Cross-border mergers and strategic alliances

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ABSTRACT

This paper develops a model with distribution costs to study firm cooperation in forming strategic alliances and mergers, under different types of foreign market entry modes, that is, export or foreign direct investment (FDI). Under both export and FDI, we find that cross-border alliances (mergers) dominate domestic alliances (mergers); and cross-border alliances and mergers are preferred to independence if and only if distribution cost is high. Under export, cross-border alliances are chosen in equilibrium if distribution cost is high. Under FDI and with high distribution cost, cross-border alliances (mergers) are chosen in equilibrium if plant setup cost is low (high). © 2010 Elsevier B.V. All rights reserved.

1. Introduction

In the past two decades we have witnessed the acceleration of globalization. Globalization takes various forms as it penetrates countries. Beyond the traditional forms, namely export and green-field foreign direct investment (FDI), it has become common nowadays for multinationals to use cross-border mergers and acquisitions (M&As) or to form cross-border strategic alliances in order to extend their businesses internationally (OECD, 2001). The value of cross-border M&As grew from USD 153 billion in 1990 to USD 1 trillion in 2000, while the number of new cross-border strategic alliances increased from around 830 in 1989 to 4520 in 1999. The Daimler-Chrysler merger, the Ford-Mazda alliance, and the Renault-Nissan alliance are just a few examples of this new trend of globalization occurring in the automobile industry. As pointed out in the OECD (2001), cross-border M&As and strategic alliances are two distinctive features of the recent industrial globalization.

Why do firms form cross-border strategic alliances or engage in cross-border mergers? What economic factors affect their incentives to form such alliances and mergers? To answer these questions, we build a two-country, multi-firm economic model, in which firms decide on the type of cooperation, namely, cross-border strategic alliances or mergers, given that they have made their individual choice in foreign market entry modes, that is, export or FDI.
Firms form strategic alliances in order to cooperate in some aspects of their business. They are often competitors in the product markets. This type of partial cooperation is in fact a common feature among most strategic alliances. Specifically, we assume that within the same alliance, the firms share their distribution networks; thus it is basically a distribution alliance. The Haier–Sampo strategic alliance is just a good example. Based on such a model, our analysis yields a number of important and empirically testable results. We find that cross-border alliances (mergers) dominate domestic alliance (mergers). A firm has a larger incentive to form a cross-border alliance or engage in a cross-border merger when distribution costs are high. In the case of FDI, the choice between cross-border alliances and mergers depends on the magnitude of FDI plant setup cost: lower plant setup cost associated with more alliances. Cross-border alliances (mergers) are strategically complementary; that is, a group of firms have a larger incentive to form an alliance (engage in a merger) when other rival firms also form alliances (engage in mergers). This last finding is consistent with the observation that cross-border alliances and mergers occur in waves.

This paper contributes to both the organizational literature on firm cooperation, such as joint venture and strategic alliances, studies include Cabral (2000), Chen and Ross (2000), Eerola and Maattanen (2004), and Morasch (2000). Like those studies, our paper also examines the incentives for forming strategic alliances. However, while those studies emphasize the alliances’ implications on entry and product market competition, in contrast, we are more interested in firms’ choice between strategic alliances and mergers, and comparing alliance incentives under different modes of foreign market entry.

While there exists no systematic study on cross-border strategic alliances, our paper is related to the small but growing body of studies on cross-border mergers in the literature of international trade and FDI. Most of these studies are concerned with the implications of trade liberalization on the profitability of cross-border mergers, the rationales for the emergence of cross-border mergers, and the various effects of cross-border mergers. However, the present paper has a different focus: it examines the choice between cross-border strategic alliances and cross-border mergers under various types of foreign market entry modes (i.e., export and FDI).

Nocke and Yeaple (2007) have also stressed the important role played by the marketing and distribution costs in affecting a firm’s foreign market entry mode. In their paper, cross-border M&A is introduced as one of the three entry modes, along with greenfield-FDI and export. In contrast, cross-border strategic alliances and mergers in our model are a type of cross-border cooperation that the firms decide after they have chosen their foreign market entry modes, that is, export and FDI (greenfield). Moreover, in light of the observations that many large firms are engaging in cross-border strategic alliances and mergers (OECD, 2001), our model focuses on oligopolistic markets, similar to Horstmann and Markussen’s (1992) model but in contrast to Nocke and Yeaple’s (2007) model, which assumes monopolistic competition. As a result, market power and strategic interaction are present in our model, but not in those with monopolistic competition.

The rest of this paper is organized as follows. Section 2 presents the model. Sections 3 and 4 analyze the firms’ cooperation decisions under export and FDI, respectively. Section 5 concludes the paper.

2. Model

Consider an industry in two identical countries, A and B. Assume that there are only two firms in each country: firms 1 and 2 in A, and firms 3 and 4 in B. All firms have the same production technology, and they produce differentiated products for both markets. Assume linear demand in each market and the two markets being segmented. Specifically, let \( x_{ik} \) be the demand and \( p_{ik} \) the price in market \( k \in \{A, B\} \) for the goods produced by firm \( i \in I = \{1, 2, 3, 4\} \), and let the inverse demand...
function be
\[ p_{ik} = 1 - x_{ik} - bX^{-i} \]
where \( i \in I, X^{-i} = \sum_{j \neq i} x_{jk}, b \in (0, 1). \)

Parameter \( b \) represents the degree of product differentiation. As our emphasis is not on how production differentiation affects the equilibrium outcomes, we set \( b = \frac{1}{2} \) to simplify the mathematical expressions.\(^9\) In the main model, we focus on export as a firm's international entry mode to sell its product to a foreign market. We will explore the FDI case in Section 4.

There are various types of costs. There should be production costs and export costs (e.g., tariffs), but in order to emphasize the role of distribution costs and simplify the mathematical expressions, we assume that these costs are zero.

We use the term distribution costs to represent all costs incurred after production, including, for example, the costs of building a salesforce, advertisement, and transportation. Assume that if a firm sells \( x \) units of its product to its domestic market, its distribution costs in that market are equal to \((D + dx)\), where \( D \) is the fixed part of the distribution costs (including, e.g., setting up a distribution center), and \( dx \) is the variable part (including, e.g., wages paid to the salespersons). However, these costs are higher when the firm's product is sold in the foreign market, which the firm is less familiar with. Accordingly, we assume that if a firm sells \( x \) units of its product in the foreign market, its distribution costs are \((1 + \gamma)(D + dx)\), where \( \gamma > 0 \). In an early version of this paper (Qi, 2007), we show that there is no loss of generality to set \( \gamma = 1 \), which simplifies many mathematical expressions in the analysis. Our emphasis on distribution costs and the above specifications about distribution costs are supported by some recent empirical studies (e.g., Maurin et al., 2002), which find that nowadays non-production activities, for example, logistics, are important in business, and domestic firms have an advantage over foreign firms in logistics activities in their own countries.\(^10\)

Distribution costs may be changed through firm cooperation. Let us first describe the distribution alliances between two domestic firms, say, firms 1 and 2, in both markets. In the domestic market, if firms 1 and 2 are independent, their (distribution) costs are \( c_i = D + dx, i = 1, 2 \). If they form a distribution alliance, the total costs of distributing their products in the domestic market, for both firms, are assumed to be \((1 + \delta)(D + dx_1 + x_2)\), where \( \delta \in [0, 1] \). This cost structure requires some discussions and justifications. We can imagine the following "re-configuration" of the firms' distribution networks. If the firms do not form an alliance, each firm establishes its own distribution center, located at point 1 and point 2, respectively, to serve consumers all over the place. If they form an alliance, they can set up just one distribution center, located at point 3 between point 1 and point 2. The new center needs to be bigger than each of the two in order to handle a larger volume of transactions [therefore \((1 + \delta)D \geq D\)], but less than the two combined [therefore \((1 + \delta)D < 2D\)] because they can share some common facilities such as a computer system. The new center is further away from each firm's ideal center, and thus each firm's marginal distribution cost is larger [thus \((1 + \delta)D \geq D\)].\(^11\) The cost saving, or synergy, if there is any, from the alliance formation is \((2D + dx_1 + dx_2) - (1 + \delta)(D + dx_1 + x_2) = (1 - \delta)D - (x_1 + x_2) \delta dx\), which is decreasing in \( \delta \). The smaller \( \delta \) is, the stronger the synergy will be. Such a cost structure change fits well with some aspects of airline alliances.\(^12\) Although we describe the distribution alliance above using a geographical example, we can reinterpret it by replacing distance with technical sophistication or complexity.\(^13\) Assume that the firms share the fixed costs equally but are responsible for their own variable distribution costs. Then, each firm's distribution costs in the domestic market are \( \frac{1}{2}(1 + \delta)D + (1 + \delta) dx\).

Distribution in the foreign market is similar to that in the domestic market. Without an alliance, the firms' distribution costs are \( 2(D + dx_1) \) and \( 2(D + dx_2) \), respectively. The total distribution cost for both firms with an alliance is \( 2(1 + \delta)(D + dx_1 + x_2)\). With equal sharing, each firm's distribution costs in the foreign market are \((1 + \delta)D + 2(1 + \delta) dx\).

The case of cross-border distribution alliance is similar to domestic alliance, and this will be described in detail in Section 3.3.

If two firms merge, they have the same distribution structure as that under a distribution alliance. In addition, they jointly choose their respective outputs to maximize their joint profits.

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\(^9\) Of course, with a general variable \( b \), we would be able to examine how the final equilibrium will change with respect to \( b \). However, as shown in our earlier version (Qi, 2007), changing the specific value of \( b \) basically will not alter the qualitative aspects of the main results derived based on the present model with \( b = \frac{1}{2} \).

\(^10\) There are very few trade models that include distribution costs, with two exceptions: Chen (2003) in his international strategic alliance model and Nocke and Yeaple (2007) in their FDI model. Nocke and Yeaple (2007) also includes the feature of asymmetric marketing costs in domestic and foreign markets.

\(^11\) The reason for the increasing marginal cost of distribution for individual firms does not have to be related to the increasing distance of transportation. It may be due to the fact that the two firms have to share the same computer system after alliance formation, which slows down handling all logistics related process, increases congestion in the new center, etc.

\(^12\) Although alliances in the airline industry were formed initially to partly respond to the restrictions on international services, they remain attractive today even after the airline services liberalization. In the 1980s, airlines organized their respective route structure in hub-and-spoke networks (similar to the distribution center story in our model). In the 1990s, some airlines started to connect their hub-and-spoke networks by forming alliances. This allowed them to increase their market reach without investing new resources to build additional new hubs and spokes (reduction in the fixed cost of distribution \( D \) in our model), but of course, they needed to provide additional services to make the connections smooth, such as coordinating flight schedules for all passengers (the increase in marginal cost of distribution \( d \) in our model). See Gellman Research Associates (1994), Morrison and Winston (1995), and US General Accounting Office (1995) for discussions on this industry.

\(^13\) When the alliance relies on only one distribution system rather than two, it is more difficult to manage the much larger system than each of the two when they are in alliance; that is, \((1 + \delta)D > D\). However, because the two firms can share some fixed investment in the distribution system (such as same control office and advertisement), synergy exists in the sense that \((1 + \delta)D < 2D\). As the same office needs to handle a larger volume of goods, distributing each unit of goods costs more, that is, \((1 + \delta)D > d\).
All four firms engage in a two-stage game. At the first stage, they make their cooperation decisions. There are three options for each firm: remains independent (I), forms a strategic alliance (A), or engages in a merger (M). Let us make a few remarks to justify our focus. First, with regard to strategic alliances, we confine to the most common type in which there are only two firms in each alliance. Similarly, for mergers, we just consider the case in which there are only two firms in a merger.

Second, although firms may also have incentive to coordinate their production output to reduce competition like a cartel, antitrust law prohibits cooperation in production. Therefore, if firms want to cooperate in their output decision, they must choose merger. For example, Article 81 of the EC Treaty states that agreements which limit or control production shall be prohibited. In contrast, agreements which contributes to improving distribution of goods would not be restricted. Although not all mergers are allowed due to competition concern, in the present model with four symmetric firms, a merger between two firms may not be challenged by the antitrust law and thus we assume that it will be allowed.

Third, even if firms form strategic alliances or merge, they still produce their original varieties of the product. In fact, it is optimal for them to do so, hence, product varieties are not reduced. We assume that each firm receives the profit from its own product sales.

Given the above discussion, we now describe the first-stage game. In each country, there is a random draw between the two firms. Due to symmetry, without loss of generality, let us assume that firm 1 and firm 4 are drawn, respectively. The drawn firms make their respective proposals of cooperation. For example, firm 1 has five choices: (i) does not make any proposal (i.e., it prefers to stay independent); (ii) proposes an alliance with firm 2 (domestic alliance); (iii) proposes a merger with firm 2 (domestic merger); (iv) proposes an alliance with firm 3 (cross-border alliance); and (v) proposes a merger with firm 3 (cross-border merger). The firm which receives a proposal can accept or reject. In both cases, the game ends. If a drawn firm does not make any proposal (case (i)), the game also ends. This game is similar to the endogenous merger game studied by Qiu and Zhou (2007). The game tree in Fig. 1 illustrates the process associated with firm 1’s proposals (that of firm 4 is similar).

Note that according to the above game, it is not excluded that a firm (say, firm 2) may receive proposals from both firms 1 and 4, but firm 2 cannot accept both due to our restriction on the number of firms in each alliance and merger.

At the second stage, the firms produce and sell their products to the markets, where they compete in Cournot fashion. We use \(i\&j\)-alliance (\(i\&j\)-merger) to represent the alliance (merger) between firms \(i\) and \(j\).

3. Equilibrium analysis

By backward induction, we should first (in Section 3.1) derive the (second-stage) product market equilibrium for any given first-stage outcome. We then calculate the profits from various configurations of cooperation (domestic alliance and merger in Section 3.2, and cross-border alliance and merger in Section 3.3). We next compare those profits in Sections 3.4 and 3.5. Finally, in Section 3.6, we derive the equilibrium of the game.

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14 Examples in the real world can be easily found. For example, after acquisitions, GM keeps producing SAAB, and Ford keeps producing Volvo. This is a common approach in M&As models with differentiated products.

15 To reduce the number of cases and simplify the analysis, we exclude the possibility of firm 1 making a proposal to another drawn firm, i.e., firm 4.

16 We will discuss in Section 5 the possible changes of our results under Bertrand competition.
3.1. Product market equilibrium

We first derive the results of the second-stage equilibrium for any given first-stage outcome. The market competition outcome depends on the number of firms in competition and their respective costs of distribution, which vary with the first-stage outcomes. Let us first derive the general results of a firm’s equilibrium output and profit in any given market. We omit subscript \( k \), which indicates market \( k \), for convenience. Denote \( c_{j,i} = \sum_{j \in \mathcal{I}} c_j \), where \( c_j \) is the marginal cost of firm \( j \) in a given market. Thus, \( c_{j,i} \) is the sum of the marginal costs of all firms except \( i \) in that market. Note that, \( \pi_i = (p_i - c_i)x_i \) is firm \( i \)'s flow profit (that is, profit excluding the fixed costs of distribution) from a market. It is easy to derive the following Cournot-Nash equilibrium for all \( i \in \mathcal{I} \). When there is no merger, four firms compete independently in each market, resulting in

\[
x_i = \frac{2}{7}(3 + c_{j,i} - 6c_i) \quad \text{and} \quad \pi_i = (x_i)^2.
\]

(1)

If firms \( i \) and \( i' \) merge, but firms \( j \) and \( j' \) do not, the two merging firms choose their outputs in each market, \( x_i \) and \( x_i' \), jointly to maximize their joint profit, \( \pi_i + \pi_{i'} \), taking the other two non-merging firms’ outputs as given. We obtain the equilibrium outcomes for firms \( i \) and \( j \), respectively, as

\[
x_i = \frac{1}{12}(3 - 9c_i + 4c_j + c_{ij}) \quad \text{and} \quad \pi_i = \frac{1}{28}(9 - 14c_i - 3c_{ij})x_i,
\]

(2)

\[
x_j = \frac{1}{32}(12 - 22c_j + 4c_i + 3c_{ij}) \quad \text{and} \quad \pi_j = (x_j)^2.
\]

(3)

where \( c_{ij} = c_i + c_j \) and \( c_{ij} = c_i + c_j \). The results for firms \( i' \) and \( j' \) can be similarly obtained. In the symmetric case where \( c_i = c_j \), we have

\[
x_i = \frac{1}{12}(3 - 5c_i + c_{ij}) \quad \text{and} \quad \pi_i = \frac{3}{4}(x_i)^2.
\]

(4)

If firm \( i \) merges with \( i' \), and firm \( j \) also merges with \( j' \), then

\[
x_i = \frac{1}{18}(4 - 11c_i + 5c_j + c_{ij}) \quad \text{and} \quad \pi_i = \frac{1}{48}(12 - 17c_i - 3c_{ij})x_i.
\]

(5)

In the symmetric case where \( c_i = c_j \), we have

\[
x_i = \frac{1}{18}(4 - 6c_i + c_{ij}) \quad \text{and} \quad \pi_i = \frac{3}{4}(x_i)^2.
\]

(6)

Similar results are obtained for other firms.

The above general formulation allows us to easily obtain the specific outputs and profits in any market based on various first-stage outcomes. We assume that the distribution costs (especially \( d \)) are sufficiently small to ensure that every firm always has positive sales in both markets.

3.2. Domestic strategic alliances and mergers

In this subsection, we focus on the incentives for domestic alliances and mergers. Take firm 1 as an example. Firm 1 has three options: (i) remains independent, (ii) forms a strategic alliance with firm 2 in distributing their products in both markets, or (iii) merges with firm 2.\(^{17}\) As firm 3 has the same cost as firm 4 in both markets, we use \( c_{3A} \) to denote each firm’s individual marginal cost in market \( k \) (= A, B).

Suppose that firms 3 and 4 do not merge (either remain independent or form an alliance). Then, from (1), firm 1’s profit from both markets with 1&2-alliance \( [c_{1A} = (1 + \delta)d, c_{1B} = 2(1 + \delta)d] \) is

\[
\hat{\pi}_{1A} = \frac{1}{18}[(3 - 5d - 5\delta d + 2c_{3A})^2 + (3 - 10d - 10\delta d + 2c_{3B})^2] - \frac{3}{2}(1 + \delta)D,
\]

(7)

where and hereafter tilde “\( \sim \)” indicates domestic alliance or merger; subscript \( A \) indicates firm 1’s decision (alliance with firm 2 in this case); and superscript \( N \) indicates the other pair of firms’ (firms 3 and 4 in this case) decision (no merger). If firms 1 and 2 merge, they have the same cost structure as in 1&2-alliance. Correspondingly, from (2), firm 1’s profit with 1&2-merger is (superscript \( M \) indicates merger)

\[
\hat{\pi}_{1M} = \frac{1}{18}[(3 - 5d - 5\delta d + 2c_{3A})^2 + (3 - 10d - 10\delta d + 2c_{3B})^2] - \frac{1}{2}(1 + \delta)D.
\]

(8)

Firm 2’s profits are the same as those of firm 1. Obviously, they both prefer alliance to merger [by comparing (7) and (8)].

Suppose that firms 3 and 4 merge. Then, from (3), firm 1’s profit from both markets with 1&2-alliance is

\[
\hat{\pi}_{1A} = \frac{1}{18}[(2d - d\delta d + 2 - 5d - 5\delta d)^2] - \frac{1}{2}(1 + \delta)D.
\]

(9)

From (6), firm 1’s profit with 1&2-merger is

\[
\hat{\pi}_{1M} = \frac{1}{18}[(2d - \delta d + 2 - 5d - 5\delta d)^2] - \frac{1}{2}(1 + \delta)D.
\]

(10)

\(^{17}\) Note that there are other options: (i) firms 1 and 2 cooperate their distribution in the domestic market only, and (ii) they cooperate distribution in the foreign market only. These are less interesting cases, and we do not consider them to reduce the total number of cases in our analysis.
Firm 2’s profits are the same as those of firm 1. Again, firms 1 and 2 both prefer alliance to merger [by comparing (9) and (10)].

Hence, regardless of firms 3 and 4’s decisions, firms 1 and 2 always prefer alliance to merger. This result is not surprising. Domestic alliance and merger both result in the same distribution structure and under export, domestic merger does not bring any additional benefit to the merging firms, compared with alliance. Thus, the profit comparison between merging and forming an alliance is the same as the merger incentive (between merger and no merger), which is well understood in the industrial organization literature. Under Cournot competition, two merging firms gain from a merger as they internalize competition, but lose from a merger due to output expansion from the non-merging firms. As a result, when product differentiation is not sufficiently large \( b = \frac{1}{2} \) in the present case, the loss outweighs the gain, and thus firms have no incentive to merge.

### 3.3. Cross-border strategic alliances and mergers

We now turn to examining incentives for cross-border alliances and mergers. Without loss of generality, let us focus on firms 1 and 3.

Firm 1 has two options with regard to cross-border cooperation: (i) forms a strategic alliance with firm 3 in distributing their products in both markets, or (ii) merges with firm 3. Recall that firm 1’s distribution cost in market A is \((D + dx_{1A})\) and that in market B is \((D + dx_{1B})\). Similarly, firm 3’s distribution cost in market B is \((D + dx_{3B})\) and that in market A is \((D + dx_{3A})\). By forming a distribution alliance, a domestic firm can help the foreign partner to distribute the latter’s export products in the local market. This helps to overcome the disadvantages a firm faces in a foreign market (such as unfamiliarity, language barriers, etc.). To capture this idea, we assume that with a distribution alliance (in both markets), the total costs of distribution for firms 1 and 3 in market A are \((1 + \delta)(D + dx_{1A} + x_{3A})\), and those in market B are \((1 + \delta)(D + dx_{1B} + x_{3B})\). By sharing the fixed costs equally and taking the responsibility of the individual’s own variable costs, each firm’s distribution costs in a market are \(\frac{1}{2}(1 + \delta)D + (1 + \delta)dx_{ik}\), where \(i \in \{1, 3\}\) and \(k \in \{A, B\}\).

If firms 1 and 3 merge, they have the same distribution structure as in the alliance case, but in addition, they coordinate their outputs in each market to reduce competition.

The same discussions also apply to 2&4-alliance and 2&4-merger.

Suppose that firms 2 and 4 do not merge (either they remain independent or form an alliance). Then, from (1), firms 1’s profit from both markets with 1&3-alliance is

\[
\pi_{1A}^N = \frac{8}{441}(3 - 5d - 5\delta d + c_{2A})^2 - (1 + \delta)D. \quad (11)
\]

From (2), firm 1’s profit from both markets with 1&3-merger is

\[
\pi_{1A}^M = \frac{3}{148}(3 - 5d - 5\delta d + c_{2A})^2 - (1 + \delta)D. \quad (12)
\]

Suppose that firms 2 and 4 merge \([c_{2A} = 2(1 + \delta)d]\). Then, from (3), firms 1’s profit from both markets with 1&3-alliance is

\[
\pi_{1A}^N = \frac{32}{441}(1 - \delta d)^2 - (1 + \delta)D. \quad (13)
\]

From (6), firm 1’s profit from both markets with 1&3-merger is

\[
\pi_{1A}^M = \frac{4}{13}(1 - \delta d)^2 - (1 + \delta)D. \quad (14)
\]

It is clear that firms 1 and 3 prefer alliance to merger if firms 2 and 4 merge.

By symmetry, firm 3’s profits under various circumstances are the same as the above. Firms 2 and 4’s profits in various situations can be obtained similarly. By comparing those profits, together with the result from the preceding subsection about domestic alliance and merger, we obtain the following lemma.

**Lemma 1.** Cross-border (domestic) strategic alliances generate more profits to the allied firms than cross-border (domestic) mergers to the merged firms.

The result that domestic alliance dominates domestic merger has been explained at the end of the preceding subsection. The same intuition explains why cross-border alliance is preferred to cross-border merger.

### 3.4. Cross-border alliances versus domestic alliances

Lemma 1 tells us that strategic alliances dominate mergers. Hence, firms will not choose mergers. We now compare cross-border alliances and domestic alliances. Again, let us focus on firm 1’s profit below.

First, domestic alliance case, i.e., the 1&2-alliance. Note that \(c_{3A} = 2d\) and \(c_{3B} = d\) if firms 3 and 4 are independent, \(c_{3A} = 2(1 + \delta)d\), and \(c_{3B} = (1 + \delta)d\) if firms 3 and 4 form an alliance. Based on (7), we have firm 1’s profit when firms 3 and 4 are independent as

\[
\pi_{1A}^N = \frac{4}{341}[(3 - 5d - 10\delta d)^2 - \frac{2}{3}(1 + \delta)D]. \quad (15)
\]
where superscript \( I \) refers to firms 3 and 4 choosing independence. Firm 1’s profit in the 3&4-alliance case is

\[
\pi_A^I = \frac{4}{27} \left[ (3-d-\delta d)^2 + (3-8d-8\delta d)^2 \right] - \frac{1}{2} (1+\delta) D. \tag{16}
\]

It is easy to see that \( \pi_A^I > \pi_A^A \), that is, firm 1’s profit is higher when firms 3 and 4 form an alliance than when they are independent. In the case of 3&4-alliance, although firms 3 and 4 save on their fixed distribution costs, their marginal costs are higher than when they are independent. Thus, firms 1 and 2 face weaker competitors in the 3&4-alliance case.

Second, cross-border alliance case, i.e., the 1&3-alliance. Note that \( c_{34} = 3d \) if firms 2 and 4 are independent, and \( c_{34} = 2(1+\delta)d \) if they form an alliance. Thus, based on (11), we have firm 1’s profit when firms 2 and 4 are independent as

\[
\pi_A^I = \frac{8}{27} (3-2d-5\delta d)^2 - (1+\delta) D. \tag{17}
\]

Firm 1’s profit when firms 2 and 4 form an alliance is

\[
\pi_A^A = \frac{8}{27} (1-d-\delta d)^2 - (1+\delta) D. \tag{18}
\]

Again, for the same reason, \( \pi_A^A > \pi_A^I \), that is, firm 1’s profit is larger when firms 2 and 4 form an alliance than when they are independent.

By comparing the profit from cross-border alliance with that from domestic alliance, we immediately have the following:

**Lemma 2.** Cross-border alliances generate more profits to the allied firms than domestic alliances.

**Proof.** See Appendix.

With domestic alliance, there is a reduction in the fixed distribution costs [from \( D \) to \( \frac{1}{2} (1+\delta) D \) in the domestic market; from 2\( D \) to \( (1+\delta) D \) in the foreign market], but there is an increase in the marginal costs of distribution [from \( d \) to \( (1+\delta)d \) in the domestic market; from 2\( d \) to \( 2(1+\delta) d \) in the foreign market]. With cross-border alliance, there is also a reduction in the fixed costs of distribution [from \( D \) to \( \frac{1}{2} (1+\delta) D \) in the domestic market; from 2\( D \) to \( \frac{1}{2} (1+\delta) D \) in the foreign market, which is larger than the case of domestic alliance], but the marginal costs of distribution increase in the domestic market [from \( d \) to \( (1+\delta) d \)] and decrease in the foreign market [from 2\( d \) to \( (1+\delta) D \)]. A firm has a disadvantage in selling its product in the foreign market. A cross-border distribution alliance can help to eliminate such a disadvantage, but a domestic alliance cannot. Thus, with cross-border alliance, the savings in fixed distribution costs is larger, and the increase in marginal distribution costs is smaller. This is the reason for cross-border alliance dominating domestic alliance.

### 3.5. Cross-border alliances versus independence

Given Lemmas 1 and 2, it is now clear that in order to derive the equilibrium, we finally need to compare cross-border alliance with independence. Suppose that all firms are independent. Then, \( c_{1A} = c_{2A} = c_{1B} = c_{4B} = d \) and \( c_{1B} = c_{2B} = c_{3A} = c_{4A} = 2d \). Using these cost values in (1), we obtain firm 1’s profit from both markets if all firms remain independent as

\[
\pi_I^I = \frac{4}{27} (19-54d + 65d^2) - 3D, \tag{19}
\]

where subscript \( I \) refers to firms 2 and 4 choosing independence, and subscript \( I \) refers to firms 1 (and firm 3) choosing independence. If firms 2 and 4 form an alliance, then \( c_{1A} = c_{3B} = d \), \( c_{1B} = c_{3A} = 2d + t \), and \( c_{2A} = c_{4B} = c_{2B} = c_{4A} = (1+\delta) d \). Using these costs in (1), we obtain firm 1’s profit from both markets if it remains independent as

\[
\pi_I^A = \frac{4}{27} [(3-2d+2\delta d)^2 + (3-9d+2\delta d)^2] - 3D. \tag{20}
\]

Comparing the above profits leads to the following lemma:

**Lemma 3.** Cross-border alliances generate more profits to the allied firms than remaining independent if and only if \( D \) is sufficiently large.

**Proof.** See Appendix.

The intuition for cross-border alliance dominating independence is as follows. There is a clear trade-off between forming and not forming an alliance. On the one hand, there exists a saving on the fixed costs of distribution in the foreign market, \( \frac{1}{2} (1+\delta) D < 2D \), which raises a firm’s profit under cross-border alliance over that under independence. On the other hand, while the marginal distribution costs decrease in the foreign market, from 2\( d \) to \( (1+\delta) d \), they increase in the domestic market, from \( d \) and \( (1+\delta) d \). In any case, it is clear that when \( D \) is larger, the benefit from the fixed cost saving is larger, and thus the incentive to form a cross-border alliance is increased.
3.6. Equilibrium

With the above comparisons of various cooperations, we are now ready to derive the equilibrium following the game tree depicted in Fig. 1. We establish the following proposition.

**Proposition 1.** There exists $D_i$ and $D_a$, with $D_a \geq D_i$. In equilibrium, if $D \geq D_a$, firms form cross-border alliances; if $D \leq D_i$, firms remain independent; and if $D \in (D_i, D_a)$, there are multiple equilibria, that is, either firms form two cross-border alliances or all firms remain independent.

**Proof.** See Appendix.

Although the intuition behind every comparison of any two types of cooperation has been provided before, it is worthwhile reiterating some key reasons behind Proposition 1. First, alliance is preferred to merger because both types of cooperation result in the same saving in distribution cost, but merger has a negative impact (the Cournot merger paradox). Even in the case of cross-border merger, there is no extra cost saving under export compared to cross-border alliance. Second, cross-border alliance is preferred to domestic alliance because the former brings more cost saving to the firm in the foreign market than the latter. Third, cross-border alliance is preferred to independence when and only when the fixed cost of distribution is large because there is a saving in the fixed cost but an increase in the marginal cost of distribution associated with cross-border alliance and so the larger the fixed cost is, the more benefits cross-border alliance brings to the allied firms.

4. Mergers and strategic alliances under FDI

Now, suppose that all firms choose FDI to enter the foreign markets. However, there is a cost of setting up a plant in the foreign county, denoted by $S$. Firms choose FDI over export as their foreign market entry mode if the trade costs (e.g., tariffs and international transport costs) are very high. The analysis in this case is similar to that under export except that under FDI, firms incur the fixed plant setup cost when investing and selling their products in the foreign market. Distribution alliances do not affect $S$, but mergers do. This difference gives rise to the possibility of mergers dominating alliances, a result absent under export. In this section, we use $\pi(f)$ to indicate profits under FDI while the expressions of $\pi$ (without $f$) are given in Section 3.

4.1. Domestic strategic alliances and mergers

Let us focus on firms 1 and 2’s alliance and merger. We derive firm 1’s profits under various situations, with the subscript indicating firms 1 and 2’s strategies and the superscript referring to firms 3 and 4’s decisions.

Suppose that firms 3 and 4 do not merge. Then, firms 1’s profit from both markets with 1&2-alliance is, by (1), $\tilde{\pi}_A^N(f) = \tilde{\pi}_N^N - S$, where $\tilde{\pi}_N^N$ is given in (7) in Section 3. When firms 1 and 2 merge, they need to build only one production plant in market B, and thus firm 1’s profit from 1&2-merger is, by (2), $\tilde{\pi}_M^N(f) = \tilde{\pi}_N^N - \frac{1}{2} S$. Hence, $\tilde{\pi}_A^N(f) > \tilde{\pi}_M^N(f)$ if and only if
\[
\frac{4}{74} (3-5d-5\delta d + 2c_{1A})^2 + (3-10d-10\delta d + 2c_{1B})^2 > S. \tag{21}
\]

Suppose that firms 3 and 4 merge. Then, firms 1’s profit from both markets with 1&2-alliance is, by (3), $\tilde{\pi}_A^M(f) = \tilde{\pi}_A^M - S$, and its profits with 1- and 2-merger is, by (5), $\tilde{\pi}_M^M(f) = \tilde{\pi}_M^M - \frac{1}{2} S$. Thus, $\tilde{\pi}_A^M(f) > \tilde{\pi}_M^M(f)$ if and only if
\[
\frac{5}{10} (2-d-\delta d)^2 + (2-5d-5\delta d)^2 > S. \tag{22}
\]

It is clear that domestic merger may be chosen over domestic alliance if the plant setup cost is high because merger would give a large cost saving to the firms.

4.2. Cross-border strategic alliances and mergers

With cross-border distribution alliance, although firms help each other in distribution, each firm still incurs the FDI plant setup cost $S$ in the foreign market. On the contrary, with cross-border merger, a firm can rely on its foreign party’s production base to produce its product, and thus save $S$ under FDI. In this subsection, we derive firm 1’s profits under various situations, with the subscript indicating firms 1 and 3’s decisions and the superscript indicating firms 2 and 4’s decisions.

Suppose that firms 2 and 4 do not merge. Then, from (1), firms 1’s profit with 1&3-alliance is $\pi_A^N(f) = \pi_A^N - S$. From (2), we have firm 1’s profit with 1&3-merger as $\pi_M^N(f) = \pi_M^N$.

---

Note: this is no longer true under FDI which is analyzed in Section 4.

We could have assumed that there is an additional cost, $\delta S$, for using the existing plant to produce both parties’ products due to the conjecture problem. However, the results would not be altered.
Suppose that firms 2 and 4 merge. Then, from (3), firms 1’s profit with 1&3-alliance is \( \pi^M_1(f) = \pi^M_1 - S \). From (5), firm 1’s profit with 1&3-merger is \( \pi^M_1(f) = \pi^M_1 \).

It is clear that the comparison between firm 1’s profit with cross-border alliance and that with cross-border merger crucially depends on \( S \), a larger \( S \) favoring merger. Let us now compare them directly. If firms 2 and 4 form an alliance, we have [noting \( c_{24} = 2(1+\delta)d \)]

\[
\pi^A_1(f) > \pi^M_1(f) \quad \text{if and only if} \quad S > S_1 = \frac{29}{8281}(1-d-\delta)^2.
\]

If firms 2 and 4 merge, we have

\[
\pi^M_1(f) > \pi^M_1(f) \quad \text{if and only if} \quad S > S_2 = \frac{5}{708}(1-d-\delta)^2.
\]

Note that \( S_1 > S_2 \). We immediately have the following results.

**Lemma 4.** Under FDI, if \( S \geq S_1 \), a firm always prefers cross-border merger to cross-border alliance. If \( S \in (S_2, S_1) \), a firm prefers cross-border alliance (merger) to cross-border merger (alliance) if the other pair of firms form a cross-border alliance (merger). If \( S \leq S_2 \), a firm always prefers cross-border alliance to cross-border merger.

The benefit of cross-border merger over alliance is the saving on plant-setup cost, but the downside is the increased output of the other firms (the well-known Cournot merger paradox). It is clear that when the plant-setup cost is very large (larger than \( S_1 \)), the benefit is so large that a firm prefers cross-border merger regardless what the other pair of firms choose, and when the plant-setup cost is very small (smaller than \( S_2 \)), the benefit is so low that a firm prefers cross-border alliance regardless what the other pair of firms choose. In the intermediate case, the comparison of benefit and cost depends on how the other pair chooses: while the benefit from plant-setup cost does not depend on how the other pair firms do, the negative impact of output expansion by the other pair of firms does. If firms 2 and 4 merge, the impact of output expansion from firms 2 and 4, as a response to firm 1 and firm 3’s merger, is weak. In this case, firm 1’s benefit from plant-setup saving is larger than the negative impact of firm 2 and firm 4’s output expansion and hence, firm 1 prefers cross-border merger.

4.3. Cross-border versus domestic cooperation

Using the above profits obtained from various situations, we now compare cross-border alliance with domestic alliance, and cross-border merger with domestic merger, respectively. The results are summarized in the following lemma.

**Lemma 5.** Under FDI, cross-border alliances (mergers) generate more profits to the allied (merged) firms than domestic alliances (mergers).

**Proof.** See Appendix.

The intuition behind the above comparison between cross-border alliance and domestic alliance is the same as that behind Lemma 2. In the case of mergers, there is an additional effect to compare, which is the saving on fixed plant setup costs, \( S \). As a cross-border merger allows the merging parties to save all plant setup costs, while a domestic merger allows the merging parties to save only half of the costs, cross-border mergers give the firms additional benefits over domestic mergers. These extra benefits reinforce the result that cross-border mergers dominate domestic mergers.

4.4. Cross-border cooperation versus independence

We now turn to comparing cooperation (alliance or merger) and independence. Since cross-border cooperation dominates domestic cooperation (Lemma 5), we just focus on cross-border cooperation. The comparisons result in the following lemma.

**Lemma 6.** Under FDI, there exists \( D_f \), such that a firm prefers cross-border alliance and cross-border merger to independence for \( D \geq D_f \).

**Proof.** See Appendix.

Cross-border alliance or merger helps firms to reduce their fixed distribution cost and so the larger the fixed cost is, the greater benefit the alliance or merger brings to the firms and so cooperation is more likely to dominate independence.

4.5. Equilibrium under FDI

Finally, with the above results of comparing various types of strategies, we derive the equilibrium based on the game tree in Fig. 1. Since there are too many factors shaping the equilibrium outcome, it is quite complicated to completely
characterize the equilibrium for all possible combinations of those factors. Hence, we focus on the most interesting cases. First, it is clear that if $D$ is sufficiently small (the condition is stronger than $D < D_f$), all firms choose independent. Second, with the focus on comparing different type of cooperation, we can confine to $D > D_f$, such that independence is never chosen by any firm.

With all the results in Lemmas 4–6, we establish the following proposition.

**Proposition 2.** Suppose that firms take FDI and $D > D_f$.

(i) Firms form cross-border alliances if $S \leq S_2$.

(ii) Firms engage in cross-border mergers if $S \geq S_1$.

(iii) There are multiple equilibria if $S_2 < S < S_1$: Either the firms form two cross-border alliances or they engage in two cross-border mergers.

**Proof.** See Appendix.

It is clear from the above proposition that the equilibrium outcome crucially depends on $S$ and $D$. From the earlier discussions, we note that for cross-border alliance and cross-border merger to dominate independence, the fixed distribution cost needs to be sufficiently large ($D > D_f$). Given that, the choice between alliance and merger is determined by the size of plant setup cost. On one hand, while cross-border merger helps to save this cost, cross-border alliance does not. Thus, cross-border merger is more likely to dominate cross-border alliance when $S$ is larger. On the other hand, there is a negative incentive to merge due to Cournot competition. Hence, for sufficiently large $S$ ($S \geq S_1$), the cost saving effect outweighs the (negative) merger effect, and thus cross-border merger is the equilibrium. For sufficiently small $S$ ($S \leq S_2$), the (negative) merger effect is larger than the cost saving effect, and thus cross-border alliance is chosen by the firms. In the case of intermediate plant setup cost ($S_2 < S < S_1$), if firms 2 and 4 form an alliance, the negative merger effect is strong, and as a result, firms 1 and 3 also form an alliance. In contrast, if firms 2 and 4 merge, the negative merger effect is weak, and thus firms 1 and 3 also merge. Complementarity is present in alliances and also in mergers.

While Proposition 2 has already described the (sufficient) conditions for cross-border mergers and cross-border alliances to emerge in the equilibrium, now we go further to characterize the equilibrium outcomes based on the entire parameter space $(S, D)$. As shown in Fig. 2, we can partition the parameter space into four regions: R1 is the high $D$ and low $S$ region; R2 is the high $S$ region; R3 is the low $D$ and low $S$ region; and R4 is the remaining region. In the proof of Proposition 3 we show how the figure is precisely drawn. Applying the earlier lemmas and Proposition 2 to this partitioned space, we establish the following proposition.

**Proposition 3.** Under FDI,

(i) firms form cross-border alliances in region R1;

(ii) firms engage in cross-border mergers in region R2;

(iii) firms remain independent in region R3; and

(iv) there are multiple equilibria in region R4.

**Proof.** See Appendix.

Our earlier discussions help to understand some of the results in Proposition 3. Here we simply need to provide further intuition to explain the other results. First, in R3, all firms remain independent. Recall that a firm benefits from forming an

![Fig. 2. Equilibrium under FDI.](image)
alliance only because it can save part of the fixed costs of distribution ($D$), and so if $D$ is sufficiently small, it will prefer independence over an alliance. A firm benefits from engaging in a cross-border merger only because it can save part of the fixed distribution costs ($D$) and the plant setup cost ($S$) in the foreign market, and so if both $D$ and $S$ are sufficiently small, it will prefer independence over a merger. Thus, independence dominates both cross-border mergers and alliances in the low-$D$ and low-$S$ region, R3.

Second, note that region R2 contains the part with very small $D$ (could be zero) but very large $S$. That is, for cross-border mergers to be the equilibrium, it is not necessary to have a large $D$. This is because if a cross-border merger brings a sufficiently large saving on the plant setup cost (very large $S$), then firms choose cross-border mergers even if the benefit from saving the distribution costs is small or even zero.

Third, in Proposition 2, we have illustrated and explained the multiple equilibria case when $S \in (S_2, S_1)$ but given $D > D_K$. This region belongs to R4. However, R4 also contains some areas where both $D$ and $S$ are small. While we may not benefit much from characterizing the various combinations of multiple equilibria in the entire region of R4, it suffices to point out that all firms choosing independence is part of the multiple equilibria in the sub-region of R4 with small $D$ and $S$.

5. Concluding remarks

This paper develops a model with distribution costs to study firm cooperation in forming strategic alliances and mergers, under different types of foreign market entry modes, that is, export or FDI. Under both export and FDI, we find that cross-border alliances (mergers) dominate domestic alliances (mergers); and cross-border alliances and mergers are preferred to independence if and only if distribution cost is high. Under export, cross-border alliances are chosen in equilibrium if distribution cost is high. Under FDI and with high distribution cost, cross-border alliances (mergers) are chosen in equilibrium if plant setup cost is low (high).

Results derived from models of oligopolistic competition are often sensitive to the type of market conduct, that is, whether it is Cournot or Bertrand competition. In the present paper, we assume Cournot competition in the product market. There would be two main differences if the product market competition is Bertrand instead of Cournot. First, the benefits from sharing distribution networks are larger under Bertrand competition. As modeled in this paper, sharing distribution network reduces the fixed costs of distribution but raises the marginal cost of distribution. Due to strategic complementarity under Bertrand competition, the negative effects of raising a firm’s marginal cost (of distribution) is smaller than under Cournot competition, which is strategic substitute. This difference makes alliances and mergers more attractive under Bertrand competition than under Cournot competition. It affects the conditions for alliances and mergers to dominate independence.

Second, merger would always dominate alliance under Bertrand competition. This result does not hold under Cournot competition, as shown in the present paper, because without the synergies from sharing distribution networks and saving plant setup costs (in the case of FDI and cross-border mergers), firms would not have the incentive to merge, which is a well known result in the merger literature (Salant et al., 1983). However, firms always have the incentive to merge under Bertrand competition even in the absence of any synergies (Deneckere and Davidson, 1985). Hence, without some countervailing effects like that in Cournot competition, mergers always give more benefits than alliances. In order to allow for the possibility of alliances in the equilibrium, we must introduce some countervailing effects to mergers under Bertrand competition. It is not uncommon to see models of mergers with some fixed costs associated with mergers. Although we have not gone through the whole analysis, with an additional fixed cost, most of the results derived in this paper can be maintained with an appropriate selection for the value of these fixed costs.

A large number of cross-border strategic alliances are marketing and distribution alliances that reduce distribution costs for the allied firms. Cross-border mergers and acquisitions are often found motivated by taking advantages of each other’s distribution networks. However, there are also a large number of cross-border production alliances that reduce the allied firms’ production costs. A natural question is how analytically, distribution costs and production costs are different in our model. That is, do production countries have the same qualitative effect on the export-FDI choice as distribution costs? The answer is no because these two types of cross-border strategic alliances create different synergies in the domestic and foreign markets. Production alliances help each of the allied firms to reduce production costs in both their domestic plants as well as their foreign plants (from FDI). However, cross-border marketing/distribution alliances reduce the allied firms’ distribution costs in their foreign markets only. How production alliances affect the firms’ export-FDI choice requires a separate scrutiny.

The present model can be extended to analyze how firms choose foreign market entry through export and FDI. Recently, Helpman et al. (2004) restudied the export-FDI choice when firms are heterogeneous. In examining the export-FDI choice, our approach has two distinguishing features. First, we can re-examine the export-FDI choice when the firms are faced with the decision of forming cross-border strategic alliances and engaging in mergers. This allows us to study the export-FDI choice within a broader decision framework. Second, we can emphasize the role of distribution costs, which are a significant part of a firm’s total costs. Unlike tariffs and plant setup costs, which affect the proximity-concentration tradeoffs in a trivial way, distribution costs are incurred in both export and FDI. Hence, although cross-border strategic alliances and mergers reduce distribution costs, their effect on the export-FDI choice is not obvious. While greenfield investment represents FDI in most studies of this literature, a few studies (e.g., Mottoo et al., 2004; Norback and Persson,
2007) have explicitly compared the choice between greenfield FDI and cross-border mergers and acquisitions, another form of FDI.

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Appendix

Proof of Lemma 2. In the case of other firms being independent,

\[ \pi_A^I - \pi_A^i = \frac{4d}{441}(30 + 30\delta - 57d - 130\delta d - 75\delta^2 d) + \frac{1}{2}(1 + \delta)D. \]  

(A1)

The right-hand side is positive for sufficiently small \( d \) (relative to \( D \)).

In the case of other firms also forming an alliance,

\[ \pi_A^I - \pi_A^i = \frac{4d(1 + \delta)}{441}[18(1 + \delta) - 47d(1 + \delta)] + \frac{1}{2}(1 + \delta)D. \]  

(A2)

The right-hand side is positive for sufficiently small \( d \) (relative to \( D \)).

Therefore, firm 1 prefers having cross-border alliance with firm 3 to having domestic alliance with firm 2. Due to symmetry, firm 3 also prefers having alliance with firm 1 to that with firm 4. Hence, all firms prefer having cross-border alliances to having domestic alliances.

Proof of Lemma 3. First, suppose that firms 2 and 4 are independent. Then, we have \( \pi_A^I - \pi_A^i = (2 - \delta)(D - D_0) \), where \( D_0 = \frac{4d}{360 - 2\delta} [30 - 60\delta - d(57 - 40\delta - 50\delta^2)] \). Hence, cross-border alliance dominates independence when \( \pi_A^I > \pi_A^i \), that is, \( D > D_0 \).

Second, suppose that firms 2 and 4 form an alliance. Then, we have \( \pi_A^I - \pi_A^i = (2 - \delta)(D - D_1) \), where \( D_1 = \frac{4d}{360 - 2\delta} [30 - 60\delta - d(67 - 80\delta - 10\delta^2)] \). Hence, cross-border alliance dominates independence when \( \pi_A^I > \pi_A^i \), that is, \( D > D_1 \).

Proof of Proposition 1. Without loss of generality, let us suppose that firm 1 and firm 4 are, respectively, drawn to make their proposals.

First, focus on firm 1. By Lemma 1, firm 1 will not propose a merger with firm 2 or firm 3 because both firm 1 and the firm that receives the proposal prefer alliance to merger. By Lemma 2, proposing an alliance with firm 3 dominates an alliance with firm 2. Thus, it remains to compare proposing an alliance with firm 3 and making no proposal.

Since firm 4 also prefers proposing to firm 2 than proposing to firm 3, firm 3 will not receive two proposals. Thus, firm 3 will accept firm 1’s proposal if it is offered because if firm 1 proposes a merger with firm 3, firm 1 must prefer merger to independence and due to symmetry, firm 3 should also have the same preference. For the same reason, firm 2 will accept firm 4’s proposal if it is offered. If firm 1 makes no proposal, there are two possible outcomes: (i) firm 4 makes no proposal and so all four firms are independent; and (ii) firms 2 and 4 form an alliance while firms 1 and 3 are independent. In the proof of Lemma 3, we have that both firms 1 and 4 choose proposing across-border alliances if \( D \geq D_1 \) and choose independence if \( D \leq D_0 \). Since \( D_0 \leq D_1 \), there are multiple equilibria if \( D \in (D_1, D_0) \), that is, either all firms remain independent or there are two cross-border alliances.

Proof of Lemma 5. Note that \( c_{24} = 2d \) and \( c_{34} = d \) if firms 3 and 4 are independent, \( c_{24} = 2(1 + \delta)d \) and \( c_{34} = (1 + \delta)d \) if firms 3 and 4 form an alliance, \( c_{24} = 3d \) if firms 2 and 4 are independent, and \( c_{24} = 2(1 + \delta)d \) if they form an alliance.

(i) Cross-border alliance versus domestic alliance.

Let us focus on firm 1’s profit. When other firms remain independent, \( \pi_1^I(f) > \pi_1^i(f) \) if and only if (A1) holds. When other firms form an alliance, \( \pi_1^I(f) > \pi_1^i(f) \) if and only if (A2) holds. As shown in the proof of Lemma 2, those two inequalities always hold for sufficiently small \( d \) relative to \( D \) and so we have \( \pi_1^I(f) > \pi_1^i(f) \), that is, cross-border alliance dominates domestic alliance when other firms do not merge.
If other firms merge, then \( \pi^4_M(f) > \pi^4_M(f) \) can be reduced to \( D > \frac{16\delta}{19}(-4+9\delta+9\delta^2) \). This inequality holds because the right hand side is negative for sufficiently small \( d \) relative to \( D \). This proves that cross-border alliance dominates domestic alliance when other firms merge.

(ii) Cross-border merger versus domestic merger.

First, if other firms remain independent, by comparing firm 1’s profit with 1&2-merger [from \( \pi^N_M(f) \)] and that with 1&3-merger [from \( \pi^{13}_N(f) \)], we obtain \( \pi^1_{M}(f) - \pi^N_M(f) = \frac{1}{19}(30+30\delta-57d-130\delta d-75\delta^2 d) + \frac{1}{2}(1+\delta)D + \frac{1}{2}S > 0 \), for sufficiently small \( d \) relative to \( D \).

Second, if other firms form an alliance, by comparing firm 1’s profit with 1&2-merger [from \( \pi^N_M(f) \)] and that with 1&3-merger [from \( \pi^{13}_N(f) \)], we have
\[
\pi^1_{M}(f) - \pi^{13}_M(f) = \frac{1}{19}(18+18\delta-47d-94\delta d-47\delta^3 d) + \frac{1}{2}(1+\delta)D + \frac{1}{2}S > 0,
\]
again for sufficiently small \( d \) relative to \( D \).

Third, if other firms merge, then we compare firm 1’s profit with 1&2-merger and that with 1&3-merger to get
\[
\pi^1_{M}(f) - \pi^{13}_M(f) = \frac{1}{19}(8+8\delta-18d-36\delta d-18\delta^2 d) + \frac{1}{2}(1+\delta)D + \frac{1}{2}S > 0,
\]
for the same reason as above.

This proves that cross-border merger dominates domestic merger.

**Proof of Lemma 6.**

(i) Cross-border alliance versus independence.

First, then firm 1’s profit from both markets is \( \pi^1(f) = \pi^1 - S \) if firms 1 and 3 are also independent but \( \pi^1(f) = \pi^{13}_N(f) - S \) with 1&3-alliance, where \( \pi^1 \) and \( \pi^{13}_N \) are given in (19) and (18), respectively. As shown in Lemma 3, \( \pi^1_M(f) > \pi^1(f) \), that is, \( D > D_m \).

Second, suppose that firms 2 and 4 form an alliance. Then, firm 1’s profit from both markets is \( \pi^1(f) = \pi^1 - S \) if firms 1 and 3 are also independent but \( \pi^1(f) = \pi^{13}_N(f) - S \) with 1&3-alliance, where \( \pi^1 \) and \( \pi^{13}_N \) are given in (20) and (18), respectively. As shown in Lemma 3, \( \pi^1_M(f) > \pi^1(f) \), that is, \( D > D_i \).

Third, suppose that firms 2 and 4 merge. Then, if firms 1 and 3 are independent, we have \( c_{1A} = c_{3B} = d \), \( c_{1B} = c_{3A} = 2d \), and \( c_{2A} = c_{2B} = c_{4A} = c_{4B} = (1+\delta)d \). Using these costs in (3), we obtain firm 1’s profit from both markets if it remains independent as \( \pi^1_M(f) = \frac{1}{19}(67d^2 + 6d^3 - 36d - 34d^2 + 13 - 100d) \), where \( D_m = 4d/1521(2-\delta) < 108 + 233d + 54d(4-5d-\delta d) \).

Define \( D_{m} = \max(D_{mA}, D_{mB}) \). Then, cross-border alliance dominates independence if \( D > D_{m} \).

(ii) Cross-border merger versus independent merger.

If firms 2 and 4 are independent, based on the previous calculation, we have the following comparison for firm 1:
\[
\pi^1_M - \pi^1 = (2-\delta)(D-D_1)+S, \quad \text{where} \quad D_1 = 1/74529(2-\delta)(261-20628d+39690\delta d+38648d^2-26460\delta d^2-33075\delta^2 d^2).
\]
If firms 2 and 4 form an alliance, based on the previous calculation, we have the following comparison for firm 1:
\[
\pi^1_M - \pi^{13}_M = (2-\delta)(D-D_3)+S, \quad \text{where} \quad D_3 = 1/74529(2-\delta)(261-20802d+40038\delta d+45553d^2-53558\delta d^2-64995\delta^2 d^2).
\]
If firms 2 and 4 merge, based on the previous calculation, we have the following comparison for firm 1:
\[
\pi^1_M - \pi^{13}_M = (2-\delta)(D-D_4) + S-S_3, \quad \text{where} \quad D_4 = 1/24336(2-\delta)(45-7002d+13734\delta d+14957d^2-17190\delta d^2-3411\delta^2 d^2).
\]
Let \( D_{m} = \max(D_{mA}, D_{mB}) \). Then, firms prefer cross-border merger than independence if \( D > D_{m} \).

Define \( D_f = \max(D_{fa}, D_{fm}) \). The result stated in the lemma is proved.

**Proof of Proposition 2.** Let us focus on firm 1’s decision.

Given \( D > D_i \), by Lemma 6, firm 1 will not choose “does not make a proposal” in the game tree (Fig. 1) and if it proposes an alliance or a merger with firm 3, the latter will accept. By Lemma 5, we know that firm 1 will prefer making a proposal to firm 3 than to firm 2. By Lemma 4, firm 1 makes an alliance (merger) proposal to firm 3 if \( S \leq S_2 \) (\( S \geq S_1 \)) regardless what firm 4 does to firm 2. However, if \( S \leq S_2, S \) firm 1 will make an alliance (merger) proposal to firm 3 given that firm 4 makes an alliance (merger) proposal to firm 2; and similarly, firm 4 will make an alliance (merger) proposal to firm 2 given that firm 1 makes an alliance (merger) proposal to firm 3. Hence, there are two equilibria, one consisting of 1&3-merger and 2&4-merger, and the other consisting of 18&3- and 28&4-alliance.

**Proof of Proposition 3.** From the proof of Lemma 6 (ii), let us define locus C1 on the (\( S,D \)) space based on \( 2-\delta)(D-D_1)+S \leq 0 \), locus C2 based on \( 2-\delta)(D-D_2)+S \leq 0 \), and locus C3 based on \( 2-\delta)(D-D_3)+S \leq 0 \). We can easily shown that \( D_1 > D_2 > D_3 \) and so C1 is everywhere above C2, which is everywhere above C3. We then draw two vertical lines, \( S = S_1 \) and \( S = S_2 \). Now we have Fig. 3. Note that Fig. 3 is drawn for the case of \( D_{fa} \geq D_1 \), where \( D_{fa} \) is defined in the proof of Lemma 6(i). If \( D_{fa} < D_1 \), we just need to have a minor adjustment of the figure, which will not alter the results qualitatively.

The proof of the proposition for R1, R2, and R3 is straightforward and thus let us focus on R4. R4 is further partitioned to R4-1, R4-2, R4-3, R4-4, and R4-5. In R4-1, since we are above locus C1, independence is dominated by mergers [see Proof of
Lemma 6(ii). Given that independence will not be chosen, we have the situation as that in Proposition 2(iii) and so there are multiple equilibria: (A,A) or (M,M), where and hereafter in this proof the first (second) letter represents the choice of firms 1 and 3 (2 and 4).

In R4-2, if firms 2 and 4 choose independence, the firms 1 and 3 prefer independence to merger because $D < D_1$ according to the Proof of Lemma 6(ii), and they also prefer independence to alliance because $D < D_2$ according to the proof of Lemma 3. Hence, (I,I) is an equilibrium. If firms 2 and 4 choose alliance, then firms 1 and 3 prefer merger to independence since $D > D_2$ according to the proof of Lemma 6(ii), but since $S \in (S_2, S_1)$, they actually prefer alliance to merger. Hence, (A,A) is an equilibrium. If firms 2 and 4 choose choice merger, then firms 1 and 3 prefer merger to independence since $D > D_2$ according to the proof of Lemma 6(ii), but since $S \in (S_2, S_1)$, they actually prefer merger to alliance. Hence, (M,M) is also an equilibrium.

Following the same analysis as above, we can prove that in R4-3, both (I,I) and (M,M) are equilibria; in R4-4, both (I,I) and (A,A) are equilibria; and in R4-5, only (I,I) is the equilibrium. Accordingly, we add the sub-region R4-5 to the region R3 in Fig. 2.

References