. \end{aligned}$$

Derivation of the decomposition for Bertrand competition is similar and hence I omit it. □

B. Proof of Proposition 3.

$$W_{B1} - W_{C1} = \frac{v(\alpha - c)^2}{\Delta_{B1}^2 \Delta_{C1}^2} F(v;\gamma), \quad \text{where}$$

$$F(v;\gamma) \equiv [v(3+\gamma-2\gamma^2)(2-\gamma)^2 - 4(1-\gamma)^2]\Delta_{C1}^2 - [v(3+\gamma)(2+\gamma)^2 - 4]\Delta_{B1}^2.$$

Define $G(v;\gamma) = F(v;\gamma)/(v\gamma^2)$. It follows that $\text{sign}(W_{B1} - W_{C1}) = \text{sign}(G)$.

Simplification and term collection yield $G(v;\gamma) = 16g_1 - 8g_2 v + g_3 v^2$, where $g_1 = (1+\gamma)(4-2\gamma-\gamma^2) > 0$, $g_2 = 16+8\gamma-8\gamma^2-2\gamma^3+\gamma^4+\gamma^5 > 0$ and $g_3 = (1+\gamma)(4-\gamma^2)^2(4-2\gamma-\gamma^2) > 0$. Since the second derivative is positive ($G_{vv} = 2g_3 > 0$), G is strictly convex in v. Moreover, $g_2^2 - g_1 g_3 > 0$ for all $\gamma \in (0,\ 1)$. Thus, given any γ, there exist two real solutions to $G = 0$, which are

$$v_1^*(\gamma) = \frac{4}{g_3}\left(g_2 - \sqrt{g_2^2 - g_1 g_3}\right) \quad \text{and} \quad v_2^*(\gamma) = \frac{4}{g_3}\left(g_2 + \sqrt{g_2^2 - g_1 g_3}\right).$$

With the help of Mathematica (Wolfram [9]), it can be shown that $v_1^*(\cdot)$ is a decreasing function with $v_1^*(1) = (32 - 4\sqrt{55})/9$, and $v_2^*(\cdot)$ is an increasing function with $v_2^*(1) = (32 + 4\sqrt{55})/9$. Moreover, $v_1^*(0) = v_2^*(0) = 1$. The above analysis leads to Figure 1. By making use of the figure with the focus on $v_2^*(\cdot)$ as required by A1 and noting $G(v_2^*(\overline{\gamma}); \overline{\gamma}) = 0$, the rest of the proof is straightforward. □

Figure 1 is about here

C. Proof of Proposition 4.

Part I. Although I only need to consider $\theta = 0$ in this proof, it proves useful to examine $\theta \in [0, 1)$ now. Following the proof of Proposition 3 and based on (6) and (13), I calculate

$$W_B - W_C = \frac{v(\alpha - c)^2}{\Delta_B^2 \Delta_C^2} F(v;\gamma,\theta), \quad \text{where}$$

$$F(v;\gamma,\theta) \equiv [v(3+\gamma-2\gamma^2)(4-\gamma^2)^2 - 4(2-\theta\gamma-\gamma^2)^2]\Delta_C^2 - [v(3+\gamma)(4-\gamma^2)^2 - 4(2-\theta\gamma)^2]\Delta_B^2.$$

Define $G(v;\gamma,\theta) = F(v;\gamma,\theta)/[v\gamma^2(4-\gamma^2)]$. It follows that $\text{sign}(W_B - W_C) = \text{sign}(G)$. After collecting terms, I obtain

$$G(v;\gamma,\theta) = 4(1+\theta)^2 g_1 - 4(1+\theta) g_2 v + g_3 v^2, \quad \text{where}$$

$$g_1 = 16 + 8\theta\gamma - 8(2 - \theta + \theta^2)\gamma^2 - 2\theta(1 + 2\theta)\gamma^3 + (3 - 2\theta + 2\theta^2)\gamma^4 + \gamma^5,$$

$$g_2 = 64 + 16(1 + \theta)\gamma - 16(5 - 2\theta)\gamma^2 - 16\gamma^3 + 4(7 - 4\theta)\gamma^4 + (7 - \theta)\gamma^5 - (3 - 2\theta)\gamma^6 - \gamma^7,$$

$$g_3 = 256 + 128\gamma - 384\gamma^2 - 160\gamma^3 + 192\gamma^4 + 72\gamma^5 - 40\gamma^6 - 14\gamma^7 + 3\gamma^8 + \gamma^9 > 0.$$

Since $G_{vv} > 0$, G is strictly convex in v. Note, $g_2^2 - g_1 g_3 > 0$ for all $\gamma \in (0,\ 1)$ and $\theta \in [0,\ 1)$. Thus, given any γ and θ, there exist two real solutions to $G = 0$. As in the proof of Proposition 3, I only need to consider the high-value solution, i.e.,

$$v_2^*(\gamma, \theta) = 2(1 + \theta)(g_2 + \sqrt{g_2^2 - g_1 g_3})/g_3.$$

Part II. Now let's concentrate on $\theta = 0$. It can be shown that $v_2^*(\gamma, 0)$ is increasing in γ and $v_2^*(1, 0) = 2/3$. Thus, if we draw a family of $G(v; \gamma, 0)$, they will be similar to those in Figure 1 except that $B = 2/3 < 1$. Since $v > 1$ by A1, $G > 0$ for all γ. The result follows. □

D. Proof of Proposition 5.

Continue from Part I of Proposition 4's proof. First, g_3 is not a function of θ, $\partial g_2/\partial\theta = \gamma(16+32\gamma-16\gamma^3-\gamma^4+2\gamma^5) > 0$, and $\partial(g_2^2 - g_1 g_3)/\partial\theta = 2\gamma^2(4-\gamma^2)^2[48(1+3\theta)+32(1+6\theta)\gamma - 56(1 + \theta)\gamma^2 - 32(1 + 4\theta)\gamma^3 + (19 + \theta)\gamma^4 + 2(5 + 14\theta)\gamma^5 - (1 - 2\theta)\gamma^6 - (1 + 2\theta)\gamma^7] > 0$. Thus, for all $\gamma \in (0, 1)$, $\partial v_2^*(\gamma, \theta)/\partial\theta > 0$. This property allows us to draw a family of G on various θs (for a given γ), as shown in Figure 2.

Second, calculate $v_2^*(\gamma, 1)$ for the given γ. Either $v = v_2^*(\gamma, 1)$ satisfies both A1 and (10) or it violates at least one of the conditions. In the latter case, all $v = v_2^*(\gamma, \theta)$ will violate at least A1 or (10). It follows that for v satisfying A1 and (10), $G > 0$, $\forall\theta \in (0, 1)$ (using Figure 2 and the argument similar to that used in the proof of Proposition 4). In the former case, using Figure 2 and the arguments similar to those used in the proof of Proposition 3, we know that outcome (ii) is the result.

Figure 2 is about here

Third, consider the case of large γ. Recall $v_2^*(1, 1) = v^*$ and $\partial v_2^*(\gamma, 1)/\partial\gamma > 0$. The continuity property of $v_2^*(\gamma, \theta)$ ensures the following: for any given $\epsilon > 0$, $v^* - \epsilon < v_2^*(\gamma, \theta) < v^*$ as long as both γ and θ are sufficiently close to 1. Thus, for large γ, $v_2^*(\gamma, 1)$ satisfies both A1 and (10), and hence outcome (ii) obtains.

Finally, consider the case of small γ. Recall $v_2^*(0, 1) = 1$. Thus, given any small $\epsilon > 0$, there exists γ_ϵ such that $v_2^*(\gamma, 1) < 1 + \epsilon$, $\forall \gamma \leq \gamma_\epsilon$. As A1 implies $v > 1$, by choosing ϵ appropriately, $v = v_2^*(\gamma, 1)$ will violate A1 and thus outcome (i) obtains. $\square$

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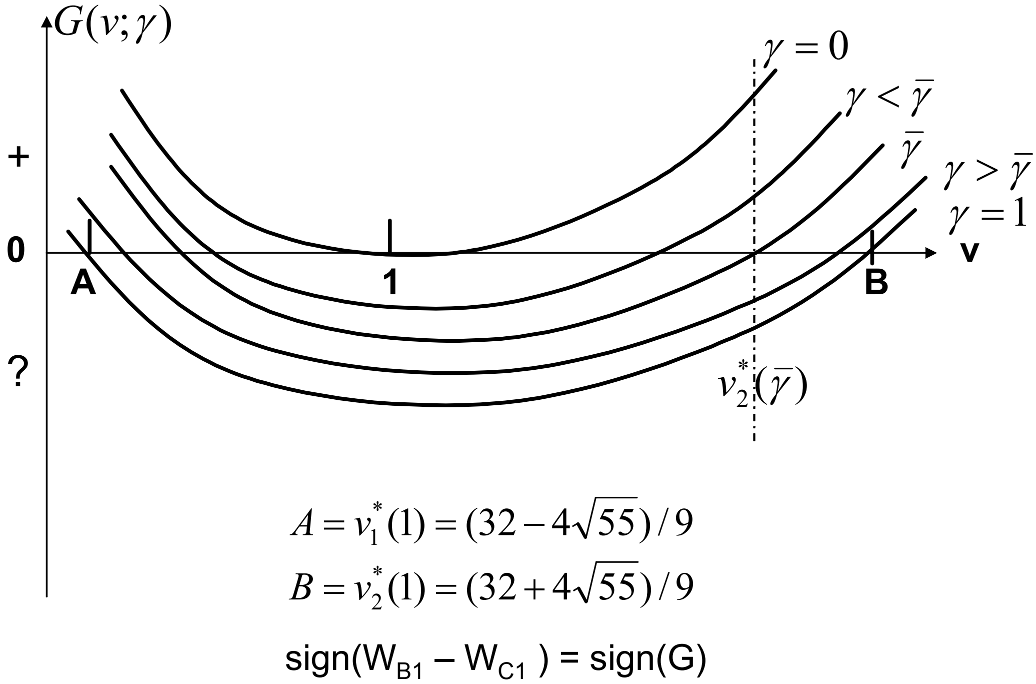
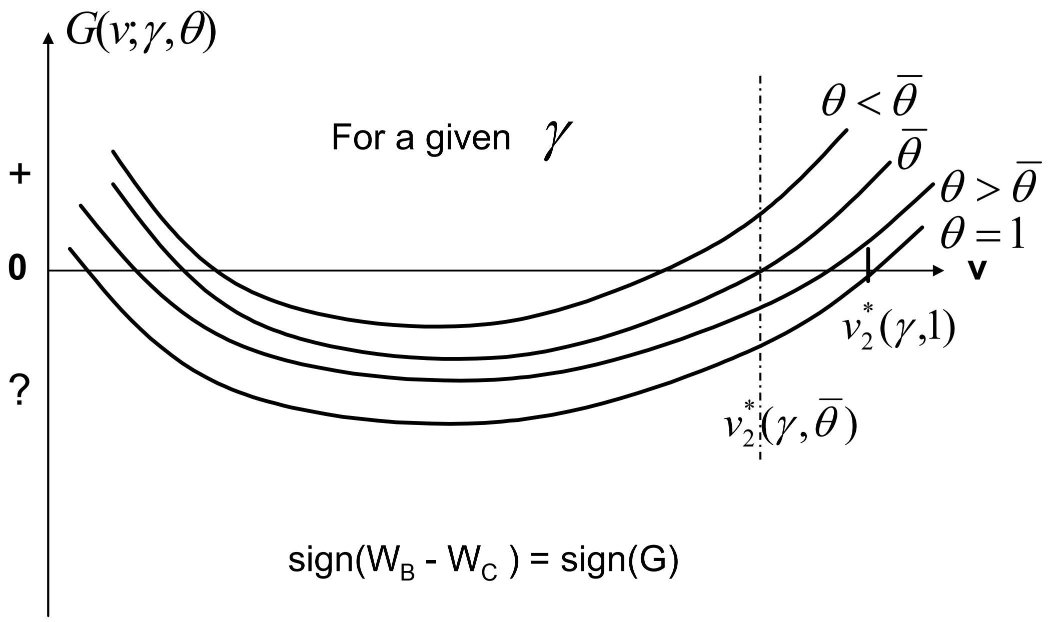

$G(v;\gamma)$
$\gamma = 0$
$\gamma < \bar{\gamma}$
$\bar{\gamma}$
$\gamma > \bar{\gamma}$
$\gamma = 1$
+
0
?
A
1
B
v
$v_2^*(\bar{\gamma})$
$A = v_1^*(1) = (32 - 4\sqrt{55})/9$
$B = v_2^*(1) = (32 + 4\sqrt{55})/9$
$\text{sign}(W_{B1} - W_{C1}) = \text{sign}(G)$

Figure 1:



$G(v;\gamma,\theta)$
For a given γ
$\theta < \bar{\theta}$
$\bar{\theta}$
$\theta > \bar{\theta}$
$\theta = 1$
+
0
?
v
$v_2^*(\gamma,1)$
$v_2^*(\gamma,\bar{\theta})$
sign(W_B - W_C) = sign(G)

Figure 2: