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BOT projects: Incentives and efficiency

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ABSTRACT

In recent years, governments have been increasingly adopting Build–Operate–Transfer (BOT) contracts for large infrastructure projects. However, BOT contracts have received little attention from economists. The apparent agency problem in BOT projects has never been analyzed. In this paper, we develop a model to examine the incentives, efficiency and regulation in BOT contracts. We show that a BOT contract with a price regulation during the concession period and a license extension after the concession period is capable of achieving full efficiency. Both license extension and price control are observed in many real-world BOT projects. We also investigate the efficiency in such contracts by considering other factors, including time consistency, price ceiling, foreign ownership, and the lack of price regulation.

1. Introduction

Large public infrastructure projects, such as roads, railways, power stations, dams, bridges and tunnels, have huge importance for a country's economic development[1] and they typically involve huge amounts of human, physical and financial resources.[2] Such projects were used to be funded primarily by governments. However, governments today are increasingly finding themselves either unwilling or unable to finance the growing number of new infrastructure activities. Instead, private companies are often empowered by governments to build and operate many large projects under the so-called BOT (build, operate and transfer) schemes. Two early examples of BOT schemes are the Suez Canal and the Panama Canal.[3] A typical BOT arrangement or contract between a government and a private firm specifies that the government licenses the firm to build (B) a project and then to operate (O) the project for a certain period, normally 5 to 30 years, and finally, at the end of the concession period, to transfer (T) the project at no cost to the government. The firm finances the project and pays the costs of construction and operation. During the concession period, the firm receives revenue from its operations.[4] When the concession period is to end, the government has the option of assuming ownership or extending the license to the firm to continue its control of and receive revenue from the project. BOT is often adopted for public utility projects such as bridges, canals, roads, tunnels, airport terminals, and power plants. Walker and Smith (1995, p. 27–30) listed 111 large BOT projects launched in over 31 countries and regions by early 1995. The BOT practice has become popular since the 1950s and the number of BOT projects is expected to increase dramatically over the next decade, especially in East Asia.

Although the term BOT is relatively new, the practice has been around for several centuries. The Europeans used to call such projects concessions, in which a government assumed many responsibilities. However, a government today is little involved in planning, construction, or financing of a BOT project, and its role normally is limited to providing loans, guarantees, tax credits, subsidies, price controls, and license renewal. The Hong Kong Cross Harbour Tunnel, opened in

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[1] There are a few empirical studies illustrating the impact of infrastructures on economic growth. See, for example, Calderon and Serven (2003) and Calderon et al. (2002). These studies suggest that a 1% increase in infrastructure spending can increase GDP by 0.2%.

[2] For example, at one time, the interest on the debt from the construction of the Suez Canal was larger than Egypt's national income. As a result, the Egyptian government was forced to sell the canal to the British.

[3] See Walker and Smith (1995, pp. 1–2) for details.

[4] In some cases, the firm is also empowered to develop the property during the concession period. When this occurs, the contractual arrangement is called a Build–Own–Operate–Transfer (BOOT) agreement.

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1972, was the first in Hong Kong to adopt a modern BOT contract as defined above. The Channel Tunnel is another BOT project granted by the governments of the United Kingdom and France. The Dulles Toll Road Extension, costing US$250 million and beginning in 1988, was reportedly the first BOT highway in the United States. The first privatized airport terminal in Canada, a BOT project, was Terminal 3 of Pearson International Airport in Toronto, which costs US$433 million and was completed in 1991.

BOT has become a major means of private–public cooperation in infrastructure projects. It is different from complete privatization, complete nationalization, or joint ventures. In particular, BOT differs from privatization in sequencing. In privatization, a government owns the entity first and then transfers (sells) it to the private sector. In contrast, in BOT, a private firm bears the cost of building a project and then owns it for a certain period of time before finally transferring it to the government at no cost. "This is the fundamental attraction of BOT. It not only takes spending off the government's balance sheet but also brings in the commercial skills of the private sector both in identifying viable projects and in running them efficiently when they are built" (Walker and Smith, 1995, p. 16) and when they are initially operated.[5] Moreover, BOT ensures that the government eventually retains strategic control of large infrastructure installations.[6]

Despite its attractiveness and popularity, BOT has received little attention from economists. In particular, although the process obviously entails agency problems, there has been basically no theoretical research on the agency problem arising from BOT projects. Since a BOT project is to be transferred to the government unconditionally after the concession period, the firm may not make a sufficient initial investment for long-lasting quality. The questions are: How can a BOT contract be designed to induce the firm to invest in the best quality? If such a BOT contract exists, does it resemble a typical BOT contract found in reality?

In this paper, we build a model that uses positive agency theory to explain the unique ownership approach to BOT projects. Our BOT model is developed based on stylized facts. We allow the government to have a license extension policy, which is dependent on observed quality. Specifically, a firm is licensed to build a project that lasts for two periods. After the project has been built, the firm owns and operates the project in the first period (the concession period). In the second period, the government may operate the project by itself or license it to the firm. Although the quality of the project is not verifiable, we assume that the government can observe it ex post. By the end of the first period, based on the observed quality of the project, the license extension policy determines whether or not and how long the firm is allowed to own the project in the second period. Without license extension in the second period, the firm will not have sufficient incentives to build an infrastructure with socially optimal quality. However, although letting the firm own the project in both periods for certain may help solve the underinvestment problem (the incentive problem), private ownership generally leads to a reduction in social welfare (the monopoly problem). Our proposed solution to this dilemma is for the government to impose price controls and provide a quality-dependent license extension in the second period. We show that such a BOT contract can induce the right incentives and achieve full efficiency.

In reality, these two measures, price controls and a possible license extension, are observed widely in BOT projects. Price controls are well-known mechanisms that are often used to improve efficiency under monopoly condition. Although license extension is not specified in a typical BOT contract in reality, it is not uncommon to find license extensions in practice. This implies that the firm and the government might have an implicit agreement ex ante. There are two ways to model this reality. First, we may take a complete contract approach; that is, we assume that the government includes the license extension in the BOT contract explicitly. Since the license extension depends on the unverifiable quality of the project, there is a government commitment problem or time inconsistency. Second, we may take an incomplete contract approach; that is, the license extension is not written in the contract, but both parties know that the government has the option to extend the license ex post. A problem that arises from this approach is ex-post rationality, i.e., whether license extension is optimal ex post. In this paper, we choose the complete contract approach and show that the license extension policy is indeed time consistent. In fact, these two approaches are equivalent in the environment considered by this paper. We can easily show that license extension satisfies ex-post rationality in the incomplete contract approach.

Empirical studies on the role of such an ownership/license mechanism are almost nonexistent. One recent paper by Brickley et al. (2006) is an exception. This paper studies the impact of contract extensions on the incentives of franchisees in business franchising. They find that "contract duration is positively and significantly related to the franchisee's physical and human capital investments." This empirical finding is consistent with our theoretical finding on infrastructure investments under a BOT scheme.

This paper is related to the literature on privatization and regulation. There is a big debate on the costs and benefits of privatization and those of public ownership. It is generally agreed that privatization enhances production and investment efficiency since private companies have stronger incentives and can run businesses more efficiently than can governments (see, for example, Dewatripont and Maskin, 1995; Kornai, 2000). However, private ownership will reduce consumer surplus[7] and there are difficulties in regulating private firms in non-competitive industries (see Laffont and Tirole, 1993). There seems to be a way to solve (at least partly) the tradeoff, which is to use public–private partnerships (PPP). In PPP, investment is made by private firms and projects are run and managed by them, while the government purchases the output (products or services) from the firms (i.e., government outsourcing) and delivers to the consumers (See Vaillancourt Rosenau, 2000; Hart, 2003; Martimort and Pouyet, 2006; Auriol and Picard, 2009). In BOT, there is private ownership during the concession period, whereas there is public ownership after the concession period. Unlike the aforementioned studies, our paper does not consider the optimality of private versus public ownership or a combination of the two. Taking the BOT structure as given, we examine the optimal contract in the presence of both the incentive and monopoly issues.

The paper is organized as follows. In Section 2, we develop a stylized BOT model. In Section 3, we derive an optimal BOT contract. In Section 4, we consider four extensions. The first extension takes into account the economic benefits and costs of ownership. With these benefits and costs being included, a license scheme needs to be redesigned to ensure both economic efficiency and time consistency. The second extension has a price ceiling in the concession period. The third extension has a foreign firm, in which case only part of the profits (i.e., the profit tax) affects domestic social welfare. In the fourth extension, we consider a BOT contract without price regulation. We conclude the paper in Section 5. All the proofs are in Appendix A.

2. BOT model

2.1. The project

Consider a situation in which a government offers a BOT contract to a (domestic) firm to build a large project (e.g., a highway) for use by

[5] Walker and Smith (1995, p.16) cite the Channel Tunnel as one example of a privately financed project addressing a need that the public sector was unwilling or unable to fulfill.

[6] Other benefits include relief of the government's financial burden, relief of the administrative burden, reduction in the size of an (inefficient) bureaucracy, and better service to the public.

[7] Estache (2002) shows that production efficiency gains from privatization are not transferred to consumers.

the citizens of the country. The firm must make an investment with a fixed cost, denoted $k(q)$, which is a strictly increasing and differentiable function of the quality, q, of the project, with $k(0)=0$ and $k'(q)>0$.[8] Let Q be the set of all possible qualities and so $q \in Q$. Once the investment has been made, the fixed cost is sunk. This describes the "B" in BOT. Assume that the quality, after being determined in the "B" stage, does not change over time.

Once it has been built, the project lasts for two periods, which are assumed to have equal length. The firm has the right to own and operate the project in the first period and earns all the profits generated in that period. This is the concession period, i.e., the "O" in BOT. At the end of the concession period, the project is transferred to the government. This is the "T" in BOT. The government may extend the ownership license to the firm in the second period.

In reality, there are many reasons for adopting the BOT approach in developing large projects. In this model, we assume that the government does not have the expertise and technology to build the project and operate it in the first period. However, once the project has been run for a long time (in the first period), the operation becomes standard and the government may be able to run it as efficiently as the firm runs it in the second period (we will relax this assumption in Section 4).

2.2. Demand

Let $x_i(p_i, q)$ be the demand function in period i for the service provided by the project, with $x_i(p_i, q) \geq 0$ and $x_i(p_i, 0)=0$, where q is the quality of the project and p_i is the price charged for the service (e.g., a toll fee for a highway). Let $A_i(q)$ be the intercept of the demand curve $x_i(p_i, q)$ at the vertical axis.[9] Demand increases with quality but decreases with price, implying the signs for the derivatives: $x_{i,q}(p, q)>0$ and $x_{i,p}(p, q)<0$ for $i=1, 2$.

2.3. Contracting

The government offers a BOT contract to the firm. In a basic BOT contract, the government empowers the firm to build the project; it guarantees that the firm owns and operates the project and receives the profits in the first period[10]; and it requires that the project is returned unconditionally to the government by the end of the first period. In addition, a contract can also contain other elements that vary under different policy environments, which are the main issues studied in this paper.

The government faces two issues in a BOT project: the firm's monopoly under conditions of private ownership, and the firm's incentive to invest in quality. Accordingly, we analyze a basic BOT contract with two additional elements to combat these two problems. First, the government can specify the prices, p_1 and p_2, in the BOT contract. Price regulation is a common practice in real-world BOT projects. In theory, price control is known to be effective in dealing with monopoly power. Second, the government could specify the quality q in the BOT contract. We study this type of contract, called the first-best contract, in Subsection 3.1. However, such a case is not realistic. Instead, as it is common in the incentive literature, we assume that the government can observe quality q ex post (i.e., after the concession period), but quality is not verifiable. This means that quality is neither contractible nor enforceable. We analyze this case in Subsection 3.2. In order to induce the firm to invest in high quality, the government can introduce a license extension policy that is a function of the observed quality. A license extension policy allows the firm to own the project continuously in the first sub-period, $\lambda(q) \leq 1$, of the second period. We consider the simplest license policy in which there is a pre-specified threshold quality level, say $q=q_0$, such that there is no license extension in the second period ($\lambda=0$) if the observed quality is below the threshold, and $\lambda=\lambda_0>0$ otherwise. This license function is depicted in Fig. 1.

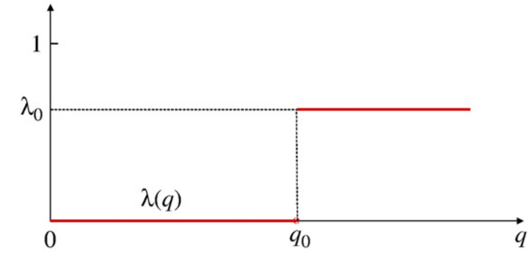

Fig. 1. The license extension policy.

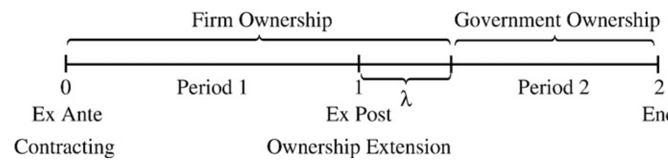

Fig. 2. The license extension scheme.

Hence, a BOT contract is represented by $(p_1, p_2, \lambda(q))$.[11] The main structure of a typical BOT agreement is illustrated in Fig. 2.[12]

Since the license extension depends on the unverifiable quality of the project, there is a government commitment problem or a time consistency problem. We will address this issue in Subsection 4.1.

2.4. Profit and welfare

Given the prices and quality, (p_1, p_2, q), the consumer surpluses are given by

$$s_1(p_1, q) = \int_{p_1}^{A_1(q)} x_1(z, q)dz, \quad s_2(p_2, q) = \int_{p_2}^{A_2(q)} x_2(z, q)dz.$$

Let the cost of providing services from the project in period i be $c_i(x)$, with $c_i(x) \geq 0$ and $c_i(0)=0$. Note that $k(q)$ is the fixed cost and $c_1(x_1)$ and $c_2(x_2)$ are the variable costs. Then, the one-period operating profits are, respectively,

$$\pi_1(p_1, q) \equiv p_1 x_1(p_1, q) - c_1[x_1(p_1, q)], \quad \pi_2(p_2, q) \equiv p_2 x_2(p_2, q) - c_2[x_2(p_2, q)]. \tag{1}$$

Some restrictions on the cost functions will be imposed later when it is more appropriate to discuss.

[8] Construction costs are crucial to the quality of large projects, such as highway projects. As Riccardo Starace, Director of Midland Expressway (BNRR), states from experience, "Spending 2–5% more on the materials for the subgrade and pavement can prolong the life by 50%" (Walker and Smith, 1995, p. 62).

[9] If $x_i(p, q)>0$ for all $p \geq 0$, we let $A_i(q)=\infty$.

[10] We will discuss the government's commitment issue later, but it is not an issue here because in this model, the government is not able to operate the project in the first period and so has no incentive to seize the project before the end of the concession period.

[11] Of course, we omit the other basic elements mentioned earlier (i.e., the firm is empowered to build and own and then transfer the project) in this contract representation. A more sophisticated contract may stipulate that the second-period regulated price is also dependent on the observed quality, i.e., $\{p_1, p_2(q), \lambda(q)\}$. However, a fixed price has already enabled us to achieve the first best as stated in Proposition 1 below. Hence, this generality in prices is unnecessary.

[12] We have assumed equal lengths for the two periods. There is no loss of generality since the demand functions can absorb a difference in period lengths. By properly adjusting the demand functions, we can always make the two periods equal in length. In fact, the second period can be infinite.

3. Analysis

3.1. The first best: the benchmark case

Consider first the benchmark case: Suppose that the project quality is contractible and so the government can offer the firm a contract that specifies both prices and quality. The first best or efficiency is achieved in this case since the incentive and monopoly problems are absent.

The government's optimization problem is

$$W^B = \max_{p_1,p_2,q} s_1(p_1,q) + \pi_1(p_1,q) + s_2(p_2,q) + \pi_2(p_2,q) - k(q), \qquad (2)$$

where the superscript B signifies the first-best outcome. The corresponding first-order conditions (FOCs) are

$$s_{1,p}(p_1,q) + \pi_{1,p}(p_1,q) = 0,$$
$$s_{2,p}(p_2,q) + \pi_{2,p}(p_2,q) = 0,$$
$$s_{1,q}(p_1,q) + \pi_{1,q}(p_1,q) + s_{2,q}(p_2,q) + \pi_{2,q}(p_2,q) = k'(q),$$

which can be simplified to

$$p_1^B = c_1'[x_1(p_1^B,q^B)], \qquad (3)$$

$$p_2^B = c_2'[x_2(p_2^B,q^B)], \qquad (4)$$

$$s_{1,q}(p_1^B,q^B) + s_{2,q}(p_2^B,q^B) = k'(q^B). \qquad (5)$$

These three equations jointly determine the first-best outcome, (p_1^B, p_2^B, q^B). Eqs. (3) and (4) are the standard efficiency formulas stating that the price must equal the marginal cost in equilibrium. Eq. (5) is the formula for quality: quality is set so that the marginal social benefit of improving the quality is equal to the marginal social cost.

3.2. The optimal BOT contract when quality is not contractible

Now, we turn to the BOT model in which quality is not contractible. We are interested in the possibility of designing a BOT contract $(p_1, p_2, \lambda(q))$ that induces the first-best outcome. Note that even if license extension is based on non-contractible (but observable) quality, in the environment described in the main model, the government will not have incentive to deviate from the pre-committed license extension, which will become clear at the end of this subsection. In Subsection 4.1, we will explore the optimal BOT contracts when the government commitment problem arises in different environments.

Given the contract, $(p_1, p_2, \lambda(q))$, the firm decides whether to reject or accept it. The firm's expected profit from the project is:

$$\Pi(p_1,p_2,q) \equiv \pi_1(p_1,q) + \lambda(q)\pi_2(p_2,q) - k(q), \qquad (6)$$

where the per-period profit functions, π_1 and π_2, are defined in Eq. (1). The firm chooses the quality to maximize its expected profit in Eq. (6).

The government's problem is to design a contract, $\{p_1, p_2, \lambda(\cdot)\}$, that maximizes social welfare, which is the sum of the consumer surplus and the firm's profits. Let p_2' be the price set by the government in the second period if the government runs the project by itself and p_2 be the price in the second period when the firm has ownership. Then, the government's problem is

$$\begin{aligned} W^* = & \max_{p_1,p_2,p_2',q,\lambda(\cdot)} s_1(p_1,q) + \pi_1(p_1,q) + \lambda(q)[s_2(p_2,q) + \pi_2(p_2,q)] \\ & + [1-\lambda(q)][s_2(p_2',q) + \pi_2(p_2',q)] - k(q) \\ \text{s.t.} \quad & \Pi(p_1,p_2,q) \geq \Pi(p_1,p_2,\overline{q}) \text{ for all } \overline{q} \in Q, \quad \text{(IC)} \\ & \Pi(p_1,p_2,q) \geq 0. \quad \text{(IR)} \end{aligned} \qquad (7)$$

Problem (7) does not rely on the troublesome first-order approach (FOA). The incentive-compatibility (IC) condition ensures that the firm will be induced to choose the desired quality, q, proposed by the government. The individual-rationality (IR) condition ensures the firm's willingness to take the project.

The following proposition characterizes the sufficient conditions under which the optimal solution to the above problem reaches the first-best outcome.

Proposition 1. *Assume*

$$\pi_1(p_1^B,q) - \pi_1(p_1^B,q^B) \leq k(q), \text{ for } q < q^B; \qquad (8)$$

$$\pi_2(p_2^B,q^B) \geq k(q^B); \qquad (9)$$

$$c_i'(x) \geq c_i'(x_i^B), \text{ for } x \geq x_i^B, \qquad (10)$$

where $x_i^B \equiv x_i(p_i^B, q_i^B)$. *Then, an optimal solution,* $\Omega^* \equiv \{p_1^*, p_2^*, p_2'^*, q^*, \lambda^*(\cdot)\}$, *to problem (7) is determined by*

$$\begin{cases} p_1^* = p_1^B, \\ p_2^* = p_2'^* = p_2^B, \\ q^* = q^B, \\ \lambda^*(q) = \begin{cases} 0, & \text{for } 0 \leq q < q^B, \\ k(q^B)/\pi_2(p_2^B,q^B), & \text{for } q \geq q^B. \end{cases} \end{cases} \qquad (11)$$

This solution is efficient.

Hence, under price regulation, with a properly designed license, the first-best outcome is achieved.

Condition (8) says that the firm's benefit from deviating from the first-best quality, q^B, is not too large. On the other hand, Condition (9) means that demand is sufficiently strong so that the second-period profit is large. This is to give large incentive to the firm to invest in the first-best quality so that it can obtain part of the second period profit due to license extension. Hence, with the aid of this license policy, these two conditions work together to induce investment at high quality.

There must be another force to prevent the firm from over investing in quality. Given the license function in Eq. (11), Condition (10) does part of the job. If the firm invests at a quality level higher than the first-best, then not only the investment cost, k, increases, but its marginal cost is also higher (Condition (10)) because higher quality raises demand, while the length of the extension in the second-period operation is not longer. Hence, the firm has no incentive to raise its quality. More precise analysis can be found in the proof. Condition (10) can be justified for many BOT projects like power stations, bridges and roads, which have a natural limit of capacity. Once the capacity is reached, the marginal cost of increasing production or services will be higher (at least not lower). Note that all the three conditions, Eqs. (8)–(10), are sufficient conditions for BOT to achieve the first-best. Condition (8) is necessary, but it is a very weak condition for obvious reasons. However, conditions (9) and (10) are not necessary.[13]

The license function defined in Eq. (11) has the properties as shown in Fig. 1. Because $\pi_2(p_2^B, q^B) \geq k(q^B)$, we have $0 \leq \lambda^*(q) \leq 1$ for all $q \in Q$.

The optimal BOT contract in Proposition 1 completely solves the problems of incentives and monopoly power: it induces the correct incentive for the firm to invest in quality and tackles the monopoly problem under the firm's ownership. Here, the license mechanism plays a critical role. In order to deal with the monopoly power under the firm's ownership, the government imposes the first-best prices. However, given low prices, the firm may not have the incentive to provide the first-best quality. To deal with this problem, the government uses the license extension scheme to induce the firm to invest the first-best quality. We now provide the intuition for how the first-best quality level is achieved (i.e., the IC condition). Note that since the firm pays all the investment cost but gets part of the social benefit (the profit, not the consumer surplus), it will not voluntarily invest to provide the socially optimal quality even if it is allowed to run the project in the entire second period. The trick of the license extension is to give the firm a "punishment" if it fails to invest at the efficient quality level (i.e., q^B), but to give it a reward if it invests at the efficient quality level. Since the government cannot really punish the firm according to the contract, what it can do is not to extend the license to the firm in the second period. Given this, in order to induce the firm to invest to provide the efficient quality, the reward must be sufficiently large. Since there is no direct monetary reward, the government relies on extending the second-period license, allowing the firm to receive more profits. The idea is to choose a sufficiently long extension period to induce the firm to provide high quality. The $\lambda^*(q^B)$ chosen in Proposition 1 is the minimum length.

Note that any value of the extension length between the minimum length and 1 (i.e., $\lambda^* \leq \lambda \leq 1$) will work. This is because the efficient quality also maximizes the firm's operating profits in each period (This is proved in the Appendix A). Thus, extending the license period will not induce the firm to increase the quality level because that would only increase its investment cost without extra benefit. Overinvestment by the firm would never occur. The key is that $\lambda(q)$ is not increasing after q^B while π_2 is decreasing.

The IR condition will be satisfied if sufficient profit from the project is earned by the firm. Condition $\pi_2(p_2^B, q^B) \geq k(q^B)$ is a sufficient condition. However, the actual condition for IR can be much weaker. If the firm also derives positive profits (excluding investment cost k) from the first period, it will help relax the above constraint. In addition, in reality, firms often receive large side benefits from accepting BOT offers, which also helps cover part of the investment costs.[14]

It is important to note that although quality is not verifiable and hence even with the BOT contract, the government is tempted not to extend the license to the firm ex post, the government will not break its commitment. When the firm has invested at quality q^B, the government has no incentive not to extend the license because, by doing so, it gains nothing. The main reason is that under our assumption, given the same output price, there is no difference in social welfare whether the project is run by the firm or by the government in the second period. Hence, the license policy is (weekly) time consistent. We will further investigate the time consistency issue in a less restrictive setting in Section 4.

Finally, after observing the quality, q, and hence the second-period demand, $x_2(p_2, q)$, the choice of the second period price, p_2^B, is also optimal ex post (ex-post efficiency). Therefore, the government has no incentive to change the second-period price (another dimension of time consistency).

4. Extensions

The main result (i.e., Proposition 1) established in the preceding section is that efficiency is obtainable by a BOT contract with an extendable license and price regulation. In this section, we reexamine this result with some modifications to the main model to better fit some realistic situations. We consider four extensions, with each modifying one assumption in the main model: ownership, price ceiling, foreign firm, and price regulation, respectively. All extensions support the main result.

4.1. Ownership: the optimal BOT contract when the government cannot commit

In this subsection, we reconsider the optimal BOT contract when ownership has economic implications. In the main model, public ownership and private ownership in the second period are assumed to be equivalent in profitability and welfare. Under this assumption, the BOT contract in Proposition 1 is (weekly) time consistent as explained before. Specifically, in the second period, the social welfare is the same whichever party owns the project. Hence, the government has no incentive to deviate from its license extension policy ex post. However, there are at least two factors that potentially affect the time consistency of the license policy. First, there is generally a cost from raising taxes. By taking over the project, the government receives income from the project, which saves the cost of raising taxes elsewhere. Moreover, transferring money from the firm to the government is also costly. Thus, social welfare under private ownership will be different from that under public ownership.[15]

Second, public ownership is generally less efficient than private ownership. Management by the government suffers from bureaucratic inefficiency due to its huge organization and various incentive and information problems.[16] Again, social welfare will be different under private ownership from under public ownership in the second period, which affects the government's commitment to the license policy.

Considering the benefits and costs of public ownership in the second period, we need to redesign the license scheme. Because public ownership and private ownership are no longer welfare equivalent, the first-best outcome may require a private ownership extension in the second period. Suppose that λ_0 is the extension length according to the first-best outcome (to be defined below). Then, the time -consistency problem only arises for the sub-period after λ_0 in the second period. Accordingly, we assume that under public ownership in period $[1+\lambda_0, 2]$, there is a savings on the deadweight loss of collecting taxes elsewhere, which can be considered as a function of

[13] It is interesting to examine empirically whether in reality many BOT projects satisfy those conditions. Note that condition $c_i'(x) \leq c_i'(x_i^B)$ for $x \leq x_i^B$, together with condition $\pi_{1,q}(p_1^B, q) \geq 0$, implies Eq. (8). If $c_i(x)$ is convex for $x \geq x_i^B$, condition (10) is satisfied.

[14] For example, in a highway project, the government may allow the firm to develop the land, such as warehouses or retail areas, at the intersection locations along the highway. In an airport, certain companies are given exclusive rights to open shops in the airport. The government may provide a purchase guarantee such as a guarantee that the government offices buy the product only from this company. In Hong Kong, the subway company is typically given land development rights to build residential apartments and commercial complexes near and above the subway stations. In some cases, where no such land is available, the government typically provides a financial transfer to the company. The derived value from such land development is usually large enough to cover part or all the investment cost. Note that we do not include this benefit in the model, but it should strengthen our results because it gives the government an additional policy tool to resolve the incentive and monopoly issues.

[15] We would like to thank Editor Dilip Mookherjee for raising this point.

[16] There is a large literature on privatization, covering both developed and developing economies. Much of the literature shows, in theoretical and empirical analyses, that privatization of state-owned firms improves profitability. In addition to those studies mentioned in the Introductory section, see also Ehrlich et al. (1994), Gupta (2005), and surveys by Megginson and Neffer (2001) and Turhan (2005).

total profit directly accrued to the government, denoted as $B[(1-\lambda_0)\pi_2]$, but there is also an efficiency loss due to the government's running of the project, which is also a function of total profit directly accrued to the government, denoted as $C[(1-\lambda_0)\pi_2]$. Both $B(\cdot)$ and $C(\cdot)$ are increasing functions. With these costs and benefits, we redo the analysis of the previous section below.

4.1.1. The first best

Consider first the benchmark case where quality is contractible. The government offers the firm a contract that specifies the prices, quality and license extension. The government's first-best problem is

$$\begin{aligned} W^E = \max_{p_1,p_2,p_2',q,\lambda_0} & s_1(p_1,q) + \pi_1(p_1,q) + \lambda_0[s_2(p_2,q) + \pi_2(p_2,q)] \\ & + (1-\lambda_0)[s_2(p_2',q) + \pi_2(p_2',q)] - k(q) \\ & + B[(1-\lambda_0)\pi_2(p_2',q)] - C[(1-\lambda_0)\pi_2(p_2',q)]. \end{aligned} \tag{12}$$

Letting superscript E stand for the first-best outcome in this case, the FOCs are:

$$p_1^E = c_1'[x_1(p_1^E,q^E)], \tag{13}$$

$$p_2^E = c_2'[x_2(p_2^E,q^E)], \tag{14}$$

$$p_2^{E\prime} = p_2^E, \tag{15}$$

$$s_{1,q}(p_1^E,q^E) + s_{2,q}(p_2^E,q^E) = k'(q^E), \tag{16}$$

$$B'[(1-\lambda_0^E)\pi_2^E] = C'[(1-\lambda_0^E)\pi_2^E], \tag{17}$$

where $\pi_2^E \equiv \pi_2(p_2^E, q^E)$. The solutions to the above FOCs, denoted as $(p_1^E, p_2^E, q^E, \lambda_0^E)$, produce the first-best outcome. Given the values of the other four variables and by assuming concavity of $B(\cdot)$ and strict convexity of $C(\cdot)$, the optimal λ_0^E is unique. Conditions (13)–(17), except (15), all mean that the marginal benefit equals the marginal cost, which are standard efficiency conditions. In particular, the marginal-cost pricing in Eqs. (13) and (14) means that the prices are set so that the marginal revenue equals the marginal cost in each period; condition (16) means that the marginal cost of quality equals the marginal cost of quality; and condition (17) means that the marginal benefit of an license extension equals the marginal cost.

4.1.2. The second best

If quality is not contractible, the firm chooses the quality of the project after accepting the contract. The firm's expected profit from the project is:

$$\Pi(p_1,p_2,q) \equiv \pi_1(p_1,q) + \lambda(q)\pi_2(p_2,q) - k(q).$$

Then, the government's second-best problem is

$$\begin{aligned} W^* = \max_{p_1,p_2,p_2',q,\lambda(\cdot)} & s_1(p_1,q) + \pi_1(p_1,q) + \lambda(q)[s_2(p_2,q) + \pi_2(p_2,q)] \\ & + [1-\lambda(q)][s_2(p_2',q) + \pi_2(p_2',q)] - k(q) \\ & + B[[1-\lambda(q)]\pi_2(p_2',q)] - C[[1-\lambda(q)]\pi_2(p_2',q)] \\ \text{s.t.} \quad & \Pi(p_1,p_2,q) \geq \Pi(p_1,p_2,\overline{q}) \text{ for all } \overline{q} \in Q, \quad \text{(IC)} \\ & \Pi(p_1,p_2,q) \geq 0. \quad \text{(IR)} \end{aligned} \tag{18}$$

Given any solution which generates the first-best outcome, i.e., $(p_1^E, p_2^E, q^E, \lambda_0^E)$, and with reference to this solution, we can derive an optimal solution to the above second-best problem and achieve the first-best outcome. This result is formally stated in the following proposition.

Proposition 2. *Assume*

$$\pi_1(p_1^E,q) - \pi_1(p_1^E,q^E) \leq k(q), \text{ for } q < q^E; \tag{19}$$

$$\lambda_0^E \pi_2(p_2^E,q^E) \geq k(q^E), \tag{20}$$

$$c_i'(x) \geq c_i'(x_i^E), \text{ for } x \geq x_i^E; \tag{21}$$

where $x_i^E \equiv x_i(p_i^E, q_i^E)$. Then, $\{p_1^*, p_2^*, p_2'^*, q^*, \lambda^*(\cdot)\}$ defined by

$$\begin{cases} p_1^* = p_1^E, \\ p_2^* = p_2'^* = p_2^E, \\ q^* = q^E, \\ \lambda^*(q) = \begin{cases} 0, & \text{for } 0 \leq q < q^E \\ \lambda_0^E, & \text{for } q \geq q^E \end{cases} \end{cases} \tag{22}$$

is an optimal solution to problem (18). *This solution is efficient and time consistent.*

The discussion on the three conditions (or assumptions) following Proposition 1 also applies to the three conditions (or assumptions) in Proposition 2. In particular, condition (20) means that the reward from the license extension is enough to cover the construction cost. As discussed in Section 3 (after Proposition 1), this is a sufficient condition to ensure that the IR constraint is met. Note that this condition is slightly different from that in Proposition 1. In Proposition 1, $\lambda^*(q^B)$ is chosen at the minimum value to ensure that the firm has the correct incentive to invest in the efficient quality, but it can be any larger value. Therefore, a weaker sufficient condition (for IR) is imposed because the government can always extend the license for the entire second period. In contrast, $\lambda^*(q^E)$ in Proposition 2 is uniquely determined by Eq. (17). Hence, the government has no flexibility to adjust the license period to affect the IR constraint. However, as discussed after Proposition 1, this sufficient condition can be weakened by recognizing the first-period profit and other possible side benefits.

An interesting question is why the first-best license extension length λ_0^E, determined by Eq. (17), is also the length to induce the firm to invest in the targeted quality. As it is proved in the Appendix A, the socially optimal quality level also maximizes the firm's operating profit in each period and so the government does not need to fine tune the license extension length to affect the firm's quality investment. So long as the reward is sufficiently large (i.e., λ_0^E is sufficiently large), the firm will follow the target to make its investment.

Is the license policy time consistent? At $t=1$, the government's problem is

$$\begin{aligned} \max_{p_2,p_2',\lambda_0} & \lambda_0[s_2(p_2,q^E) + \pi_2(p_2,q^E)] + (1-\lambda_0)[s_2(p_2',q^E) + \pi_2(p_2',q^E)] \\ & + B[(1-\lambda_0)\pi_2(p_2',q^E)] - C[(1-\lambda_0)\pi_2(p_2',q^E)]. \end{aligned}$$

The FOCs of this problem for the three choice variables are the same as Eqs. (14), (15) and (17). Hence, (p_2^E, λ_0^E) with $p_2' = p_2^E$ is optimal ex post. This means that the ex ante solution in Proposition 2 is time consistent. The intuition is as follows. As discussed before, the firm never overinvests and so the government's job is to choose a sufficiently long license extension to induce the correct investment level (or to discourage underinvestment). Thus, the government ex ante chooses the ex post optimal license extension length, which plays two roles at the same time: balancing the benefit and cost of public ownership and inducing investment at the target quality. Because of this flexibility, by choosing the ex post optimal license policy in the ex ante contract, the time consistency problem is solved.

4.2. Ramsey pricing and price ceiling during the concession period

We now return to the main model to explore another extension. Recall that in Section 3, we consider the case in which the firm must charge the price set by the government. Let us call this case strict price regulation. In this subsection, we relax this type of price regulation by considering a price ceiling. That is, a price ceiling replaces the strict price regulation in the concession period. Moreover, the price ceiling is assumed to be given exogenously, i.e., it is not a policy variable. In particular, we allow the price ceiling to be any level higher than the welfare-maximizing price. However, the second-period price is still determined by the government no matter who runs the project. This realistic extension is motivated by the concern that in the case of strict price regulation, the welfare-maximizing price (the marginal-cost pricing) might be too low such that the firm's profit is not sufficient to cover the investment cost. In that case, the IR constraint is difficult to meet without using side benefits. Allowing the firm to charge a price higher than the marginal cost (approaching the Ramsey price) helps to alleviate the IR constraint.

When quality is contractible, we have the first-best problem, but this notion of first best is weaker than that in Section 3 in the sense that the firm's monopoly power in the first period is not fully checked. Let $\bar{p}_1$ be the price ceiling. Since, in the first-best case, the government runs the project in the second period, the firm's problem is

$$\Pi = \max_{p_1 \le \bar{p}_1} \pi_1(p_1, q) - k(q).$$

Without loss of generality, assume that the price ceiling is less than the firm's profit-maximizing price level. Then, the first-period price hits the ceiling. Hence, the government's optimization problem becomes

$$W^C \equiv \max_{p_2, q} s_1(\bar{p}_1, q) + \pi_1(\bar{p}_1, q) + s_2(p_2, q) + \pi_2(p_2, q) - k(q), \tag{23}$$

where the superscript C denotes the first-best outcome in the case of price ceiling. The FOCs imply that

$$p_2^C = c_2^{'}[x_2(p_2^C, q^C)], \tag{24}$$

$$s_{1,q}(\bar{p}_1, q^C) + \pi_{1,q}(\bar{p}_1, q^C) + s_{2,q}(p_2^C, q^C) + \pi_{2,q}(p_2^C, q^C) = k^{'}(q^C), \tag{25}$$

which determine the first-best price, p_2^C, and quality, q^C.

We now consider the second-best problem in which quality is not contractible. Given a contract, $(p_1, p_2, \lambda(\cdot))$, the firm's profit is

$$\Pi(p_1, p_2, q) \equiv \pi_1(p_1, q) + \lambda(q)\pi_2(p_2, q) - k(q). \tag{26}$$

In the second period, no matter who manages the project, since the objective is the same, the government will set the same price for the whole period. This allows us to eliminate $\lambda(\cdot)$ from the objective function. Hence, the government's optimization problem is

$$\begin{aligned} W^* \equiv & \max_{p_1, p_2, q, T, \lambda(\cdot)} s_1(p_1, q) + \pi_1(p_1, q) + s_2(p_2, q) + \pi_2(p_2, q) - k(q) \\ \text{s.t. } & \Pi(p_1, p_2, q) \ge \Pi(p_1^{'}, p_2, q^{'}), p_1^{'} \le p_1, q^{'} \in Q, \quad \text{(IC)} \\ & \Pi(p_1, p_2, q) \ge 0. \quad \text{(IR)} \end{aligned} \tag{27}$$

Assume that $\pi_2(p_2^C, q^C) \ge k(q^C)$, which ensures the existence of a properly designed license policy. Again, the first-best outcome is obtainable by a properly designed license policy under a price ceiling.

Proposition 3. *Suppose that the demand function, $x_i(p_i, q)$, is concave in q, the cost function, $c_i(x)$, has a constant marginal cost, (e.g., $c_i(x) = c_i x$), and the price ceiling is between the marginal cost and the firm's profit-maximizing price. Then, an optimal solution, $(p_1^*, p_2^*, T^*, \lambda^*(\cdot))$, of problem (27) is defined by*

$$\begin{cases} p_1^* = \bar{p}_1, p_2^* = p_2^C, \\ q^* = q^C, \\ T^* \le \pi_1(p_1^C, q^C), \\ \lambda^*(q) = \begin{cases} 0 & \text{for } q \in [0, q^C), \\ k(q^C) / \pi_2(p_2^C, q^C) & \text{for } q \ge q^C. \end{cases} \end{cases} \tag{28}$$

This solution is efficient.[17]

The license function in (28) is similar to that in (11). With $\pi_2(p_2^C, q^C) \ge k(q^C)$, we have $0 \le \lambda^*(q) \le 1$ for all $q \in Q$.

Proposition 3 shows that, even without strict price regulation in the concession period, the incentive problem can be resolved completely by a similar license policy as in Proposition 1. However, there is a crucial difference between Propositions 1 and 3. In Proposition 1, with a strict price control in both the concession period and the extended period, the government can completely eliminate monopoly power. In Proposition 3, with a limited price control in the form of an exogenously given price ceiling, the firm is allowed to exercise a certain degree of monopoly power, which is captured by the difference between the price ceiling and the welfare-maximizing price. As a result, full efficiency is generally not achievable in Proposition 3, where full efficiency in the context of our model means a complete resolution of both monopoly power and the agency problem. However, the incentive problem can be completely solved.

4.3. Foreign firms and BOT contracts

Suppose now that the firm in the BOT is a foreign firm. This is a realistic extension because BOT projects usually welcome bids from all over the world. The winners may be foreign firms, especially on BOT projects in developing countries. This change in the model has strong implications for the welfare function and hence the first-best outcome. The firm faces an economy-wide profit tax, which is exogenously given as $\tau \in (0, 1)$. Suppose that the government can also levy a lump-sum tax, T, on the firm. It is clear that the country's welfare is different from the case of domestic firm only when the profit, if it exists, accrues to the foreign firm and so does not belong to the domestic country. Therefore the issue becomes interesting only when positive profit exists in both periods. Accordingly, we assume that the firm has non-negative profit (including the fix investment cost) in the first period. This would also allow us to avoid subsidy to the foreign firm in the first period in the general welfare formula below.

If quality, q, is contractible, since only part of the firm's profit, via taxes, goes to domestic welfare, in the absence of the moral hazard problem regarding quality, the government will surely run the project by itself in the second period. Thus, the first-best problem is

$$W^F = \max_{p_1, p_2, q} s_1(p_1, q) + \tau[\pi_1(p_1, q) - k(q)] + s_2(p_2, q) + \pi_2(p_2, q),$$

[17] Since Proposition 3 is a generalization of Proposition 1 (when the price ceiling is the welfare-maximizing price and with a tax, T), we can replace the conditions in Proposition 1 by the conditions in Proposition 3, which is Tirole's approach (Tirole 1988, p.178). A constant marginal cost in Proposition 3 is actually unnecessary; this assumption substantially reduces the complexity of the proof.

where the superscript F denotes the first-best outcome when a foreign firm is in the BOT. The FOCs define the first-best outcome, (p_1^F, p_2^F, q^F), which is jointly determined by

$$\begin{cases} p_1^F = c_1'[x_1(p_1^F, q^F)] + (1-\tau)\dfrac{x_1(p_1^F, q^F)}{x_{1,p_1}(p_1^F, q^F)}, \\ p_2^F = c_2'[x_2(p_2^F, q^F)], \\ s_q(p_1^F, q^F) + s_q(p_2^F, q^F) = \tau k'(q^F) - \dfrac{\tau(1-\tau)x_1(p_1^F, q^F)}{x_{1,p_1}(p_1^F, q^F)} x_{1,q}(p_1^F, q^F). \end{cases} \tag{29}$$

Note that the pricing law for the first period is different from that without a foreign firm. The price is lower (than the marginal cost) in this case because the government is less concerned about the firm's (it is the foreign firm) profit.

Now suppose that quality is not contractible but there is price regulation. Similar to the analysis in Section 3, the government's problem becomes

$$\begin{aligned} W^* = & \max_{p_1, p_2, p_1', q, T, \lambda(\cdot)} s_1(p_1, q) + \tau[\pi_1(p_1, q) - k(q)] + \lambda(q)[s_2(p_2, q) + \tau\pi_2(p_2, q)] \\ & + [1-(q)][s_2(p_2', q) + \pi_2(p_2', q)] + T \\ \text{s.t.} & \textstyle\prod(p_1, p_2, q) \ge \prod(p_1, p_2, \overline{q}) \text{ for all } \overline{q} \in Q, \quad \text{(IC)} \\ & \textstyle\prod(p_1, p_2, q) \ge 0, \quad \text{(IR)} \end{aligned} \tag{30}$$

where the firm's expected profit is given as

$$\textstyle\prod(p_1, p_2, q) = (1-\tau)[\pi_1(p_1, q) + \lambda(q)\pi_2(p_2, q) - k(q)] - T.$$

We show that a license policy can be properly designed to induce efficient investment.

Proposition 4. *Assume*

$$\pi_1(p_1^F, q) - \pi_1(p_1^F, q^F) \le k(q), \text{ for } q < q^F; \tag{31}$$

$$c_i'(x) \ge c_i'(x_i^F), \text{ for } x \ge x_i^F; \tag{32}$$

$$\pi_1(p_1^F, q^F) \ge k(q^F); \tag{33}$$

$$\pi_2(p_2^F, q^F) \ge k(q^F) - (1-\tau)\frac{[x_1(p_1^F, q^F)]^2}{x_{1,p_1}(p_1^F, q^F)}, \tag{34}$$

where $x_i^F \equiv x_i(p_i^F, q_i^F)$. An optimal solution to problem (30) is $\Omega^F = (p_1^*, p_2^*, q^*, T^*, \lambda^*(\cdot))$, where

$$\begin{cases} p_1^* = p_1^F,\ p_2^* = p_2'^* = p_2^F, \\ q^* = q^F, \\ \lambda^*(q) = \begin{cases} 0 & \text{for } 0 \le q < q^F, \\ \dfrac{k(q^F) - (1-\tau)\dfrac{[x_1(p_1^F, q^F)]^2}{x_{1,p_1}(p_1^F, q^F)}}{\pi_2(p_2^F, q^F)} & \text{for } q \ge q^F, \end{cases} \\ T^* = (1-\tau)\lambda^*(q^F)\pi_2(p_2^F, q^F). \end{cases} \tag{35}$$

This solution is efficient.

Ω^F given in Eq. (35) is uniquely defined. The functional form of the license function is similar to that in Eq. (11). Because of Eq. (34), we have $0 \le \lambda^*(q) \le 1$ for all $q \in Q$. Proposition 4 shows that this license policy under price regulation and a lump-sum tax, T^*, can achieve the first-best outcome.

Note, Condition (34) says that the second-period profits should cover the investment cost, $k(q^F)$, plus a positive amount. This condition is stronger than the corresponding one, Eq. (9), in Proposition 1. Recall that these two conditions are to provide additional incentive for the firm to invest in quality (with the license extension). However, in the foreign firm case, the first period price is further reduced (below the marginal cost, as shown by Eq. (29)). Hence, to restore the high incentive for quality investment, more incentive from the second period should be given to compensate the profit reduction in the first period.

At the beginning of this subsection, we argue that it is more interesting to focus on the case where the firm earns non-negative profit, $\pi_1(p_1^F, q^F) \ge k(q^F)$, in the first period. Now we show that this condition is in fact important for us to derive a policy to achieve the first-best outcome. This profitability condition is due to $T^* \ge 0$, which means a lump-sum tax, not a subsidy. By construction of our policy, T^* is chosen to achieve efficiency (the maximum welfare) for the domestic economy. Given that the prices and equality are (or must be) the first best, T^* must have the form in (35); since this T^* takes all the second-period profit away from the firm, the firm's first-period profit must be able to cover its investment/fixed cost to ensure the participation condition being met.

The use of the lump-sum tax is to maintain the first-best welfare level. Unlike the case when a domestic firm is in the BOT, the welfare when the project is run by a foreign firm is different from that when it is run by the government. To induce incentives, the foreign firm is given a chance to run the project in the second period, but that will reduce the welfare. A way out is to let the lump-sum tax restore the welfare level without distorting the incentives.

4.4. BOT without price regulation

In this subsection, we analyze the case in which the government does not regulate the prices when the firm runs the project, but still allows for a license extension. We intend to use this case to highlight the role of price regulation in the main model.

Given $\lambda(\cdot)$, with the power to decide its own prices, the firm's problem is to choose $\{p_1, p_2, q\}$ to maximize its expected profit[18]:

$$\hat{\textstyle\prod} = \max_{p_1, p_2, q} \pi_1(p_1, q) + \lambda(q)\pi_2(p_2, q) - k(q), \tag{36}$$

where a hat ($\wedge$) indicates no price regulation. Knowing that the firm will choose prices and quality according to Eq. (36), the government chooses $\{p_2', \lambda(\cdot)\}$ to maximize social welfare, where p_2' is the price in the period under government ownership. That is, the government's problem is

$$\begin{aligned} \hat{W} = & \max_{p_1, p_2, p_2', q, \lambda(\cdot)} s_1(p_1, q) + \pi_1(p_1, q) + \lambda(q)[s_2(p_2, q) + \pi_2(p_2, q)] \\ & + [1-\lambda(q)][s_2(p_2', q) + \pi_2(p_2', q)] - k(q) \\ \text{s.t.} & \textstyle\prod(p_1, p_2, q) \ge \prod(\overline{p}_1, \overline{p}_2, \overline{q}), \text{ for all } \overline{q} \in Q,\ \overline{p}_1, \overline{p}_2 \ge 0, \quad \text{(IC)} \\ & \textstyle\prod(p_1, p_2, q) \ge 0. \quad \text{(IR)} \end{aligned} \tag{37}$$

Proposition 5 below gives the solution. It shows that, when there is no price regulation, (i) the first best is not achievable, and (ii) the government will not extend the firm's ownership.

Proposition 5. *An optimal solution, $\hat{\Omega} \equiv \{\hat{p}_1, \hat{p}_2', \hat{q}, \hat{\lambda}(\cdot)\}$ to problem (37) is defined by*

$$\begin{cases} \pi_{1,p}(\hat{p}_1, \hat{q}) = 0, \\ \hat{p}_2' = c_2'[x(\hat{p}_2', \hat{q})], \\ \pi_{1,q}(\hat{p}_1, \hat{q}) = k'(\hat{q}), \\ \hat{\lambda}(\cdot) = 0, \end{cases} \tag{38}$$

if $\pi_1(\hat{p}_1, \hat{q}) \ge k(\hat{q})$. In particular, $\hat{\lambda}(\cdot)$ is the optimal contract without price regulation, which induces the firm to choose $(\hat{p}_1, \hat{q})$. This solution is inefficient.

[18] We could also consider the case in which the license extension depends on prices. However, this case is equivalent to the case with price regulation in the last section.

Condition $\pi_1(\hat{p}_1, \hat{q}) \geq k(\hat{q})$ ensures the IR condition. Without a license extension, the firm needs sufficient profits, $\pi_1(\hat{p}_1, \hat{q})$ in the first period to cover the investment cost $k(\hat{q})$.

In the proof of Proposition 5, we do not impose any specific form on the license function, $\lambda(\cdot)$, such as that in Fig. 1. Nevertheless, $\hat{\lambda}(\cdot)=0$. Therefore, the optimal contract is a pure BOT contract in which the firm operates in the first period only.

Why $\hat{\lambda}(\cdot)=0$? By Eq. (36), the FOCs for the prices are

$$\pi_{1,p}(p_1, q) = 0, \quad \pi_{2,p}(p_2, q) = 0.$$

These equations imply that

$$\hat{p}_1 > c_1'[x_1(\hat{p}_1, \hat{q})], \quad \hat{p}_2 > c_2'[x_2(\hat{p}_2, \hat{q})];$$

that is, the prices are set higher than the marginal costs of production. This monopoly pricing is in sharp contrast to the marginal-cost pricing for the BOT contract under price regulation in Eqs. (3) and (4). The two well-known distortions associated with a monopoly (Spence, 1975) are present in the case without price regulation: (a) the price is higher than the marginal cost, and (b) quality is not at the socially optimal level. When the government has two independent control instruments, price regulation and a license policy, for the regulated BOT, the two problems can be solved completely. In the absence of price regulation, only one control instrument (i.e., the license policy) is left and, hence, it is not surprising that the first-best outcome cannot be restored. However, what is surprising from Proposition 5 is that the remaining instrument has no role to play, i.e., $\lambda(\cdot)=0$. The reason is as follows. Recall that with price regulation, the government can easily eliminate the price distortion. But that will also reduce the firm's incentive to invest in quality. To restore the incentive, the firm is offered a license extension to run the project in the second period. Without price regulation, price distortion cannot be eliminated directly. If the quality chosen by the firm is lower than q^B, then the license policy may help to reduce or eliminate this quality distortion. However, this will induce the firm to charge a higher price, making the price distortion worse. If the quality chosen by the firm is higher than q^B, then the license policy will make both the price and quality distortions worse. On the other hand, since the government will set the marginal-cost pricing in the second period when it runs the project (see (38)), not extending the license allows the price distortion to be eliminated earlier. Thus, the government chooses $\lambda(\cdot)=0$ to minimize the total distortion.

5. Concluding remarks

Incentives and monopoly power are two key issues inherent in BOT projects. This study analyzes the effectiveness of two policy instruments, i.e., price regulation and license extension, in dealing with those issues. Through price regulation, the government can impose marginal-cost pricing during the concession period. Using a license policy, the government can induce the firm to invest at the socially optimal level. Hence, the first-best outcome can be reached by a properly designed BOT contract with price regulation and license extension.

The above results are obtained using a simplified model that aims at capturing only the key features of BOT contracts. There are many potentially interesting extensions of this model. In Section 4, we have studied four extensions. In the first case, we take into account different economic implications of public versus private ownership. We properly redesign the license policy to obtain the efficient result. In the second case, a price ceiling replaces strict price regulation in the concession period and Ramsey pricing is used. Again, the first-best outcome is achievable. In the third case, the firm taking the BOT project is a foreign firm. Full efficiency can still be achieved. In the last case, if for whatever reason, the price is not regulated during the concession period, there is no role for a license policy to play. This supports the importance of regulation-cum-license extension found in the main model.

We may also consider random demand. With uncertain demand, an efficient solution needs to include a proper risk-sharing scheme between the private firm and the government, in addition to the incentive and monopoly problems. We can show that a BOT contract can still achieve efficiency.

Another possible extension is to consider a case in which a minimum level of quality at the end of the concession period is specified in a contract, provided that such a minimum level is verifiable. A realistic situation is that the quality of a project deteriorates over time. If a proper penalty is introduced and some risk in achieving a targeted level of quality is involved, a minimum quality requirement may force the firm to invest sufficiently in quality.

The BOT approach has been popular across the world. It will be interesting to see more studies in this area so that we can better understand BOT schemes and better design the contracts.

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Appendix A

This appendix includes proofs of all propositions.

A.1. Proof of Proposition 1

This proof is only a special case of the proof of Proposition 2 with $B(\cdot)=C(\cdot)=0$.

A.2. Proof of Proposition 2

Step 1. Identify the optimal solution.

Our strategy is to identify a contract that achieves the first-best outcome. This contract is an optimal contract since there is no other contract that can do better than the first-best outcome. We find such a contract in three steps. First, let $p_2' = p_2$. Then, the government problem becomes

$$\begin{aligned} W^* = \max_{p_1, p_2, q, \lambda(\cdot)} & s(p_1, q) + \pi(p_1, q) + s_2(p_2, q) + \pi_2(p_2, q) - k(q) \\ & + B[(1-\lambda(q))\pi_2(p_2, q)] - C[(1-\lambda(q))\pi_2(p_2, q)] \\ \text{s.t. } & \Pi(p_1, p_2, q) \geq \Pi(p_1, p_2, \overline{q}) \text{ for all } \overline{q} \in Q, \quad (IC) \\ & \Pi(p_1, p_2, q) \geq 0. \quad (IR) \end{aligned} \tag{39}$$

Second, given a function $\lambda: Q \rightarrow [0, 1]$, consider the following function:

$$\begin{aligned} W(p_1, p_2, q) \equiv & s(p_1, q) + \pi(p_1, q) + s_2(p_2, q) + \pi_2(p_2, q) - k(q) \\ & + B[(1-\lambda(q))\pi_2(p_2, q)] - C[(1-\lambda(q))\pi_2(p_2, q)]. \end{aligned}$$

This function $W(p_1, p_2, q)$ cannot possibly have a higher value than the efficient level of social welfare in problem (12), since whatever the social welfare value $W(p_1, p_2, q)$ can obtain, the objective function of problem (12) can obtain. But, if we choose the license function $\lambda^*(q)$ defined in Eq. (22), then $W(p_1^E, p_2^E, q^E)$ achieves the efficient social welfare level in Eq. (12). The question is whether or not this tuple $(p_1^E, p_2^E, q^E, \lambda^*(\cdot))$ can also satisfy the two constraints in Eq. (39). If it does, this tuple is an optimal solution of Eq. (39), since it is not possible to have a higher welfare value than this. The next two steps show that the two constraints are indeed satisfied.

Step 2. Verify the IC condition.

We now show that, at $t=0$, given $\lambda^*(\cdot)$ as defined in Eq. (22), the IC condition in Eq. (39) is satisfied. First, with the contract $\{p_1^E, p_2^E, \lambda^*(\cdot)\}$, the firm's profit function becomes

$$\Pi(p_1^E, p_2^E, q) = \pi_1(p_1^E, q) + \lambda^*(q)\pi_2(p_2^E, q) - k(q).$$

We have

$$\begin{aligned}\Pi(p_1^E, p_2^E, q^E) - \Pi(p_1^E, p_2^E, q) &= [\pi_1(p_1^E, q^E) - \pi_1(p_1^E, q)] + [\lambda^*(q^E)\pi_2(p_2^E, q^E) - \lambda^*(q)\pi_2(p_2^E, q)] \\ &\quad + [k(q) - k(^E)].\end{aligned} \tag{40}$$

Since $\lambda^*(q^E)\pi_2(p_2^E, q^E) \geq k(q^E)$ by assumption, we have

$$\Pi(p_1^E, p_2^E, q^E) - \Pi(p_1^E, p_2^E, q) \geq [\pi_1(p_1^E, q^E) - \pi_1(p_1^E, q)] - \lambda^*(q)\pi_2(p_2^E, q) + k(q). \tag{41}$$

For $q < q^E$, we have $\lambda^*(q) = 0$, implying

$$\Pi(p_1^E, p_2^E, q^E) - \Pi(p_1^E, p_2^E, q) \geq [\pi_1(p_1^E, q^E) - \pi_1(p_1^E, q)] + k(q).$$

Hence, by Eq. (19),

$$\Pi(p_1^E, p_2^E, q^E) \geq \Pi(p_1^E, p_2^E, q), \quad \text{for} \quad q < q^E.$$

On the other hand, for $q \geq q^E$, by Eq. (21) and $x_{i,q}(p_i^E, q) \geq 0$, we have

$$\begin{aligned}\frac{\partial \pi_i(p_i^E, q)}{\partial q} \equiv \pi_{i,q}(p_i^E, q) &= \{p_i^E - c_i'[x_i(p_i^E, q)]\}x_{i,q}(p_i^E, q) \\ &= \{c_i'[x_i(p_i^E, q^E)] - c_i'[x_i(p_i^E, q)]\}x_{i,q}(p_i^E, q) \leq 0,\end{aligned} \tag{42}$$

for both $i = 1$ and 2. It implies

$$\pi_1(p_1^E, q) \leq \pi_1(p_1^E, q^E) \text{ and } \pi_2(p_2^E, q) \leq \pi_2(p_2^E, q^E), \quad \text{for} \quad q \geq q^E.$$

Then, by Eq. (40), we have, for $q \geq q^E$,

$$\begin{aligned}\Pi(p_1^E, p_2^E, q^E) - \Pi(p_1^E, p_2^E, q) &\geq [\lambda^*(q^E)\pi_2(p_2^E, q^E) - \lambda^*(q)\pi_2(p_2^E, q)] + [k(q) - k(q^E)] \\ &\geq [\lambda^*(q^E) - \lambda^*(q)]\pi_2(p_2^E, q^E) + [k(q) - k(q^E)] = [k(q) - k(q^E)] \geq 0.\end{aligned}$$

where we have used the fact that $\lambda^*(q) = \lambda^*(q^E)$ for $q \geq q^E$. Hence, we have

$$\Pi(p_1^E, p_2^E, q^E) \geq \Pi(p_1^E, p_2^E, q), \quad \text{for any} \quad q \in Q.$$

That is, given $\lambda^*(\cdot)$ defined in Eq. (22) and prices p_1^E and p_2^E, the firm will voluntarily choose q^E as the optimal quality. The IC condition is thus satisfied.

Step 3. Verify the IR condition.

We now verify the IR condition. Since $k(0) = x_1(p_1, 0) = c(0) = 0$, condition (19) implies

$$\pi_1(p_1^E, q^E) \geq \pi_1(p_1^E, 0) - k(0) = 0.$$

Further, we have $\lambda_0^E \pi_2(p_2^E, q^E) \geq k(q^E)$ by assumption. Hence,

$$\Pi(p_1^E, p_2^E, q^E) = \pi_1(p_1^E, q^E) + \lambda_0^E \pi_2(p_2^E, q^E) - k(q^E) \geq 0.$$

That is, the IR condition is satisfied.

Therefore, $(p_1^E, p_2^E, q^E, \lambda^*(\cdot))$ is indeed a solution of Eq. (18). The welfare level is the same as the first-best welfare W^E, implying that the solution is efficient.

Step 4. The solution is time consistent.

Finally, we also need to ensure time consistency for our solution. This is shown and explained in detail in Section 4.1.

A.3. Proof of Proposition 3

Our strategy is to find a solution that achieves the first-best outcome. Since the first-period price must hit the ceiling, the government's problem is

$$\begin{aligned}W^* \equiv \max_{p_2, q, T} &\; s_1(\overline{p}_1, q) + \pi_1(\overline{p}_1, q) + s_2(p_2, q) + \pi_2(p_2, q) - k(q) \\ \text{s.t.} &\; \Pi(\overline{p}_1, p_2, q) \geq \Pi(\overline{p}_1, p_2, q'),\ q' \in Q,\ \Pi(\overline{p}_1, p_2, q) \geq 0.\end{aligned} \tag{43}$$

This problem can be separated into three problems. The first problem is to solve Eq. (43) without the IC and IR conditions, which is the same as that in Eq. (23) and the solution is $\{\bar{p}_1, p_2^C, \lambda^*(\cdot)\}$. The second problem is to show that $\{\bar{p}_1, p_2^C, \lambda^*(\cdot)\}$ ensures the IC condition in Eq. (43):

$$\Pi(\overline{p}_1, p_2^C, q^C) \geq \Pi(\overline{p}_1, p_2^C, q), \quad q \in Q. \tag{44}$$

The third problem is to choose T to satisfy the IR condition.

We now show Eq. (44). With the contract $\{\bar{p}_1, p_2^C, \lambda^*(\cdot)\}$, the firm's profit is

$$\Pi(\overline{p}_1, p_2^C, q) = \pi_1(\overline{p}_1, q) + \lambda^*(q)\pi_2(p_2^C, q) - k(q) - T.$$

Then, for $q \leq q^C$, using Eq. (28), $\bar{p}_i \geq c_i$ and the fact that $x_{i,q}(p, q) \geq 0$, we find

$$\begin{aligned}\Pi(\overline{p}_1, p_2^C, q^C) - \Pi(\overline{p}_1, p_2^C, q) &= (\overline{p}_1 - c_1)[x_1(\overline{p}_1, q^C) - x_1(\overline{p}_1, q)] + \lambda^*(q^C)(p_2^C - c_2)x_2(p_2^C, q^C) \\ &\quad - \lambda^*(q)(p_2^C - c_2)x_2(p_2^C, q) - k(q^C) + k(q) \\ &\geq (p_2^C - c_2)x_2(p_2^C, q^C)[\lambda^*(q^C) - \lambda^*(q)] + k(q) - k(q^C) \\ &= \pi_2(p_2^C, q^C)[\lambda^*(q^C) - \lambda^*(q)] + k(q) - k(q^C) \\ &= k(q) \geq 0,\end{aligned}$$

which implies Eq. (44) for $q \leq q^C$.

For $q > q^C$, given the license function in Eq. (28), $\lambda(q)$ is constant: $\lambda(q) = \lambda_0$ for $q \geq q^C$, where $\lambda_0 \in (0, 1)$. If $\hat{q}$ maximizes the firm's profit and $\hat{q} \geq q^C$, then it is determined by

$$\pi_{1,q}(\overline{p}_1, \hat{q}) + \lambda_0 \pi_{2,q}(p_2^C, \hat{q}) = k'(\hat{q}).$$

For any $q \geq 0$, denote the social benefit and the company's revenue, respectively, as

$$B(p_1, p_2, q) \equiv s_1(p_1, q) + \pi_1(p_1, q) + s_2(p_2, q) + \pi_2(p_2, q),$$

$$\begin{aligned}R(p_1, p_2, q) \equiv \pi_1(p_1, q) + \lambda_0 \pi_2(p_2, q) &= (p_1 - c_1)x_1(p_1, q) \\ &\quad + \lambda_0(p_2 - c_2)x_2(p_2, q).\end{aligned}$$

Then, $\hat{q}$ and q^C are defined, respectively, by

$$R_q(\overline{p}_1, p_2^C, \hat{q}) = k'(\hat{q}), \quad B_q(\overline{p}_1, p_2^C, q^C) = k'(q^C).$$

Since $s_{i,q}(p, q) \geq 0$ and $\pi_{2,q}(p, q) \geq 0$, we have

$$R_q(p_1, p_2, q) \leq B_q(p_1, p_2, q), \quad \text{for all} \quad (p_1, p_2, q) \geq 0.$$

Since $x(p, q)$ is concave in q, $R(\bar{p}_1, p_2^C, q)$ is concave in q. As shown in Fig. 3, since the curve of B_q is always above the curve of R_q and the latter curve is decreasing and the curve of $k'(q)$ is increasing, we must have $q^C \geq \hat{q}$. This means that the profit function defined in Eq. (26) cannot possibly achieve its maximum in $[q^C, +\infty)$. Hence, condition (44) is satisfied, given the contract in Eq. (28).

Finally, since

$$\Pi(p_1^C, p_2^C, q^C) = \pi_1(p_1^C, q^C) - T,$$

we can take an arbitrary value $T^C \in R$ such that

$$T^C \le \pi_1(p_1^C, q^C).$$

Then, the IR condition is satisfied. This completes the proof.

A.4. Proof of Proposition 4

Our strategy is to construct a mechanism such that the first-best outcome in Eq. (29) is obtained. We first impose $p_1 = p_1^F$ and $p_2 = p_2' = p_2^F$, where p_1^F and p_2^F are defined in Eq. (29). Then, the second-best problem (30) becomes

$$\begin{aligned} &\max_{q,\lambda(\cdot)} s_1(p_1^F, q) + \tau[\pi_1(p_1^F, q) - k(q)] + \lambda(q)[s_2(p_2^F, q) + \tau\pi_2(p_2^F, q)] \\ &\quad + [1-\lambda(q)][s_2(p_2^F, q) + \pi_2(p_2^F, q)] \\ &\text{s.t.} \quad \prod(p_1^F, p_2^F, q) \ge \prod(p_1^F, p_2^F, \overline{q}) \text{ for all } \overline{q} \in Q, \quad \text{(IC)} \\ &\qquad \prod(p_1, p_2, q) \ge 0. \quad \text{(IR)} \end{aligned} \tag{45}$$

With $\lambda(q) = \lambda^*(q)$, we verify the IC condition first. We have

$$\begin{aligned} &\frac{\prod(p_1^F, p_2^F, q^F) - \prod(p_1^F, p_2^F, q)}{1-\tau} \\ &= [\pi_1(p_1^F, q^F) - \pi_1(p_1^F, q)] + [\lambda^*(q^F)\pi_2(p_2^F, q^F) - \lambda^*(q)\pi_2(p_2^F, q)] \\ &\quad + [k(q) - k(q^F)]. \end{aligned} \tag{46}$$

Since $\lambda^*(q^F)\pi_2(p_2^F, q^F) \ge k(q^F)$ by the definition of $\lambda^*(q)$ in Eq. (35), we have

$$\frac{\Pi(p_1^F, p_2^F, q^F) - \Pi(p_1^F, p_2^F, q)}{1-\tau} \ge [\pi_1(p_1^F, q^F) - \pi_1(p_1^F, q)] - \lambda^*(q)\pi_2(p_2^F, q) + k(q). \tag{47}$$

For $q < q^F$, we have $\lambda^*(q) = 0$, implying

$$\frac{\Pi(p_1^F, p_2^F, q^F) - \Pi(p_1^F, p_2^F, q)}{1-\tau} \ge [\pi_1(p_1^F, q^F) - \pi_1(p_1^F, q)] + k(q).$$

Hence, by Eq. (31),

$$\Pi(p_1^F, p_2^F, q^F) \ge \Pi(p_1^F, p_2^F, q), \quad \text{for} \quad q < q^F.$$

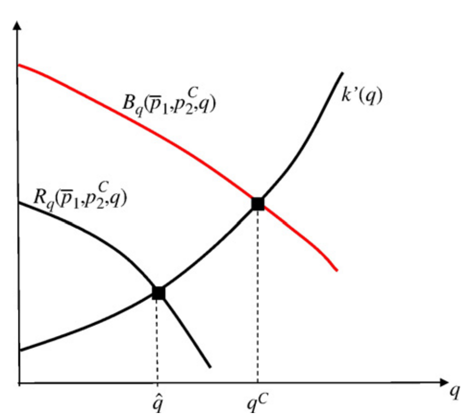

Fig. 3. Marginal social benefit and marginal revenue.

On the other hand, for $q \ge q^F$, by Eq. (32) and $x_{i,q}(p_i^F, q) \ge 0$, we have

$$\begin{aligned} \frac{\partial \pi_1(p_1^F, q)}{\partial q} &\equiv \pi_{1,q}(p_1^F, q) = \{p_1^F - c_1'[x_1(p_i^F, q)]\} x_{1,q}(p_1^F, q) \\ &= \{c_1'[x_1(p_1^F, q^F)] + (1-\tau)\frac{x_1(p_1^F, q^F)}{x_{1,p_1}(p_1^F, q^F)} - c_1'[x_1(p_1^F, q)]\} x_{1,q}(p_1^F, q) \\ &\le (1-\tau)\frac{x_1(p_1^F, q^F)}{x_{1,p_1}(p_1^F, q^F)} x_{1,q}(p_1^F, q) \le 0, \end{aligned} \tag{48}$$

and

$$\begin{aligned} \frac{\partial \pi_2(p_2^F, q)}{\partial q} &\equiv \pi_{2,q}(p_1^F, q) = \{p_2^F - c_2'[x_2(p_2^F, q)]\} x_{2,q}(p_2^F, q) \\ &= \{c_2'[x_2(p_2^F, q^F)] - c_2'[x_2(p_2^F, q)]\} x_{2,q}(p_2^F, q) \le 0. \end{aligned} \tag{49}$$

They imply

$$\pi_1(p_1^F, q) \le \pi_1(p_1^F, q^F) \text{ and } \pi_2(p_2^F, q) \le \pi_2(p_2^F, q^F), \quad \text{for } q \ge q^F.$$

Then, by Eq. (46), we have, for $q \ge q^F$,

$$\begin{aligned} \Pi(p_1^F, p_2^F, q^F) - \Pi(p_1^F, p_2^F, q) &\ge [\lambda^*(q^F)\pi_2(p_2^F, q^F) - \lambda^*(q)\pi_2(p_2^F, q)] + [k(q) - k(q^F)] \\ &\ge [\lambda^*(q^F) - \lambda^*(q)]\pi_2(p_2^F, q^F) + [k(q) - k(q^F)] \\ &= [k(q) - k(q^F)] \ge 0. \end{aligned}$$

where we have used the fact that $\lambda^*(q)$ is fixed for $q \ge q^F$. Hence, we have

$$\Pi(p_1^F, p_2^F, q^F) \ge \Pi(p_1^F, p_2^F, q), \quad \text{for any} \quad q \in Q.$$

That is, the IC constraint in Eq. (45) holds for $(p_1^F, p_2^F, q^F, \lambda^*(\cdot))$.

We now turn to the lump-sum tax. For the T^F defined in Eq. (35), we find

$$\begin{aligned} W^* &= s_1(p_1^F, q^F) + \tau[\pi_1(p_1^F, q^F) - k(q^F)] + \lambda^*(q^F)[s_2(p_2^F, q^F) + \tau\pi_2(p_2^F, q^F)] \\ &\quad + [1-\lambda^*(q^F)][s_2(p_2^F, q^F) + \pi_2(p_2^F, q^F)] + (1-\tau)\lambda^*(q^F)\pi_2(p_2^F, q^F) \\ &= s_1(p_1^F, q^F) + \tau[\pi_1(p_1^F, q^F) - k(q^F)] + s_2(p_2^F, q^F) + \pi_2(p_2^F, q^F) \\ &= W^F. \end{aligned}$$

That is, the second-best welfare is equal to the first-best welfare.

Finally, we verify the IR condition. We have

$$\begin{aligned} &\prod(p_1^F, p_2^F, q^F) \\ &= (1-\tau)[\pi_1(p_1^F, q^F) + \lambda^*(q^F)\pi_2(p_2^F, q^F) - k(q^F)] - (1-\tau)\lambda^*(q^F)\pi_2(p_2^F, q^F) \\ &= (1-\tau)[\pi_1(p_1^F, q^F) - k(q^F)]. \end{aligned}$$

By the assumption that $\pi_1(p_1^F, q^F) \ge k(q^F)$, the IR condition is satisfied. We can see that the assumption is necessary.

A.5. Proof of Proposition 5

We examine the IR condition at the end and so let us drop the IR condition for the moment. Then, problem (37) can be written as

$$\begin{aligned} \hat{W} = \max_{p_1, p_2, p_2', q, \lambda(\cdot)} \; & s_1(p_1, q) + \pi_1(p_1, q) + \lambda(q)[s_2(p_2, q) + \pi_2(p_2, q)] \\ & + [1-\lambda(q)][s_2(p_2', q) + \pi_2(p_2', q)] - k(q) \\ \text{s.t. } & \pi_{1,p}(p_1, q) = 0, \\ & \pi_{2,p}(p_2, q) = 0, \\ & q \in \underset{\overline{q}}{\operatorname{argmax}} \; \pi_1(p_1, \overline{q}) + \lambda(\overline{q})\pi_2(p_2, \overline{q}) - k(\overline{q}). \end{aligned} \tag{50}$$

Let $\hat{q}$ be the optimal quality. We will allow an arbitrary $\lambda(\cdot)$, not to be restrictive to the special form in Fig. 1. For simplicity, assume that $\lambda(\cdot)$ is differentiable except at $\hat{q}$; at $\hat{q}$, the left limit $\lambda'(\hat{q}^-)$ and the right limit $\lambda'(\hat{q}^+)$ of the derivative exist [such as the $\lambda(\cdot)$ in Fig. 1]. Then, given prices p_1 and p_2 and license function $\lambda(\cdot)$, one sufficient condition for the optimality of $\hat{q}$ from the firm's viewpoint is

$$\pi_{1,q}(p_1,q) + \lambda(q)\pi_{2,q}(p_2,q) + \lambda'(q)\pi_2(p_2,q) \geq k'(q) \text{ if } q < \hat{q},$$
$$\pi_{1,q}(p_1,q) + \lambda(q)\pi_{2,q}(p_2,q) + \lambda'(q)\pi_2(p_2,q) \leq k'(q) \text{ if } q > \hat{q}.$$

Hence, one necessary condition for the optimality of $\hat{q}$ is

$$\pi_{1,q}(p_1,\hat{q}) + \lambda(\hat{q})\pi_{2,q}(p_2,\hat{q}) + \lambda'(\hat{q}^-)\pi_2(p_2,\hat{q}) \geq k'(\hat{q}),$$
$$\pi_{1,q}(p_1,\hat{q}) + \lambda(\hat{q})\pi_{2,q}(p_2,\hat{q}) + \lambda'(\hat{q}^+)\pi_2(p_2,\hat{q}) \leq k'(\hat{q}),$$

where $\lambda'(\hat{q}^-)$ and $\lambda'(\hat{q}^+)$ are respectively the left and right limits of $\lambda'(q)$ at $\hat{q}$, defined by

$$\lambda'(\hat{q}^-) \equiv \lim_{q\to\hat{q}^-} \lambda'(q), \quad \lambda'(\hat{q}^+) \equiv \lim_{q\to\hat{q}^+} \lambda'(q).$$

Thus, the third constraint in Eq. (50) can be replaced by the above two sets of inequalities:

$$\begin{aligned}\hat{W} = &\max_{p_1,p_2,p_2',q,\lambda(\cdot)} s_1(p_1,q) + \pi_1(p_1,q) + \lambda(q)[s_2(p_2,q) + \pi_2(p_2,q)] \\ &+ [1-\lambda(q)][s_2(p_2',q) + \pi_2(p_2',q)] - k(q) \\ &\text{s.t. } \pi_{1,p}(p_1,q) = 0, \\ &\quad \pi_{2,p}(p_2,q) = 0, \\ &\quad \pi_{1,q}(p_1,q) + \lambda(q)\pi_{2,q}(p_2,q) + \lambda'(q)\pi_2(p_2,q) \geq k'(q), \\ &\quad \pi_{1,q}(p_1,q) + \lambda(q)\pi_{2,q}(p_2,q) + \lambda'(q^+)\pi_2(p_2,q) \leq k'(q).\end{aligned} \tag{51}$$

From the government's objective function in (51), we can see that, given any q, the expected social welfare is higher if the government shortens the license period, $\lambda(q)$. Since $\pi_{i,p}(\hat{p}_i, \hat{q}) = 0$ implies $\hat{p}_i > c_i'[x_i(\hat{p}_i, \hat{q})]$ (notice that $x_{i,p}(p_i, q) < 0$ by decreasing demand), we have

$$\pi_{i,q}(\hat{p}_i,\hat{q}) = \{\hat{p}_i - c'[x_i(\hat{p}_i,\hat{q})]\}x_{i,q}(\hat{p}_i,\hat{q}) > 0.$$

Suppose that $(\hat{p}_1, \hat{p}_2, \hat{p}_2', \hat{q}, \hat{\lambda}(\cdot))$ is an optimal solution to problem (51) but $\hat{\lambda}(\hat{q}) > 0$ Then, with $\pi_{i,q}(\hat{p}_i, \hat{q}) > 0$ we can find positive numbers, $\varepsilon > 0$, $\xi > 0$ and $\Delta > 0$, such that $\hat{\lambda}(q) - \varepsilon > 0$ for $q \in [\hat{q} - \Delta, \hat{q} + \Delta]$ and

$$\begin{aligned}&\pi_{1,q}(p_1,\hat{q}) + [\hat{\lambda}(\hat{q}) - \varepsilon]\pi_{2,q}(p_2,\hat{q}) + [\hat{\lambda}'(\hat{q}^-) + \xi]\pi_2(p_2,\hat{q}) \geq k'(\hat{q}), \\ &\pi_{1,q}(p_1,\hat{q}) + [\hat{\lambda}(\hat{q}) - \varepsilon]\pi_{2,q}(p_2,\hat{q}) + \hat{\lambda}'(\hat{q}^+)\pi_2(p_2,\hat{q}) \leq k'(\hat{q}).\end{aligned} \tag{52}$$

We now construct another license function $\bar{\lambda}(q)$ such that

$$\bar{\lambda}(\hat{q}) = \hat{\lambda}(\hat{q}) - \varepsilon \quad \text{and} \quad \bar{\lambda}'(\hat{q}^-) = \hat{\lambda}'(\hat{q}^-) + \xi.$$

Since, for $(q \neq \hat{q})$, $\bar{\lambda}(q)$ does not have to satisfy the last two conditions in problem (51) (the two conditions are required for $q = \hat{q}$ only), it is easy to see that we can construct a license function $\bar{\lambda}(\cdot)$ that has the above property. By Eq. (52), this $\bar{\lambda}(q)$ satisfies the last two conditions in Eq. (51) at $q = \hat{q}$, which is what is required by the conditions in Eq. (51). In other words, this $\bar{\lambda}(\cdot)$ induces the firm to choose the same quality as the original $\hat{\lambda}(\cdot)$ does. However, the social welfare in Eq. (51) has a higher value with $(\hat{p}_1, \hat{p}_2, \hat{p}_2', \hat{q}, \bar{\lambda}(\cdot))$ than with $(\hat{p}_1, \hat{p}_2, \hat{p}_2', \hat{q}, \bar{\lambda}(\cdot))$. This contradicts the fact that $(\hat{p}_1, \hat{p}_2, \hat{p}_2', \hat{q}, \hat{\lambda}(\cdot))$ is an optimal solution to (51). By this contradiction, we conclude that we must have $\hat{\lambda}(\hat{q}) = 0$.

With the knowledge of $\lambda(\hat{q}) = 0$, we consider the following problem:

$$\begin{aligned}\hat{W}' = &\max_{p_1,p_2,p_2',q,\lambda(\cdot)} s_1(p_1,q) + \pi_1(p_1,q) + s_2(p_2',q) + \pi_2(p_2',q) - k(q) \\ &\text{s.t. } \pi_{1,p}(p_1,q) = 0, \\ &\quad \pi_{2,p}(p_2,q) = 0, \\ &\quad \pi_{1,q}(p_1,q) + \lambda'(q^-)\pi_2(p_2,q) \geq k'(q), \\ &\quad \pi_{1,q}(p_1,q) + \lambda'(q^+)\pi_2(p_2,q) \leq k'(q), \\ &\quad \lambda(q) = 0.\end{aligned} \tag{53}$$

Since any solution to Eq. (51) satisfies the four conditions of Eq. (53) and $\lambda(\cdot)$ does not appear in the objective function of Eq. (53), we must have $\hat{W}' \geq \hat{W}$. However, since problem (53) imposes a more restrictive constraint on $\lambda(\cdot)$, we must have $\hat{W}' \leq \hat{W}$. Therefore, $\hat{W}' = \hat{W}$, i.e., problems (51) and (53) are equivalent.

We can always assume an increasing $\lambda(\cdot)$, since it does not make sense to punish the firm for investing more in quality when the firm already has an incentive to invest less. Since $\lambda(\hat{q}) = 0$ and $\lambda'(\cdot) \geq 0$ along any sample path, we must have $\lambda'(\hat{q}^-) = 0$ By the third condition in Eq. (53), this implies that $\pi_{1,q}(p_1, q) \geq k'(q)$. Then, with conditions $\pi_{1,q}(p_1, q) \geq k'(q)$ and $\lambda'(q^+) \geq 0$, the fourth condition in Eq. (53) implies that $\lambda'(q^+) = 0$. Therefore, by the same argument as before, problem (53) is equivalent to the following problem:

$$\begin{aligned}\hat{W} = &\max_{p_1,p_2,p_2',q} s_1(p_1,q) + \pi_1(p_1,q) + s_2(p_2',q) + \pi_2(p_2',q) - k(q) \\ &\text{s.t. } \pi_{1,p}(p_1,q) = 0, \\ &\quad \pi_{2,p}(p_2,q) = 0, \\ &\quad \pi_{1,q}(p_1,q) = k'(q).\end{aligned}$$

In the above problem, the three conditions determine (p_1, p_2, q). Also, since p'_2 does not affect the IC conditions, we immediately find the FOC for p'_2: $p'_2 = c'_2[x_2(p'_2, q)]$.

Finally, since $\Pi(\hat{p}_1, \hat{p}_2, \hat{q}) = \pi_1(\hat{p}_1, \hat{q}) - k(\hat{q})$, by condition $\pi_1(\hat{p}_1, \hat{q}) \geq k(\hat{q})$, the IR condition is satisfied. This completes the proof.

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