

# Policy on international R&D cooperation: Subsidy or tax?

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## Abstract

In this paper we derive the non-cooperative, optimal policy towards international R&D cooperation. Two types of R&D cooperation are considered: collaboration and coordination. When firms cooperate, the familiar strategic behavior, which prevails in R&D competition, is reduced, or eliminated, or even completely reversed. However, we prove that R&D subsidy is still an optimal policy for individual governments in the case of R&D coordination and, more strikingly, the subsidies are larger for higher degrees of coordination. Government policies do not help the firms to commit. In the case of R&D collaboration, both R&D subsidy and tax are possible. With linear demands, however, tax is never optimal; moreover, we show that the optimal policy is subsidy regardless of the strategic nature (substitute or complement) of the strategy variables, a result that contradicts the traditional wisdom. © 1998 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The number of agreements on international research and development (R&D) cooperation has been increasing at an unprecedented rate. A survey article by

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Chesnais (1988) contains much information on various forms of R&D cooperation in a wide range of industries. For example, it reports from the FOR (Italy) data that between 1982 and 1985 there were 1061 inter-firm agreements in high technology industries, where the product markets are characterized by imperfect competition. Among those agreements, a large portion were for R&D cooperation, and more than half were for international cooperation.<sup>1</sup> Government support for both domestic and international R&D cooperation, including public investment, subsidy, and antitrust law modification, has also become more frequent.<sup>2</sup>

It is not only evident that governments often assist their domestic producers via trade and industrial policies when the latter are competing with foreign rivals in markets characterized by imperfect competition, but also clear that theories have provided justification for such policy interventions. Export subsidies can be used to shift rents strategically between rival firms;<sup>3</sup> however, such outright subsidies on exports are strictly forbidden by the World Trade Organization (WTO). In contrast, subsidizing domestic R&D is allowed by the WTO<sup>4</sup> and, as demonstrated by Spencer and Brander (1983), via an R&D policy a government can achieve the same strategic outcomes otherwise obtained under direct export subsidies. While Spencer and Brander, and all subsequent studies, have provided useful insights on subsidies to domestic firms participating in *international R&D competition*, it remains unclear what would be a 'good' policy towards *international R&D cooperation*. The present study addresses this issue and is motivated by the following two questions. First, should a government subsidize the domestic firm's R&D activity when the firm cooperates with its foreign rival? The answer to this question would test the robustness of the Spencer–Brander-type strategic R&D subsidy to R&D cooperation. The second question concerns the magnitude of subsidies: if a subsidy is optimal, then is it larger for cooperative R&D or for noncooperative R&D?

Our analysis is built upon two existing sets of literature. One is the strategic trade literature, which focuses on how governments can alter the market games played by firms of various countries in favor of their domestic firms through

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<sup>1</sup> See also Murphy (1991) for a list of international R&D cooperation involving at least one American firm. Even rival firms competing in the same final product markets may cooperate in R&D investment. For example, IBM and Toshiba formed a research joint venture, named Display Technologies Inc., to develop liquid crystal color displays, which will be used to produce their final products (see 'Such good friends with IBM', *Fortune* Oct. 4, 1993.)

<sup>2</sup> It was estimated that Airbus benefited from public subsidies totalling US\$2.5 billion during 1968–1982 (Krugman, 1984).

<sup>3</sup> See Brander (1995) for the most recent survey of the strategic trade literature.

<sup>4</sup> In the most recent GATT talk, the Uruguay Round, the scope for R&D subsidies was substantially widened (Cline, 1995).

trade and/or R&D policies. The other is the cooperative R&D literature, which justifies cooperative R&D in a variety of settings and investigates the effects of this cooperation. In terms of analysis, the former literature has a dominant model of Spencer and Brander (1983), while the latter has at least two popular models of cooperative R&D.

To focus on the effect of international R&D cooperation on strategic government policy, we choose a model that differs from Spencer and Brander only in the assumption about R&D competition and cooperation. In particular, as in Spencer and Brander, we consider deterministic cost-reducing R&D; but unlike Spencer and Brander, who consider only competition of firms in R&D, we consider two possible types of R&D cooperation between a domestic firm and a foreign firm. In the first type of R&D cooperation, the two firms share the benefit of their R&D investments (see, Katz, 1986; Veugelers and Kesteloot, 1994). Specifically, each firm's marginal cost of production depends on both its own and its counterpart's R&D investments.<sup>5</sup> In the second type of R&D cooperation, the two firms coordinate to reduce R&D overinvestment (see, d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; Suzumura, 1992). Specifically, each firm chooses its R&D investment to maximize a weighted sum of both its own and its counterpart's profits. To distinguish these two types of cooperation, we refer to the first type as R&D collaboration and the second type as R&D coordination.

We are already familiar with the rationale for a government R&D subsidy when the domestic firm and the foreign firm make the cost-reducing R&D investment in a competing fashion. Each firm has two motives for R&D investments: (1) to raise its profit directly by lowering the cost of production (profit motive), and (2) to raise its profit indirectly by discouraging its rival's production (strategic motive). This latter motive leads to R&D overinvestment. An R&D subsidy by the domestic government enhances the firm's cost advantage through discouraging the foreign rival's R&D investment, which in turn enables the domestic firm to snatch a larger market share at the expense of the foreign firm.

If the domestic and foreign firms engage in R&D collaboration (i.e., the first type of R&D cooperation), each firm still has two (profit and strategic) motives for R&D investment. However, the strategic motive could be either positive or negative depending on the degree of R&D collaboration. Consider the case of full collaboration, namely, each firm's R&D investment reduces by the same

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<sup>5</sup> International R&D spillover is the focus of Coe and Helpman (1995). They show that a country's total factor productivity depends not only on the domestic R&D stock but also on its trading partners' R&D stock. R&D spillover could be a natural and inevitable result of economic interaction or it could also be a chosen outcome by various parties that conduct R&D. By spillover in this study, we refer to the latter type.

amount its own and its counterpart's marginal costs of production. In this case, by committing to a higher R&D investment, a firm cannot gain a cost advantage over its counterpart. As a result, no market share will be shifted by any government R&D subsidy. This seems to indicate that there will be no role for government R&D subsidy to shift profits in the firms' interest. However, we show that, under reasonable conditions, the governments still have the unilateral incentives to subsidize their firms' R&D. Furthermore, we show that, in a specific version of the model, the government R&D subsidies first decrease and then increase with respect to the degree of international R&D collaboration.

If the domestic and foreign firms engage in R&D coordination (i.e., the second type of R&D cooperation), each firm has a new motive for R&D investment. Specifically, coordinating firms are not totally selfish in the sense that they care not only for their individual profits but also for the counterparts' profits. Thus, each firm has the incentive to reduce its R&D investment so as to mitigate the negative effect on its counterpart's profit (the coordination motive).<sup>6</sup> This works in the opposite direction of the strategic motive. Hence, depending on the degree of R&D coordination, either the strategic motive or the coordination motive could dominate, leading to, respectively, R&D overinvestment or underinvestment. In the case of full coordination, the firms' strategic motive is dominated by their coordination motive, and firms have incentive to underinvest in R&D. This seems to indicate that there will be no role for government R&D subsidy to shift profits in the firms' interest. Surprisingly, we find that individual governments still have unilateral incentives to subsidize their firms' R&D. In a specific version of the model, we further show that the government R&D subsidies are increasing with respect to the degree of international R&D coordination.

While unilateral R&D subsidies may still be optimal for international R&D cooperation, the rationales are quite different from that of the existing strategic trade literature. In the absence of R&D cooperation, the foreign firm's R&D investment adversely affects the domestic firm's profit, but an R&D subsidy to the domestic firm discourages the foreign firm's R&D investment. Together, these two effects provide the rationale for government R&D subsidies. On the contrary, in the case of international R&D collaboration, thanks to the induced R&D spillovers, the foreign firm's R&D investment may positively affect the domestic firm's profit, while a domestic R&D subsidy may encourage the foreign firm's R&D activity. Nevertheless, these effects also call for R&D subsidies.

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<sup>6</sup> Overinvestment is harmful to producers, but not to consumers. This type of coordination could be viewed and is often criticized as collusion (see, Brodley, 1990; Jorde and Teece, 1990; Shapiro and Willig, 1990).

In the case of international R&D coordination, the effects are similar to the R&D competition case in that an R&D subsidy lowers the foreign firm's R&D investment, which in turn raises the domestic firm's profit. But underinvestment occurs, in the sense that each firm undertakes less R&D investment than that would maximize its profit. This leaves room for additional R&D subsidies.

In addition to the above-discussed different rationales for government policies, from the analysis and results of the present study, we identify two departures from the existing strategic trade literature.<sup>7</sup> First, the literature has stressed that the role of a government's strategic policy is to help its firm to commit to certain action (e.g., larger output or higher price). In the case of R&D coordination, however, we show that the R&D subsidy drives the subsidized firm's action to the opposite direction of its commitment (i.e., underinvestment). The second departure is about the relationship between the type of strategic policy and the strategic nature of the strategy variables. It is generally concluded that the optimal policy is a subsidy (a tax) for strategic substitutes (complements). In contrast, we, in the case of R&D collaboration, are able to derive the result that an R&D subsidy is optimal even when R&D investments are strategic complements.

The plan of the paper is as follows. Section 2 focuses on R&D collaboration, while Section 3 focuses on R&D coordination. In each case, we analyze the interaction between firms, that between governments, and that between governments and firms. In particular, we analyze the firms' incentive to make R&D investments and the governments' R&D policies. The relation between the government R&D policies and the degree of international R&D collaboration/coordination is addressed in the specific versions of the models. The paper concludes in Section 4.

## 2. R&D collaboration

### 2.1. *The model*

The model is similar to Spencer and Brander (1983) except that we consider R&D collaboration. There are two firms located in two different countries. The firms produce homogeneous products and compete in a third country's market by setting quantity (Cournot competition). Demand for the product follows a general inverse demand function:

$$p = p(q_1 + q_2), \quad p' < 0,$$

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<sup>7</sup>The following two points can be found in Brander and Spencer (1985) and Eaton and Grossman (1986).

where  $q_1$  and  $q_2$  are firms 1 and 2's outputs, respectively, and  $p$  is the product price.

Before making output decisions, the firms invest in R&D to lower their respective production costs. Moreover, the firms may engage in R&D collaboration. Specifically, the firms share the benefits of their R&D investments in that firm  $i$ 's marginal cost of production depends on its own R&D investment ( $x_i$ ) and firm  $j$ 's investment ( $x_j$ ), or,

$$MC_i = C - c(x_i) - \lambda_R c(x_j),$$

where  $C$  is large enough such that  $MC_i$  is positive,  $c$  is increasing and concave, and  $\lambda_R \in [0, 1]$  capturing all possible degrees of R&D collaboration. Note that the two firms have identical technology to begin with and are equally efficient in R&D activities.<sup>8</sup>

Finally, the government in each country has an R&D policy toward its firm's R&D activity. As in the current strategic trade literature, we consider R&D tax or subsidy proportional to the firm's R&D expenditure.

Firms collaborate in R&D by setting, through negotiation, the spillover parameter  $\lambda_R$ . Although how  $\lambda_R$  is set is an important issue, it is not in the interest of this paper. In this study,  $\lambda_R$  is exogenously given, allowing us to focus on our central question, i.e., how government policy responds to R&D collaboration.

## 2.2. A three-stage game

We describe the interaction between the firms and governments using a non-cooperative, three-stage game. In the first stage, the governments simultaneously announce their R&D subsidy (tax if negative) rates,  $s_1$  and  $s_2$ . In the second stage, each firm chooses its R&D level, knowing its spillovers to the other firm. Finally, in the third stage, the firms compete in the product market by setting quantities. To obtain a subgame perfect equilibrium, this multi-stage game is solved by backward induction.

Given  $(x, s)$ , where  $x = (x_1, x_2)$  and  $s = (s_1, s_2)$ , firm  $i$  in the last stage chooses  $q_i$  to maximize its profit:

$$\pi_i = [p(q_i + q_j) - MC_i] q_i - x_i (1 - s_i).$$

The following two first-order conditions,

$$\frac{\partial \pi_i}{\partial q_i} = p' q_i + p - MC_i = 0, \quad i = 1, 2, \tag{1}$$

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<sup>8</sup> When identical, the two firms have the most to gain from R&D collaboration, as neither can beat the other without suffering badly. In addition, this assumption makes our analysis much simpler.

determine the Cournot–Nash equilibrium output and profits, denoted by

$$q_1^*(x, \lambda_R), \quad q_2^*(x, \lambda_R), \quad \pi_1^*(x, s_1, \lambda_R), \quad \pi_2^*(x, s_2, \lambda_R),$$

and the welfare, which is defined as the firm’s profit net of subsidy payment and denoted by

$$W_1^*(x, s_1, \lambda_R) = \pi_1^*(x, s_1, \lambda_R) - s_1 x_1,$$

$$W_2^*(x, s_2, \lambda_R) = \pi_2^*(x, s_2, \lambda_R) - s_2 x_2.$$

Let  $a_{ij} \equiv \partial^2 \pi_i / \partial q_j \partial q_i$ . Then  $a_{ii} < 0$  by the second-order conditions to the above profit maximization, and  $a_{ij} < 0$  for  $i \neq j$  because of strategic substitution. Moreover, we impose the stability condition  $\Delta_1 \equiv a_{11} a_{22} - a_{12} a_{21} > 0$ .

In the second stage, given  $s$ , firm  $i$  chooses  $x_i$  to maximize its profit:

$$\pi_i^*(x, s_i, \lambda_R) = [p(q_i^* + q_j^*) - (C - c(x_i) - \lambda_R c(x_j))] q_i^* - x_i(1 - s_i).$$

The following two first-order conditions,

$$\frac{\partial \pi_i^*}{\partial x_i} = \left[ p'(q_i^* + q_j^*) \frac{\partial q_j^*}{\partial x_i} + c'(x_i) \right] q_i^* - (1 - s_i) = 0,$$

$$i, j = 1, 2 \text{ and } i \neq j, \tag{2}$$

jointly determine the firms’ optimal R&D investments and the resulting profits, denoted by

$$x_i^*(s, \lambda_R), \quad \pi_i^{**}(s, \lambda_R).$$

Let  $b_{ij} \equiv \partial^2 \pi_i^* / \partial x_j \partial x_i$ . Assume that both the second-order and the stability conditions are satisfied, i.e.,

$$b_{ii} < 0 \quad \text{and} \quad \Delta_2 \equiv b_{11} b_{22} - b_{12} b_{21} > 0. \tag{3}$$

The sign of  $b_{ij}$  is indeterminate. By definition,  $x_1^*$  and  $x_2^*$  are strategic substitutes if  $b_{ij} < 0$ , and strategic complements if  $b_{ij} > 0$ .

Finally (in the first stage), each government selects a subsidy rate (unrestricted in sign), knowing its effects on the subsequent two stages of the game, to maximize its country’s welfare:

$$W_i^{**}(s, \lambda_R) = \pi_i^{**}(s, \lambda_R) - s_i x_i^*(s, \lambda_R).$$

Let  $s_i^*(\lambda_R)$  denote the optimal subsidy rates, which are simultaneously determined by the following first-order conditions:

$$\frac{\partial W_i^{**}}{\partial s_i} = \frac{\partial \pi_i^{**}}{\partial s_i} - x_i^* - s_i \frac{\partial x_i^*}{\partial s_i} = 0, \quad i = 1, 2. \tag{4}$$

As with the other two stages' optimization problems, we assume the satisfaction of the second-order and stability conditions:

$$e_{ii} < 0, \quad \Delta_3 \equiv e_{11}e_{22} - e_{12}e_{21} > 0,$$

where  $e_{ij} \equiv \partial^2 W_i^{**} / \partial s_i \partial s_j$  and  $i, j = 1, 2$ .

### 2.3. Comparative statics results

Having described the three-stage game, we now examine how government R&D policy affects R&D activities and how they, in turn, affect the market outcomes. We look at the latter effect first. Partially differentiating Eq. (1) with respect to  $x_1$  yields

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \partial q_1^* / \partial x_1 \\ \partial q_2^* / \partial x_1 \end{pmatrix} = \begin{pmatrix} -c'(x_1) \\ -\lambda_R c'(x_1) \end{pmatrix}.$$

By differentiating Eq. (1) with respect to  $x_2$  we can also obtain a similar equation system. Solving these two equation systems, we get

$$\frac{\partial q_i^*}{\partial x_i} = \frac{1}{\Delta_1} (-a_{jj} + \lambda_R a_{ij}) c'(x_i) \quad \text{and} \quad \frac{\partial q_i^*}{\partial x_j} = \frac{1}{\Delta_1} (a_{ij} - \lambda_R a_{jj}) c'(x_j). \quad (5)$$

For  $\lambda_R = 0$ , we have  $\partial q_i^* / \partial x_i = -a_{jj} c'(x_i) / \Delta_1 > 0$  and  $\partial q_i^* / \partial x_j = a_{ij} c'(x_j) / \Delta_1 < 0$ . However, for  $\lambda_R = 1$ , we have  $\partial q_i^* / \partial x_i = (a_{ij} - a_{jj}) c'(x_i) / \Delta_1$  and  $\partial q_i^* / \partial x_j = (a_{ij} - a_{jj}) c'(x_j) / \Delta_1$ , both of which are positive under the symmetric setting (namely,  $a_{11} = a_{22}$ ,  $a_{12} = a_{21}$ , and  $\Delta_1 > 0$  implies  $a_{ij} - a_{jj} > 0$ ). Intuitively, as  $x_i$  increases, both firms' marginal costs are reduced. For  $\lambda_R < 1$ , the reduction of firm  $i$ 's marginal cost is greater than that of firm  $j$ 's; however, the difference is smaller when  $\lambda_R$  gets larger. Lowering one's marginal cost tends to raise one's output and lower the rival's output. Thus, with a small  $\lambda_R$  one's own effect dominates the spillover effect and so an increase in  $x_i$  leads to higher  $q_i^*$  and lower  $q_j^*$ . When  $\lambda_R$  is large, the spillover effect becomes strong and so both firms' marginal costs become so low that both  $q_i^*$  and  $q_j^*$  are higher.

Next, we examine the policy implications for R&D activities. Differentiating Eq. (2) with respect to  $s_1$  gives

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} \partial x_1^* / \partial s_1 \\ \partial x_2^* / \partial s_1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}.$$

Similar equations can be obtained by differentiating Eq. (2) with respect to  $s_2$ . Solving these two equation systems, we get

$$\frac{\partial x_i^*}{\partial s_i} = -\frac{b_{jj}}{\Delta_2} > 0 \quad \text{and} \quad \frac{\partial x_j^*}{\partial s_i} = -\frac{b_{ji}}{\Delta_2}. \quad (6)$$

It is clear that the sign of  $\partial x_j^*/\partial s_i$  is the same as that of  $b_{ji}$ . The intuition is as follows. First,  $s_i$  always raises  $x_i$ . Second, an increase in  $x_i$  lowers  $x_j$  for strategic substitutes and raises  $x_j$  for strategic complements. Thus, whether  $s_i$  raises or reduces  $x_j$  completely depends on the strategic nature of  $x_i$  and  $x_j$  (captured by  $b_{ji}$ ).

In summary, while government  $i$ 's R&D subsidy raises its firm's R&D investment, it may not reduce the rival firm's R&D investment. Moreover, by investing in R&D, firm  $i$  may not be able to restrict its rival firm's output.

#### 2.4. R&D incentives

It is established in the strategic trade literature that firms have strategic motive to overinvest in R&D, and, consequently, governments subsidize their firms to enhance further their strategic motive. Thus, in order to analyze government R&D policy in the presence of R&D collaboration, we first examine the firms' incentives for R&D investment. Recall that  $\partial \pi_i/\partial q_i = 0$  by Eq. (1) and so

$$\frac{\partial \pi_i^*}{\partial x_i} = \underbrace{\frac{\partial \pi_i}{\partial x_i}}_{\text{profit motive}} + \underbrace{\frac{\partial \pi_i \partial q_j^*}{\partial q_j \partial x_i}}_{\text{strategic motive}}. \quad (7)$$

Thus, a firm has both the profit and strategic motives for R&D investment. The former comes from the fact that R&D investment allows the firm to raise its profit by lowering the production cost. The latter is that the firm's R&D investment indirectly affects its profit through its influence on its rival's output. In the existing strategic trade literature, it has been known that the strategic motive is always positive and because of that, overinvestment occurs.<sup>9</sup> However, in the presence of R&D collaboration, the strategic motive could be either positive or negative. Specifically, the sign of the strategic motive is the same as that of  $\partial q_j^*/\partial x_i$ , which depends on  $\lambda_R$  as shown in Section 2.3. When  $\lambda_R$  is small,  $\partial q_j^*/\partial x_i$  is negative and firm  $i$  overinvests in R&D; when  $\lambda_R$  is large,  $\partial q_j^*/\partial x_i$  is positive and firm  $i$  underinvests in R&D.

#### 2.5. R&D policy

We have just shown that, in the presence of R&D collaboration, firms may no longer have the strategic motive to overinvest in R&D. The ensuing question is: would the governments still have the unilateral interests to subsidize their firms' R&D when the latter engage in R&D collaboration?

<sup>9</sup> See Spencer and Brander (1983) for more detailed discussions.

Recall that the equilibrium R&D subsidies (or taxes) are determined by Eq. (4). Note that

$$\frac{\partial \pi_i^{**}}{\partial s_i} = \frac{\partial \pi_i^*}{\partial x_i} \frac{\partial x_i^*}{\partial s_i} + \frac{\partial \pi_i^*}{\partial x_j} \frac{\partial x_j^*}{\partial s_i} + \frac{\partial \pi_i^*}{\partial s_i}, \tag{8}$$

where the first term on the right-hand side is zero by Eq. (2) and the third term is  $x_i^*$  by the envelope theorem. Substituting Eq. (8) into Eq. (4), we have

$$s_i^* \frac{\partial x_i^*}{\partial s_i} = \frac{\partial \pi_i^*}{\partial x_j} \frac{\partial x_j^*}{\partial s_i}. \tag{9}$$

Eqs. (6) and (9) together imply that the sign of  $s_i^*$  coincides with that of  $(\partial \pi_i^* / \partial x_j) (\partial x_j^* / \partial s_i)$ . In the absence of international R&D collaboration, higher  $s_i$  leads to lower  $x_j^*$  (i.e.,  $\partial x_j^* / \partial s_i < 0$ ) and higher  $x_j$  leads to lower  $\pi_i^*$  (i.e.,  $\partial \pi_i^* / \partial x_j < 0$ ), altogether implying  $s_i^* > 0$ . However, in the presence of R&D collaboration, the signs of  $\partial x_j^* / \partial s_i$  and  $\partial \pi_i^* / \partial x_j$  depend on  $\lambda_R$  and other factors.

From  $\pi_i^*(x, s_i, \lambda_R)$  we have

$$\frac{\partial \pi_i^*}{\partial x_j} = \frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial x_j} + \frac{\partial \pi_i}{\partial x_j} = \left[ p' \frac{\partial q_j^*}{\partial x_j} + \lambda_R c'(x_j) \right] q_i^*. \tag{10}$$

$x_j$  affects  $\pi_i^*$  negatively, as it decreases the market price through inducing higher  $q_j^*$ . The first term in the square bracket of Eq. (10) captures this effect. However, in the presence of R&D collaboration,  $x_j$  could also affect  $\pi_i^*$  positively through indirectly reducing firm  $i$ 's marginal cost. The second term captures this effect. Depending on the relative magnitude of these two terms,  $\partial \pi_i^* / \partial x_j$  could be either positive or negative.

On the other hand, the sign of  $\partial x_j^* / \partial s_i$  is determined by  $b_{ji}$ . When  $\lambda_R$  is small,  $x_1$  and  $x_2$  are strategic substitutes and  $b_{ji}$  is negative; when  $\lambda_R$  is large,  $x_1$  and  $x_2$  are strategic complements and  $b_{ji}$  is positive. Overall, we have:

*Proposition 1. In the case of international R&D collaboration, (i) when  $\lambda_R$  is small enough, governments subsidize R&D investments; (ii) when  $\lambda_R$  is large enough, governments may or may not subsidize R&D investments.*

As in the existing strategic trade literature, the only role played by a government's R&D policy is to help its firm commit to overinvestment or underinvestment, according to the strategic motive. That is, a government will subsidize or tax its firm's R&D if by doing so it could affect the rival firm's R&D investment in such a way that its own firm's profit will increase. In the absence of international R&D collaboration, a subsidy does the job, as higher  $s_i$  leads to

lower  $x_j^*$  and higher  $x_j$  leads to lower  $\pi_i^*$ . However, in the presence of international R&D collaboration, the strategic incentive does not necessarily call for government R&D subsidies. When it does, the rationale is different: first, higher  $s_i$  leads to higher  $x_j^*$  (i.e.,  $\partial x_j^*/\partial s_i > 0$ ) and second, higher  $x_j$  leads to higher  $\pi_i^*$  (i.e.,  $\partial \pi_i^*/\partial x_j > 0$ ), both of which are made possible by the induced R&D spillovers.

## 2.6. Degree of government policy

We have shown in the preceding subsection the possibility of both subsidy and tax on international R&D collaboration. It is this subsection's purpose to give an example that emphasizes the case for subsidy and from which we can examine how the government subsidy responds to the changing degree of collaboration. In particular, would the governments subsidize more or less as  $\lambda_R$  becomes larger?

To serve the above purpose, we assume that demand and cost functions take the following forms:<sup>10</sup>

$$p = a - (q_1 + q_2) \quad \text{and} \quad c(x_i) = \sqrt{\frac{x_i}{\beta}}, \quad (11)$$

where parameters  $a$  and  $\beta$  are positive,  $\alpha \equiv a - C > 0$ , and  $C$  is sufficiently large so that  $MC_i$  is non-negative for all relevant  $x_i$  and  $x_j$ .

Let  $y_i \equiv \sqrt{x_i/\beta}$ . Then, choosing  $y_i$  is equivalent to choosing  $x_i$  in each firm's R&D decision. In Appendix A, we will show that the optimal R&D responses are

$$y_i = \frac{(2 - \lambda_R) [\alpha - (1 - 2\lambda_R) y_j]}{9\beta(1 - s_i) - (2 - \lambda_R)^2}, \quad (12)$$

where the denominator is positive as required by the second-order condition in this stage's maximization. Thus,

$$\frac{\partial y_i}{\partial y_j} \begin{cases} < 0 & \text{if } \lambda_R < 1/2, \\ = 0 & \text{if } \lambda_R = 1/2, \\ > 0 & \text{if } \lambda_R > 1/2. \end{cases}$$

The strategic nature between  $x_i$  and  $x_j$  changes from substitute to complement when  $\lambda_R$  increases from below half to above half.

<sup>10</sup> This model is similar to the one used by d'Aspremont and Jacquemin (1988) except we include government policy. Notice that our qualitative result does not change if the slope of the demand curve is a constant  $b$  rather than the specific case, i.e.,  $b = 1$ .

The following lemma gives the necessary and sufficient condition for the existence of a symmetric equilibrium and characterizes the non-cooperative government R&D policy.

*Lemma 1.* *The model as specified by Eq. (11) has a unique solution to the optimal government policy if and only if  $\beta \geq 1$ . Specifically, the equilibrium R&D subsidy by each government is*

$$s^* = \frac{1}{6\beta} (3\beta - 1 + \lambda_R^2 - \sqrt{(1 - \lambda_R^2)^2 - 2(5 - 8\lambda_R + 5\lambda_R^2)\beta + 9\beta^2}); \quad (13)$$

and the equilibrium R&D investment by each firm is

$$x^* = \frac{(2 - \lambda_R)^2 \alpha^2 \beta}{[9\beta(1 - s^*) - (1 + \lambda_R)(2 - \lambda_R)]^2}. \quad (14)$$

*Proof.* See Appendix A.1.

In this specific model, the governments always subsidize their firms' R&D except when  $\lambda_R = \frac{1}{2}$  at which there will be no tax or subsidy.

*Proposition 2.* *In the case of international R&D collaboration,  $s^* \geq 0$  for all  $\lambda_R \in [0, 1]$ . Moreover, the optimal R&D subsidy rate,  $s^*$  (given in Eq. (13)), is decreasing in  $\lambda_R$  for  $\lambda_R < \frac{1}{2}$  ( $ds^*/d\lambda_R < 0$ ) but increasing in  $\lambda_R$  for  $\lambda_R > \frac{1}{2}$  ( $ds^*/d\lambda_R > 0$ ). At  $\lambda_R = \frac{1}{2}$ ,  $s^* = 0$  and  $ds^*/d\lambda_R = 0$ .*

*Proof.* See Appendix A.2.

It is worth noting that when  $\lambda_R > \frac{1}{2}$ ,  $x_i$  and  $x_j$  become strategic complements, but the governments still subsidize their firms' R&D investments. This contradicts the general conclusion about export subsidy/tax: subsidize (tax) when the competing variables are strategic substitutes (complements).<sup>11</sup>

### 3. R&D coordination

#### 3.1. The model

In this section, we analyze the policy implication of another popular model of R&D cooperation, namely, R&D coordination, pioneered by d'Aspremont and

<sup>11</sup> See Eaton and Grossman (1986).

Jacquemin (1988). The set-up differs from that of Section 2.1 in two ways. First, in the production stage, firm  $i$ 's marginal cost of production depends only on its own R&D investment and is denoted by  $MC_i = C - c(x_i)$ . Second, in the investment stage, firm  $i$  chooses  $x_i$  to maximize  $(\pi_i + \lambda_F \pi_j)$ , where  $\lambda_F \in [0, 1]$ , capturing all possible degrees of R&D coordination.<sup>12</sup> In the extreme case of full coordination (when  $\lambda_F = 1$ ), each firm chooses its investment level to maximize their joint profits.

### 3.2. A three-stage game

For simplicity of exposition, we will keep the same notations as in Section 2 when there is no need for change. In the third stage, for given  $(x, s)$ , the equilibrium production levels,  $q_1^*$  and  $q_2^*$ , are determined by Eq. (1) except with  $MC_i = C - c(x_i)$ . We define  $a_{ij}$  as in Section 2 and make the same assumptions.

In the second stage, firm  $i$  chooses  $x_i$  to maximize  $(\pi_i^* + \lambda_F \pi_j^*)$ . The following first-order conditions determine the equilibrium R&D investments:

$$\frac{\partial \pi_i^*}{\partial x_i} + \lambda_F \frac{\partial \pi_j^*}{\partial x_i} = 0. \quad (2a)$$

Let  $\tilde{\pi}_i \equiv (\pi_i^* + \lambda_F \pi_j^*)$ ,  $b_{ii} \equiv \partial^2 \tilde{\pi}_i / \partial x_i^2$ ,  $b_{ij} \equiv \partial^2 \tilde{\pi}_i / \partial x_j \partial x_i$ . We make the same assumptions about  $b_{ij}$  as in Section 2. Furthermore, in this section, the firms' R&D investments are always strategic substitutes, which implies  $b_{ij} < 0$ .

Finally, the equilibrium R&D subsidies (or taxes) are determined by Eq. (4). Notation  $e_{ij}$  is retained and the same assumptions about  $e_{ij}$  are made as in Section 2.

### 3.3. Comparative statics results

Following the analysis in Section 2.3, we can probe the effect of firms' R&D investments on the product market competition. Without the induced R&D spillover of Section 2, the effect becomes unambiguous. Specifically,

$$\frac{\partial q_i^*}{\partial x_i} = -\frac{1}{\Delta_1} a_{jj} c'(x_i) > 0 \quad \text{and} \quad \frac{\partial q_i^*}{\partial x_j} = \frac{1}{\Delta_1} a_{ij} c'(x_j) < 0. \quad (5a)$$

Intuitively, with higher R&D investment, firm  $i$  gains a cost advantage over firm  $j$ , resulting in higher  $q_i^*$  and lower  $q_j^*$ .

<sup>12</sup> For this approach, see papers by d'Aspremont and Jacquemin (1988), Kamien et al. (1992), and Suzumura (1992). They only consider  $\lambda_F = 1$ , however.

The effect of government R&D policy on the firms' R&D investments can be shown to be the same in formulation as that of Section 2. What is different is that  $b_{ij} < 0$  in this section whereas the sign of  $b_{ij}$  depends on  $\lambda_R$  in Section 2. Thus, we have

$$\frac{\partial x_i^*}{\partial s_i} = -\frac{b_{jj}}{\Delta_2} > 0 \quad \text{and} \quad \frac{\partial x_j^*}{\partial s_i} = \frac{b_{ji}}{\Delta_2} < 0. \tag{6a}$$

In summary, government  $i$ 's subsidy raises firm  $i$ 's R&D investment and reduces firm  $j$ 's investment, which subsequently improves firm  $i$ 's competitiveness in the market.

### 3.4. R&D incentives

Following the analysis in Section 2.4, we decompose

$$\frac{\partial \tilde{\pi}_i}{\partial x_i} = \frac{\partial(\pi_i + \lambda_F \pi_j)}{\partial x_i} + \frac{\partial(\pi_i + \lambda_F \pi_j)}{\partial q_i} \frac{\partial q_i^*}{\partial x_i} + \frac{\partial(\pi_i + \lambda_F \pi_j)}{\partial q_j} \frac{\partial q_j^*}{\partial x_i}.$$

Using the market stage first-order conditions and the fact that  $\pi_j$  is not a direct function of  $x_i$ , we can simplify the above equation as

$$\frac{\partial \tilde{\pi}_i}{\partial x_i} = \underbrace{\frac{\partial \pi_i}{\partial x_i}}_{\text{profit motive}} + \underbrace{\frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial x_i}}_{\text{strategic motive}} + \underbrace{\lambda_F \frac{\partial \pi_j}{\partial q_i} \frac{\partial q_i^*}{\partial x_i}}_{\text{coordination motive}}. \tag{7a}$$

Thus, when firms engage in R&D coordination ( $\lambda_F > 0$ ), there are three motives for R&D investment. The first two are the familiar profit and strategic motives. For the R&D coordination considered in this section, the strategic motive is always positive (by Eq. (5a)). This is in contrast to the results in Section 2. The third term on the right-hand side of Eq. (7a) represents a new motive, which we call the coordination motive. Intuitively, when firm  $i$  increases its R&D investment  $x_i$ , it decreases firm  $j$ 's profit (by Eq. (5a)). As firm  $i$  cares about firm  $j$ 's profit, its R&D incentive is diminished.

The coordination motive to underinvest in R&D is at odds with the strategic motive to overinvest in R&D, which can be better illustrated in the symmetric equilibrium. In the symmetric case (i.e.,  $s_1 = s_2$ ) we have  $a_{11} = a_{22}$  and  $a_{12} = a_{21}$ , which implies  $\partial \pi_i / \partial q_j = \partial \pi_j / \partial q_i$  and  $a_{jj} / a_{ji} > 1$ ; by Eq. (5a), we also have  $\partial q_i^* / \partial x_i = -(a_{jj} / a_{ji}) \partial q_j^* / \partial x_i$ . The net effect of these two motives can be simplified and signed:

$$\begin{aligned} & \underbrace{\frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial x_i}}_{\text{strategic motive}} + \underbrace{\lambda_F \frac{\partial \pi_j}{\partial q_i} \frac{\partial q_i^*}{\partial x_i}}_{\text{coordination motive}} \\ &= \left( 1 - \lambda_F \frac{a_{jj}}{a_{ji}} \right) \frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial x_i} \begin{cases} > 0 & \text{for } \lambda_F = 0, \\ < 0 & \text{for } \lambda_F = 1. \end{cases} \end{aligned} \tag{15}$$

For a sufficiently small  $\lambda_F$ , the strategic motive dominates the coordination motive, and firm  $i$  has incentive to overinvest in R&D. For a sufficiently large  $\lambda_F$ , the coordination motive dominates the strategic motive, and firm  $i$  has incentive to underinvest in R&D. Thus, similar to the case of R&D collaboration, firms may underinvest or overinvest in R&D depending on the degree of R&D coordination.

### 3.5. R&D policy

As in Section 2.5, the equilibrium R&D subsidies (or taxes) are determined by equation system given by Eq. (4). By Eq. (8), we can simplify Eq. (4) as

$$s_i^* \frac{\partial x_i^*}{\partial s_i} = \frac{\partial \pi_i^*}{\partial x_i} \frac{\partial x_i^*}{\partial s_i} + \frac{\partial \pi_i^*}{\partial x_j} \frac{\partial x_j^*}{\partial s_i}. \quad (9a)$$

Compared with Eq. (9) of Section 2, Eq. (9a) has an additional term,  $(\partial \pi_i^* / \partial x_j) (\partial x_j^* / \partial s_i)$ . The reason that this term does not show up in Section 2.5 is simple. Under the R&D cooperation of Section 2, firm  $i$  chooses  $x_i$  to maximize  $\pi_i^*$  and thus this term is equal to zero by the first-order condition.

Moreover, we can show

$$\frac{\partial \pi_j^*}{\partial x_i} = \frac{\partial \pi_j}{\partial q_i} \frac{\partial q_i^*}{\partial x_i} + \frac{\partial \pi_j}{\partial q_j} \frac{\partial q_j^*}{\partial x_i} + \frac{\partial \pi_j}{\partial x_i} = p' q_j^* \frac{\partial q_i^*}{\partial x_i} < 0,$$

which implies by Eq. (2a) that  $\partial \pi_i^* / \partial x_i > 0$ . By Eq. (6a), both terms on the right-hand side of Eq. (9a) are positive. We have

*Proposition 3. With international R&D coordination, governments always subsidize R&D investments. That is,  $s_i^* > 0$  for all  $\lambda_F$ .*

There are two reasons for the governments to subsidize their firms' R&D investments. First, the foreign firm's R&D investment adversely affects the domestic firm's profit (i.e.,  $\partial \pi_i^* / \partial x_j < 0$ ), and a government's R&D subsidy to the domestic firm discourages the foreign firm's R&D investment (i.e.,  $\partial x_j^* / \partial s_i < 0$ ). The combined effect (the second term on the right-hand side of Eq. (9a)) is the familiar strategic incentive for R&D policies. For the R&D coordination considered in this section, the strategic incentive always leads to R&D subsidies. This is similar to those studies in the literature, but is in contrast to the case of the R&D collaboration considered in Section 2.

Second, when firms engage in R&D coordination, they undertake fewer investments than those that would maximize their own respective profits (this is because  $\partial \pi_i^* / \partial x_i > 0$  at the equilibrium  $x_i^*$ ). This allows a government's R&D subsidy to raise the domestic firm's profit directly by encouraging the firm's R&D investment (namely,  $(\partial \pi_i^* / \partial x_i) (\partial x_i^* / \partial s_i)$  – the first term on the right-hand

side of Eq. (9a) – is positive). Note that the first term is equal to zero when the domestic firm chooses the R&D level to maximize its own profit (namely,  $\partial\pi_i^*/\partial x_i = 0$ ) as in the R&D competition case and in the R&D collaboration discussed in Section 2.

In R&D coordination, a firm has an incentive to lower R&D investment, but the government wishes to change the firm’s behavior by providing R&D subsidies to the firm. Thus, the governments’ policies do not help their firms to commit. This is a new message for the literature.

### 3.6. Coordination at the government level

From the discussion following Proposition 3, it seems that the governments’ incentives for R&D subsidies is partly due to the fact that the governments do not have coordination as their firms do. To investigate the robustness of Proposition 3, we examine the case of government coordination in this subsection.

Suppose now that in the first stage, government 1 chooses  $s_1$  to maximize  $\bar{W}_1$  while government 2 chooses  $s_2$  to maximize  $\bar{W}_2$ , where  $\bar{W}_i$  is defined as

$$\bar{W}_i = W_i^{**} + \lambda_G W_j^{**}, \quad i \neq j.$$

Here  $\lambda_G \in [0, 1]$ , capturing all possible degrees of government coordination. Accordingly, the two first-order conditions that jointly determine the optimal policies become

$$\frac{\partial \bar{W}_i}{\partial s_i} = \frac{\partial (W_i^{**} + \lambda_G W_j^{**})}{\partial s_i} = 0, \tag{4a}$$

which can be rearranged to get

$$s_i \frac{\partial x_i^*}{\partial s_i} + \lambda_G s_j \frac{\partial x_j^*}{\partial s_i} = \frac{\partial (\pi_i^* + \lambda_G \pi_j^*)}{\partial x_i} \frac{\partial x_i^*}{\partial s_i} + \frac{\partial (\pi_i^* + \lambda_G \pi_j^*)}{\partial x_j} \frac{\partial x_j^*}{\partial s_i}.$$

The above equation can be further simplified using Eqs. (2a) and (6a):

$$\left( s_i - s_j \lambda_G \frac{b_{ji}}{b_{jj}} \right) \frac{\partial x_i^*}{\partial s_i} = \left( \frac{\lambda_F - \lambda_G}{\lambda_F} \right) \frac{\partial \pi_i^*}{\partial x_i} \frac{\partial x_i^*}{\partial s_i} + (1 - \lambda_G \lambda_F) \frac{\partial \pi_i^*}{\partial x_j} \frac{\partial x_j^*}{\partial s_i}. \tag{9b}$$

Let us examine the right-hand side of Eq. (9b) to explain the governments’ policy incentives. When the governments coordinate in setting R&D policies (i.e.,  $\lambda_G > 0$ ), they care about not only their own welfare but also their rival’s welfare. It follows that, though the firms still underinvest in R&D, the governments have less incentive to subsidize their firms’ R&D. In particular, when the governments coordinate more than their firms do (i.e.,  $\lambda_G > \lambda_F$ ), they instead have

incentive to tax their firms' R&D. The first term on the right-hand side of Eq. (9b) indicates this. Specifically,

$$\left( \frac{\lambda_F - \lambda_G}{\lambda_F} \right) \frac{\partial \pi_i^*}{\partial x_i} \frac{\partial \pi_i^*}{\partial s_i} \begin{cases} > 0 & \text{for } \lambda_G < \lambda_F, \\ = 0 & \text{for } \lambda_G = \lambda_F, \\ < 0 & \text{for } \lambda_G > \lambda_F. \end{cases} \quad (16)$$

On the other hand, the governments' strategic incentive to shift profits (the second term on the right-hand side of Eq. (9b)) remains so long as the governments are not fully coordinate ( $\lambda_G \neq 1$ ). This is the case even when the firms fully coordinate in R&D (i.e.,  $\lambda_F = 1$ ). Thus, we have

*Proposition 4.* *In the absence of full coordination at the government level, i.e.,  $\lambda_G < 1$ , the optimal government R&D policy is to subsidize so long as the governments do not cooperate more than the firms do. More precisely,  $s^* > 0$  if  $\lambda_G \leq \lambda_F$  and  $\lambda_G \neq 1$ .*

*Proof.* See Appendix A.3.

The full coordination case,  $\lambda_G = 1$ , deserves further scrutiny. When  $\lambda_G = 1$ , the first term on the right-hand side of Eq. (9b) is non-positive while the second term is non-negative. However, we can show that the first term dominates the second term unless the firms also have full coordination (i.e.,  $\lambda_F = 1$ ) in which case the two terms cancel out each other.

*Proposition 5.* *Fully coordinating governments should tax R&D unless their firms are also fully cooperating in R&D activities. With full coordination at both governmental and firm levels, the optimal policy is laissez faire. More precisely, the optimal R&D policies are*

$$s^* \begin{cases} = 0 & \text{for } \lambda_F = \lambda_G = 1, \\ < 0 & \text{for } \lambda_F < \lambda_G = 1. \end{cases}$$

*Proof.* See Appendix A.4.

### 3.7. Degree of government policy

Similar to Section 2.6, here we examine the relationship between the degree of R&D coordination and that of R&D subsidy. In this Section, we return to our basic framework of Section 3.1 (i.e., no policy coordination).

As in Section 2.6, assume linear demand and the specific function  $c(x_i)$  as in Eq. (11). The following lemma gives the necessary and sufficient condition for

the existence of a symmetric equilibrium and characterizes the non-cooperative government R&D policy.

*Lemma 2.* The model as specified by Eq. (11) has a unique solution to the optimal government policy if and only if  $\beta \geq 2.96$ . Specifically, the equilibrium R&D subsidy by each government is

$$s^* = \frac{1}{12\beta}(-2 + 6\beta + 3\lambda_F\beta - \sqrt{4 - 4(10 - \lambda_F - 2\lambda_F^2)\beta + 9(2 - \lambda_F)^2\beta^2}); \quad (17)$$

and the equilibrium R&D investment by each firm is

$$x^* = \frac{(2 - \lambda_F)^2\alpha^2\beta}{(\lambda_F - 2 + 9\beta - 9\beta s^*)^2}. \quad (18)$$

*Proof.* See Appendix A.5.

Furthermore, we can derive the following property of the equilibrium subsidy.

*Proposition 6.* The optimal R&D subsidy rate,  $s^*$  (given in Eq. (17)), is increasing in  $\lambda_F$ , i.e.,  $ds^*/d\lambda_F > 0$ .

*Proof.* See Appendix A.6.

#### 4. Conclusions

It has become increasingly prevalent that rival firms of different countries engage in R&D cooperation. This presents challenges to the existing strategic trade literature, which focuses on government R&D policies for firms engaging in international R&D competition. This paper investigates the role of strategic government policies, if there is any, in the presence of international R&D cooperation.

In the absence of R&D cooperation, firms have incentive to overinvest in R&D, which is then further enhanced by their governments' R&D subsidies. However, under the two types of R&D cooperation (i.e., R&D collaboration and R&D coordination as we call them in this paper), firms may have incentives to overinvest or underinvest in R&D depending on the degree of R&D cooperation. Furthermore, we show that governments may still have the unilateral incentives to subsidize their firms' R&D though the rationales for

such government R&D subsidies are quite different from those in the existing literature.

We have also pointed out the two departures of the present study from the existing strategic trade literature. First, optimal government policies may not necessarily reinforce the firms' commitments. Second, subsidies could be still optimal when the strategy variables are strategic complements.

The present analysis is by no means complete. While we have investigated government R&D policies using two popular models of international R&D cooperation, it is interesting to probe the robustness of our main results to other forms of R&D cooperation. In addition, while R&D subsidies or taxes are important policy tools, governments can also use R&D cooperation policies – i.e., whether to allow R&D cooperation – as a tool. Only by addressing these two extensions, can unambiguous policy suggestions be obtained.

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## Appendix A.

### A.1. Proof of Lemma 1

Due to linearity of the demand, the equilibrium outputs and profits in the third stage can be easily calculated,

$$q_i^* = \frac{1}{3}[\alpha + (2 - \lambda_R) y_i - (1 - 2\lambda_R) y_j],$$

$$\pi_i^* = \frac{1}{9}[\alpha + (2 - \lambda_R) y_i - (1 - \lambda_R) y_j]^2 - (1 - s_i) \beta y_i^2, \quad i \neq j.$$

In the second stage, the firms maximize their individual profits by choice of  $x_i$ , which is equivalent to the corresponding profit maximization by choice of  $y_i$ . In what follows, we choose the latter to simplify our expressions. From the first-order conditions, we can compute the firms' second stage reaction functions, which are given in Eq. (12). Thus, the second-stage

equilibrium is

$$y_i^* = \frac{(2 - \lambda_R) [9(1 - s_j) \beta - 3(2 - \lambda_R) (1 - \lambda_R)] \alpha}{[9(1 - s_1) \beta - (2 - \lambda_R)^2] [9(1 - s_2) \beta - (2 - \lambda_R)^2] - (2 - \lambda_R)^2 (1 - 2\lambda_R)^2},$$

$i \neq j.$

Turning to the first stage, government  $i$  chooses  $s_i$  to maximize its country's welfare, which is given by

$$W_i^{**}(s_1, s_2) = \frac{1}{9}(\alpha + (2 - \lambda_R)y_i^* - (1 - 2\lambda_R)y_j^*)^2 - \beta y_i^{*2}, \quad i \neq j.$$

To simplify the analysis, let us suppose that government  $i$  chooses  $t_i \equiv (1 - s_i)$  rather than  $s_i$  to maximize its welfare. The two ways are clearly equivalent. We first derive the first-order conditions. Suppose the equilibrium exists. Then, for symmetry, the two optimal subsidy rates are equal, i.e.,  $s^* = s_1^* = s_2^*$ , which means  $t^* = t_1^* = t_2^*$  and

$$y^* = y_1^* = y_2^* = \frac{(2 - \lambda_R)\alpha}{9\beta t^* - (2 - \lambda_R)(1 + \lambda_R)}. \tag{A.1}$$

Thus, using these symmetric results in the first-order conditions we can simplify them to

$$9\beta t^{*2} - 3(1 + 3\beta - \lambda_R^2) t^* + (2 - \lambda_R)^2 = 0.$$

There are two solutions to the above equation, one of which being

$$t^* = \frac{1}{6\beta} [1 + 3\beta - \lambda_R^2 + \sqrt{9\beta^2 - 2(5 - 8\lambda_R + 5\lambda_R^2) \beta + (1 - \lambda_R^2)^2}]. \tag{A.2}$$

It is straightforward to verify that  $t^* \in (0, 1)$ . Thus, Eq. (13) follows. Eq. (A.1) also gives rise to Eq. (14). The other solution has the same expression as Eq. (A.2) except the plus sign in front of the square root is replaced with a minus sign. But this latter solution minimizes rather than maximizes the welfare and therefore we delete it.

It remains to show the existence of a unique equilibrium. To guarantee real value solutions in Eq. (A.2), we require

$$f(\lambda_R, \beta) \equiv 9\beta^2 - 2(5 - 8\lambda_R + 5\lambda_R^2) \beta + (1 - \lambda_R^2)^2 \geq 0.$$

Note that, given any  $\lambda_R$ , there exist two real value solutions to  $f(\lambda_R, \beta) = 0$ :

$$\beta_1(\lambda_R) = \frac{1}{9} [5 - 8\lambda_R + 5\lambda_R^2 - 2(2 - \lambda_R)(1 - 2\lambda_R)],$$

$$\beta_2(\lambda_R) = \frac{1}{9} [5 - 8\lambda_R + 5\lambda_R^2 + 2(2 - \lambda_R)(1 - 2\lambda_R)].$$

Note also that  $f(\lambda_R, \beta)$  is convex in  $\beta$ . Thus, the necessary and sufficient condition for  $f(\lambda_R, \beta) \geq 0$  for all  $\lambda_R$  is

$$\beta \geq \beta^* \equiv \max_{\lambda_R \in [0, 1]} \beta_2(\lambda_R) = \beta_2(0) = 1.$$

In fact, the inequality,  $f(\lambda_R, \beta) \geq 0 \forall \lambda_R$ , also holds for all  $\beta \leq \min_{\lambda_R \in [0, 1]} \beta_1(\lambda_R) = \frac{1}{4}$ . However, we eliminate this range of  $\beta$  because these values violate the stability condition to the second stage optimization. Note that the stability condition for optimal  $y_i$  requires  $\beta > \frac{4}{9}$  in the case of no subsidy and no spillovers.  $\square$

### A.2. Proof of Proposition 2

It is straightforward to show  $s^* \geq 0$  for all  $\lambda_R \in [0, 1]$ . We now check the monotonicity of  $s^*$ .

*Step 1:* Note from Eq. (13) that  $\text{sign}(\partial s^*/\partial \lambda_R) = \text{sign}(\partial s_a/\partial \lambda_R)$ , where

$$s_a \equiv \lambda_R^2 - \sqrt{(3\beta + 1 - \lambda_R)^2 - 4\beta(2 - \lambda_R)^2}.$$

By calculation, we further obtain  $\text{sign}(\partial s_a/\partial \lambda_R) = \text{sign}(s_b)$ , where

$$s_b \equiv \lambda_R \sqrt{(3\beta + 1 - \lambda_R)^2 - 4\beta(2 - \lambda_R)^2} + 5\beta\lambda_R + \lambda_R - \lambda_R^3 - 4\beta.$$

*Step 2:*  $\partial s_b/\partial \lambda_R = (5\beta + 1 - 3\lambda_R^2) + s_c/\sqrt{(3\beta + 1 - \lambda_R)^2 - 4\beta(2 - \lambda_R)^2}$ , where

$$s_c \equiv (3\beta + 1 - \lambda_R^2)^2 - 4\beta(2 - \lambda_R)^2 - 2\lambda_R^2(3\beta + 1 - \lambda_R^2) + 4\beta\lambda_R(2 - \lambda_R).$$

First, the first term in bracket is positive since  $\beta \geq 1$ . Second, by rearrangement, we obtain  $s_c = (9\beta - 1)(\beta - 1) + 20\beta\lambda_R(1 - \lambda_R) + 4\lambda_R(\beta - \lambda_R) + 3\lambda_R^4 \geq 0$ . Moreover,  $s_b = 0$  when  $\lambda_R = \frac{1}{2}$ .

Combining steps 1 and 2, we conclude that  $s_b < 0$  when  $\lambda_R < \frac{1}{2}$  and  $s_b > 0$  when  $\lambda_R > \frac{1}{2}$ . The conclusion in the proposition then follows.  $\square$

### A.3. Proof of Proposition 4

The above analysis shows that the right-hand side of Eq. (9b) is positive under the conditions of Proposition 3. In the symmetric equilibrium, we have  $s_i = s_j$ . By Eq. (3), we also have  $b_{jj} - \lambda_G b_{ji} \leq b_{jj} - b_{ji} < 0$ , which implies  $(1 - \lambda_G b_{ji}/b_{jj}) > 0$ . Proposition 3 follows immediately.  $\square$

### A.4. Proof of Proposition 5

The case for  $\lambda_F = \lambda_G = 1$  has been proven in the analysis leading to Proposition 3. For symmetric equilibrium,  $\partial \pi_i^*/\partial x_i = \partial \pi_j^*/\partial x_j$ . By Eqs. (2a) and (6a),

respectively, we also have

$$\frac{\partial \pi_i^*}{\partial x_j} = -\frac{1}{\lambda_F} \frac{\partial \pi_j^*}{\partial x_j} \quad \text{and} \quad \frac{\partial x_j^*}{\partial s_i} = -\frac{b_{ji}}{b_{jj}} \frac{\partial x_i^*}{\partial s_i}.$$

Thus, the right-hand side of Eq. (9b) can be simplified as

$$-\frac{(1 - \lambda_F)}{\lambda_F} \left(1 - \frac{b_{ji}}{b_{jj}}\right) \frac{\partial \pi_i^*}{\partial x_i} \frac{\partial x_i^*}{\partial s_i} < 0.$$

Thus,  $s^* < 0$ .  $\square$

### A.5. Proof of Lemma 2

Due to linearity of the demand, the equilibrium outputs and profits in the third stage can be easily calculated,

$$q_i^* = \frac{1}{3}(\alpha + 2y_i - y_j), \quad \pi_i^* = \frac{1}{9}(\alpha + 2y_i - y_j)^2 - (1 - s_i) \beta y_i^2, \quad i \neq j.$$

In the second stage, depending on whether they compete or cooperate in R&D investment, the firms maximize their individual or joint profits by choice of  $x_i$ , which is equivalent to the corresponding profit maximization by choice of  $y_i$ . In what follows, we choose the latter to simplify our expressions. From the first-order conditions, we can compute the firms' second-stage optimal decisions:

$$y_i^* = \frac{3(2 - \lambda_F) [3(1 - s_j) \beta - 2 - \lambda_F] \alpha}{[9(1 - s_1) \beta - 4 - \lambda_F] [9(1 - s_2) \beta - 4 - \lambda_F] - 4(1 + \lambda_F)^2}, \quad i \neq j.$$

Turning to the first stage, government  $i$  chooses  $t_i$  to maximize its country's welfare, which is given by

$$W_i^{**}(s_1, s_2) = \frac{1}{9}(\alpha + 2y_i^* - y_j^*)^2 - \beta y_i^{*2}, \quad i \neq j.$$

Then, the first-order conditions are, for  $i \neq j$ ,

$$\begin{aligned} &(\alpha + 2y_i^* - y_j^*) [(2 - \lambda_F) \alpha - (4 + \lambda_F - 9\beta t_j)(2y_i^* - y_j^*)] \\ &+ 9\beta(4 + \lambda_F - 9\beta t_j) y_i^{*2} = 0. \end{aligned} \tag{A.3}$$

Suppose the equilibrium exists. Then, for symmetry, the two optimal subsidy rates are equal, i.e.,  $s^* = s_1^* = s_2^*$ , which means  $t^* = t_1^* = t_2^*$  and

$$y^* = y_1^* = y_2^* = \frac{(2 - \lambda_F) \alpha}{\lambda_F - 2 + 9\beta t^*}. \tag{A.4}$$

Thus, the first-order conditions, Eq. (A.3), can be simplified to

$$18\beta t^{*2} - 3(2 + 6\beta - 3\lambda_F\beta) t^* + (8 - 2\lambda_F - \lambda_F^2) = 0.$$

There are two solutions to the above equation, one of which being

$$t^* = \frac{1}{12\beta} \left[ (2 + 6\beta - 3\lambda_F\beta) + \sqrt{4 - 4(10 - \lambda_F - 2\lambda_F^2)\beta + 9(2 - \lambda_F)^2\beta^2} \right]. \tag{A.5}$$

It is straightforward to verify that  $t^* \in (0, 1)$ . Thus, Eq. (17) follows. Eq. (A.4) also gives rise to Eq. (18). The other solution has the same expression as Eq. (A.5) except the plus sign in front of the square root is replaced with a minus sign. But this latter solution minimizes rather than maximizes the welfare and therefore we delete it.

It remains to show the existence of a unique equilibrium. To guarantee real-value solutions in Eq. (A.5), we require

$$f(\lambda_F, \beta) \equiv 4 - 4(10 - \lambda_F - 2\lambda_F^2)\beta + 9(2 - \lambda_F)^2\beta^2 \geq 0. \tag{A.6}$$

Note that, given any  $\lambda_F$ , there exist two real value solutions to  $f(\lambda_F, \beta) = 0$ :

$$\beta_1(\lambda_F) = \frac{2(5 + 2\lambda_F) - 4\sqrt{(1 + \lambda_F)(4 + \lambda_F)}}{9(2 - \lambda_F)},$$

$$\beta_2(\lambda_F) = \frac{2(5 + 2\lambda_F) + 4\sqrt{(1 + \lambda_F)(4 + \lambda_F)}}{9(2 - \lambda_F)}.$$

Note also that  $f(\lambda_F, \beta)$  is convex in  $\beta$ . Thus, the necessary and sufficient condition for  $f(\lambda_F, \beta) \geq 0$  for all  $\lambda_F$  is

$$\beta \geq \beta^* \equiv \max_{\lambda_F \in [0, 1]} \beta_2(\lambda_F) = \frac{14 + 4\sqrt{10}}{9} \approx 2.96.$$

In fact, the inequality,  $f(\lambda_F, \beta) \geq 0 \forall \lambda_F$ , also holds for all  $\beta \leq \min_{\lambda_F \in [0, 1]} \beta_1(\lambda_F) = \frac{1}{9}$ . However, we eliminate this range of  $\beta$  because these values violate the stability condition to the second-stage optimization. For stable second-stage equilibrium,  $\beta$  must be greater than  $\frac{2}{3}$ . One can obtain this critical value from expression given by Eq. (6) of Henriques (1990), who discusses the stability conditions in a model similar to ours.  $\square$

### A.6. Proof of Proposition 6

It can be easily verified that  $f(\lambda_F, \beta)$  as defined by Eq. (A.6) is a strictly decreasing function of  $\lambda_F$  for any given  $\beta \geq \beta^*$ . Thus, from Eq. (A.5),  $t^*$  clearly

decreases in  $\lambda_F$ . Therefore,

$$\text{sign} \left( \frac{ds^*}{d\lambda_F} \right) = -\text{sign} \left( \frac{dt^*}{d\lambda_F} \right) = + . \quad \square$$

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